STEM

Using Maths Tasks:
Teacher's Notes

Air Travel: Teacher's Notes

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Overview

This task challenges pupils to use a wide range of mathematical formulae and techniques in relation to the science behind air travel. They use the equations of motion to calculate certain characteristics of flight.

Prior Learning

To complete all aspects of the task successfully, pupils will need to have the knowledge, understanding and skills listed below.

In Mathematics:

- Using and understanding compound measures
- Converting metric to imperial units and vice versa
- Rounding to appropriate degrees of accuracy
- Substituting values into formulae
- Manipulating formulae
- Using Pythagoras' Theorem.

In Physics:

- Knowledge and understanding of speed and acceleration
- Knowledge and understanding of units of measurement for speed, acceleration, distance, force and mass
- Knowledge and understanding of forces and Newton's second law:

  force = mass x acceleration

- Using the following equation:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

General

Throughout the task, pupils base their calculations on two aircraft (Bombardier's Q400 and CRJ700). All the information they need about each aeroplane is provided.

You may wish to include more detail about the laws of motion to explain the science to your pupils. However, if you are using the task to assess Using Mathematics against the Levels of Progression, your pupils will need to work more independently to identify and use the mathematics involved.

Throughout the task, pupils are expected to:

- identify and evaluate what mathematics to use, and how, when solving the problems;
- identify which units of measurement to use;
- convert between imperial and metric units where necessary (for example miles and kilometres, pounds and kilograms);
- manipulate formulae to find solutions; and
- be able to comment on their findings.

Activity 1

Flight Times 1

In this activity, pupils need to use the formula

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

to work out how long it would take each aeroplane to travel to two destinations: Berlin and Rome. They should identify that the Q400 cannot travel to Rome, as it is too far away.

They are also expected to comment on why this formula would not be suitable for calculating accurate distances.

Activity 2

Fuel Consumption

Pupils calculate the fuel efficiency of each aeroplane, working out fuel consumption as kilometres per kilogram. They take into account the range of each aeroplane (how far it can travel when carrying its maximum amount of fuel).

They then identify how much fuel each would need to travel to Berlin and Rome, explaining that the Q400 cannot fly to Rome due to its fuel capacity. They also calculate the difference in fuel cost for each aeroplane.

Activity 3

Flight Times 2

Developing on from Activity 1, Activity 3 allows pupils to investigate the effect of using a law of motion to work out the final speed of a plane. The acceleration is given in m/s^2. In 3.1, to use the formula, pupils will need to convert the time to seconds and find the answer first in metres per second.

For 3.2, they need to consider a journey in three parts – acceleration, time at cruising height, and deceleration – to work out total flight time. They use the given equation of motion to calculate the first and third parts.

When they have a solution, they should compare their answer with the length of time they calculated for the CRJ700 to fly to Rome in Activity 1.

Activity 4

Forces

This activity looks at how the engines of a plane exhibit a force that creates acceleration, depending on the mass of the aeroplane.

It also gives you an opportunity to show how other equations of motion can be used in calculations for air travel.

Activity 5

Safe Landing

For question 5.1, pupils need to use Pythagoras' Theorem. They are given the height and the diagonal landing path of a Q400 aeroplane, and they need to calculate the horizontal distance that the aeroplane travels along a runway.

In 5.2, they first work out the distance it takes the aeroplane to come to rest, using its change in speed and the time it takes to stop. Then, by adding the two distances together, they calculate the runway length that a Q400 aeroplane needs to land.

Key

The methods shown here for working out and arriving at solutions are examples only. Pupils do not need to replicate them exactly. However, they will need to show how they worked out their answers.

1.1

	Q400:	**CRJ700:**
(a)	830 miles ÷ 5 × 8 = 1328 km	830 miles ÷ 5 × 8 = 1328 km
	830 miles ÷ 414 mph ≈ **2 hours**	830 miles ÷ 544 mph ≈ **1 hour 32 mins**
(b)	1230 miles ÷ 5 × 8 = 1968 km	1230 miles ÷ 5 × 8 = 1968 km
	Not possible: range only 1859 km	1230 ÷ 544 ≈ **2 hours 16 mins**

1.2 *(Example)* It would be better to use a formula that took changes in speed into account because:

- the speed wouldn't be constant from start to finish; and
- the aeroplane wouldn't fly in a direct path and may have to circle after taking off and before landing.

