STEM FUTURES

Using Maths Tasks

STEM Works!

Northern Ireland Curriculum

Air Travel

In these activities, you work in the aviation industry. You will need to use your mathematical skills to help your team solve some problems.

About the Industry

Setting up an airline company involves a lot of risk. It's very expensive, and it's a very competitive market. Companies need to provide a service that's safe, offers value for money, and makes a profit.

One of an airline company's biggest decisions is to choose which aeroplanes to buy. It can be a complicated process. For example, they might need to take into account the number of passengers that an aeroplane can hold. They'd think about and compare different flight ranges, fuel capacity, fuel efficiency and take-off speeds. To perform calculations like these, solve problems and help make big decisions, scientists, mathematicians and engineers need to use mathematics.

About Your Task

Your company is planning to buy several new aircraft, and your team is researching the options. You need to compare different specifications to see which aircraft is the best choice for your business. Your role is to investigate some mathematical problems relating to:

- flight times;
- fuel efficiency;
- acceleration and motion; and
- force during take-off.

You'll need to refer to some information about two commercial aircraft manufactured by Bombardier: the CRJ700 and the Q400 (see Table 1). As you can see from the table, these are in a mixture of metric units (such as kilometres and kilograms) and imperial ones (such as miles and pounds).

Table 1: Specifications for the Q400 and CRJ700

Q400	CRJ700
Passengers – 78 (up to)	Passengers – 78 (up to)
Maximum range – 1859 km	Maximum range – 2655 km
Maximum fuel capacity – 11 724 lb	Maximum fuel capacity – 19 450 lb
Maximum speed – 414 mph	Maximum speed – 544 mph
Maximum take-off mass – 61 700 lb	Maximum take-off mass – 72 750 lb

ACTIVITY 1

Flight Times 1

To begin your investigation, you need to work out how long it would take the Q400 and the CRJ700 to fly two different routes. Your company will need to ensure that flights are on time and give reliable information about arrival and departure times to its passengers.

Assume that each aeroplane will fly at its maximum speed for the whole journey.

1.1 How long would it take for each aeroplane to travel from Belfast International Airport:

(a) to Berlin Schönefeld Airport (approximately 830 miles)?

(b) to Rome Ciampino Airport (approximately 1230 miles)?

1.2 Can you think of any possible problems with your method if you want to calculate accurate flight times? If so, what are they?

ACTIVITY 2

Fuel Consumption

Next, you need to investigate how much fuel each aeroplane would use – its rate of fuel consumption – for each journey. That way, your company can calculate the cost of each flight and how much fuel each aeroplane would need to carry.

For cars, fuel consumption is usually given as mpg (number of miles travelled per gallon of fuel). For the Q400 and CRJ700, you should work out the fuel consumption as kilometres per kilogram of fuel (km/kg).

The cost of fuel for both aeroplanes is £1.80 per kilogram.

2.1 Which aeroplane is more fuel efficient: the Q400 or the CRJ700?

2.2 How much fuel is needed to fly each aeroplane to:

(a) Berlin?

(b) Rome?

2.3 How much money could you save by flying the more fuel efficient aeroplane to Berlin?

2.4 Can you think of any other factors that might affect the amount of fuel an aeroplane would need during a flight? What are they, and what effect would they have?

2.5 Taking into account all of your answers so far, which aeroplane would you buy if you owned your own airline company?

ACTIVITY 3

Flight Times 2

Continuing your investigations, you need to carry out some more accurate calculations to work out journey times. This time, you'll take into account that an aeroplane doesn't always travel at a constant speed. For each journey, it will:

- begin from a stationary position;
- accelerate along the runway;
- take off;
- keep accelerating while it climbs up;
- reach a cruising height, where it keeps to a constant speed;
- start to decelerate as it prepares to land; and
- land and slow down on the runway until it stops.