2.1

Q400:	**CRJ700:**
Range = 1859 km	Range = 2655 km
Fuel capacity = 11 724 lbs ÷ 2.2 = 5329.1 kg	Fuel capacity = 19 450 lbs ÷ 2.2 = 8840.9 kg
Fuel consumption =	Fuel consumption =
1859 km ÷ 5329.1 kg = 0.35 km/kg	2655 km ÷ 8840.9 kg = **0.3 km/kg**

The Q400 travels farther per kilogram of fuel, so it is more fuel efficient.

2.2

	Q400:	**CRJ700:**
(a)	Berlin: 830 miles = 1328 km	Berlin: 830 miles = 1328 km
	1328 km ÷ 0.35 km/kg = **3794.3 kg**	1328 km ÷ 0.3 km/kg = **4426.7 kg**
(b)	(Flight to Rome not possible; see 1.1(b).)	Rome = 1230 miles = 1968 km
		1968 km ÷ 0.3 km/kg = **6560 kg**

2.3

Q400:	**CRJ700:**
3794.3 kg × £1.80 per kg = **£6829.74**	4426.7 kg × £1.80 per kg = **£7968.06**

Difference in cost: £7968.06 - £6829.74 = £1138.32

It's £1138.32 cheaper to use the Q400 aeroplane.

2.4 Examples might include the following:

- the weight of the plane;
- the speed of the plane;
- the wind;
- the actual distance travelled may be longer;
- the plane may need to carry more than the exact amount for unexpected detours.

2.5 Pupils discuss and justify their choice in line with their answers for Activities 1 and 2.

3.1

5 minutes = 300 seconds

$v = u + at$

$v = 0 + 0.7 \times 300$

$v = 210$ m/s

210 m/s ÷ 1000 × 3600 = 756 km/h — v = 756 km/h

756 km/h × 5 ÷ 8 = 472.5 mph — v = **472.5 mph**

The maximum speed of the Q400 is 414 mph.
The maximum speed of the CRJ700 is 544 mph.
Only the CRJ700 can reach a speed of 472.5 mph.

3.2

Maximum speed in m/s:

544 mph × 8 ÷ 5 = 870.4 km/h
870.4 km/h × 1000 = 870 400 m/h
870 400 m/h ÷ 3600 ≈ 241.8 m/s

544 mph ≈ **241.8 m/s**

Acceleration time:

$v = u + at$
241.8 m/s = 0 + 0.52 m/s^2 × t
241.8 m/s ÷ 0.52 m/s^2 = t
t ≈ 465 s
t ≈ **7 mins 45 secs**

Time at maximum speed:

s = d/t
544 = 1094 ÷ t
t = 1094 ÷ 544
t ≈ **2 hrs**

Deceleration time:

544 mph ≈ 241.8 m/s
0 = 241.8 m/s + (-0.18) m/s^2 × t
(-241.8) m/s ÷ (-0.18) m/s^2 = t
t ≈ 1343.3 secs
t ≈ **22 mins 24 secs**

Total journey time:

t ≈ 7 mins 45 secs + 2 hrs + 22 mins 24 secs
t ≈ 2 hrs, 30 mins, 9 secs

Compare this with the time of 2 hours 16 mins calculated in Activity 1.

3.3

(Example) Other variables might include the following:

- The acceleration would not be constant and could vary depending on the stage of flight.
- The bearings for take-off and landing could add to the distance travelled.
- The aeroplane may not be able to travel at its maximum speed for a long period of time.
- The plane could have a tail wind or head wind.

4.1

CRJ700's maximum take-off mass = 72 750 lbs
72 750 lbs ÷ 2.2 = 33 068.18 kg

Half of the maximum take-off mass = 16 534.09 kg

F = ma
F = 16 534.09 kg × 3 m/s^2
F = 49 602.27 N
F ≈ 49.6 kN

Yes, the engines will be able to make 49.6 kN.
Their maximum force is 61.3 kN.

4.2

CRJ700's maximum take-off mass = 33 068.18 kg

F = ma
F = 33 068.18 kg × 3 m/s^2
F = 99 204.54 N
F ≈ 99.2 kN

No. The aeroplane engines will not be able to make 99.2 kN.
Their maximum force is 61.3 kN.

4.3

CRJ700's maximum take-off mass = 33 068.18 kg
Maximum force = 61.3 kN

F = ma
61 300 N = 33 068.18 kg × a
a = 61 300 ÷ 33 068.18
a ≈ 1.85 m/s^2

The aeroplane will not be able to exceed an acceleration of 1.85 m/s^2.