If we know an aeroplane's rate of acceleration after take-off and how long it accelerates for, we can calculate its final speed using the equation of motion below.

$$v = u + at$$

v – final speed
u – initial speed
a – acceleration
t – time

3.1 Calculate the final speed of each aeroplane in miles per hour if they both accelerated from 0 mph at an overall rate of 0.7 m/s² for 5 minutes.

Is this possible – can both aeroplanes reach this speed?

You've been given some more information about the CRJ700 aeroplane's journey from Belfast to Rome:

- From a stationary position, the CRJ700 accelerates at a rate of 0.52 m/s² until it reaches its maximum speed.
- It stays at its maximum speed for 1094 miles.
- It then decelerates at a rate of 0.18 m/s² until it lands.

3.2 Calculate how long the journey would take. Is this surprising in any way?

3.3 Although this equation of motion is useful, it still doesn't give you a completely accurate flight time. Can you think of any other factors that might affect an aeroplane's journey time? What are they?

ACTIVITY 4

Forces

When it flies, an aeroplane experiences many different forces. Some of the greatest forces are experienced by the engines during take-off, when the aeroplane is accelerating.

For an aeroplane to fly, it needs to generate thrust. This is the forward force that is created when air is pushed over the wing.

You can use the equation below to calculate the force created by the thrust of the engines.

$$F = ma$$

F – force
m – mass of aeroplane
a – acceleration

In this case, the engines of the CRJ700 aeroplane cannot generate more than 61.3 kN of thrust.

4.1 A CRJ700 is waiting on the runway with half of its maximum take-off mass. It then accelerates at a rate of 3 m/s^2. Will the aeroplane's engines be able to make this much force?

4.2 This time, imagine that the CRJ700 accelerates at a rate of 3 m/s^2 with a full, maximum take-off mass. Will the aeroplane's engines be able to make this much force?

It is important to make sure an aeroplane has enough speed before it takes off. A heavier aeroplane needs more force to accelerate to the required speed.

4.3 When the CRJ700 is carrying its maximum take-off mass, what is the greatest amount of acceleration its engines can produce?

ACTIVITY 5

Safe Landing

When landing an aeroplane, the pilot needs to make sure they can land safely and bring the aeroplane to a stop before it reaches the end of the runway. To do this, pilots decelerate the plane and adjust their flight paths so that they approach the runway at a certain angle. This is the 'angle of approach': the angle between the ground and the plane's flight path as it descends.

The Q400 aeroplane needs a runway that is 1269 metres long to allow for a safe landing.

5.1 A Q400 aeroplane is coming in to land. When it is 60 metres above the ground, it is perpendicular to the start of the runway. The plane then travels a distance of 870 metres on a diagonal path to the runway. How far down the runway does the aeroplane land?

There are also other equations of motion that you can use. For example, you can calculate the distance (s) an aeroplane travels if you know the initial speed (u) and final speed (v), as well as the time it takes the aeroplane to change speeds (t).

$$s = \tfrac{1}{2}(u + v)\,t$$

5.2 Once the Q400 aeroplane has landed on the runway, it is still travelling at 120 mph. It then continues to brake and comes to rest after 15 seconds. Will the aeroplane stop on the runway?

Evaluating Your Learning and Success Criteria

Now work in pairs or small groups to review and evaluate your own and each other's work. How successful were you in this task? Think about how well you demonstrated that you can:

- use mathematical knowledge and concepts accurately;
- work systematically and check your work;
- use mathematics to solve problems and make decisions;
- use mathematical understanding and language to ask and answer questions, talk about and discuss ideas, and explain ways of working;
- round to an appropriate number of decimal figures;
- formulate linear equations;
- manipulate simple algebraic expressions, equations and formulae; and
- understand and apply Pythagoras' Theorem.

As part of your learning evaluation, you could consider:

- how your approach to the task compared with others;
- your strengths and weaknesses;
- what you might do differently if you were to repeat the task;
- what aspects of your work you can improve upon; and
- setting targets and a focus for development.