5.1

Pythagorus' Theorem: $a^2 + b^2 = c^2$

Horizontal length = a
Perpendicular height = 60 m
Diagonal path (hypotenuse) = 870 m

$a^2 + 60^2 = 870^2$
$a^2 + 3600 = 756\,900$
$a^2 = 756\,900 - 3600$
$a^2 = 753\,300$
$a = \sqrt{753\,300}$
a ≈ 867.9 m

The aeroplane will land 867.9 m along the runway.

5.2

initial speed = 120 mph
120 mph × 8 ÷ 5 = 192 km/h
192 km/h × 1000 ÷ 3600 = 53.3 m/s

final speed = 0 mph = 0 m/s
time = 15 s

$s = \frac{1}{2}(u + v)t$
$s = \frac{1}{2} \times (53.3 + 0) \times 15$
$s = \frac{1}{2} \times 53.3 \times 15$
s = 400 m

It takes a further 400 m before the aeroplane stops after landing on the runway.
867.9 m + 400 m = 1267.9 m

The runway is 1269 m long.

So the aeroplane will stop 1.1 m before the end of the runway.

Air Travel Assessment Grid

If you use Air Travel to assess the Cross-Curricular Skill of Using Mathematics, the following assessment grid highlights the Requirements, Knowledge and Understanding that your pupils will cover in the task. It gives examples of how pupils might demonstrate these at Levels 6 and 7.

Pupils should be enabled to:	Level 6	Level 7
Context	**Through discussion, solving routine and non-routine problems with increasing independence in a wide range of familiar and unfamiliar contexts and situations, pupils can:**	**Through discussion, solving routine and non-routine problems with increasing independence in a wide range of familiar and unfamiliar contexts and situations, pupils can:**
• **choose the appropriate materials, equipment and mathematics to use in a particular situation;**	• **consider and identify a range of materials/equipment, mathematical techniques and problem-solving strategies required to meet the purpose of activities;** In Activity 1, pupils recognise that they need to convert the distance in miles for each journey into km to see if the aeroplane can make the journey. They also independently identify the formula needed to solve the questions: s = d/t. In Activity 2, they covert the maximum weight (lbs) of fuel into kg in order to find fuel efficiency in km/kg. In Activity 3, pupils identify that they need to convert m/s into mph (although they need guidance on how to convert m/s into mph). In Activity 4, pupils convert the plane's weight from lbs to kg to work out the mass of the plane.	• **consider and identify, with some justification, the materials/equipment, mathematical techniques and problem-solving strategies required;** In Activity 3, pupils explain and fully demonstrate how to convert m/s into mph – by first converting m/s to km/s, then km/s into km/h, then km/h into mph (or a similar approach) – in order to calculate the speed of the plane. In Activity 5, pupils identify that Pythagoras' Theorem is required to help solve question 5.1 and explain why. For question 5.2 they also explain that it would be best to convert the plane's speed from mph into m/s, as the distance required has been given in metres. Pupils use their answers from 5.1 to help answer 5.2.

• **use mathematical knowledge and concepts accurately;** • **work systematically and check their work;**	• **use a range of appropriate mathematical techniques and notation;** Pupils round to a given number of decimal places. They clearly show their working out when converting. They include the correct units in their answers. • **work systematically and efficiently to a given degree of accuracy;** Pupils show a systematic approach that highlights how they have obtained conversions and answers, for example referring to conversion rates and calculations that use them. Pupils also round their answers to a degree of accuracy that is given to them. • **review their work, using appropriate checking procedures and evaluating their effectiveness at each stage;** Pupils have few or no errors in their working out. There should be evidence of whether the pupil checked their calculations or not. For example, in Activity 1 pupils need to make sure the planes can actually fly to the destinations in the first place.	• **use a range of appropriate mathematical techniques and notation;** Pupils round to a degree or accuracy of their own choosing, which is done appropriately. This could be done at each step of the calculations or at the end – but reference must be made to the accuracy. Pupils also use unit notation throughout any conversion of m/s to mph and vice versa. In Activity 3, pupils clearly show how they calculated 3.3 by breaking the problem up into three separate calculations before adding up the three stages of the journey. • **critically review to what extent they succeeded in carrying out activities, checking if the level of accuracy and their findings are appropriate and making an assessment of any limitations;** Pupils explain why the formulae or their answers may not be accurate in the real world of air travel and why. They suggest what could be done to make the calculations more accurate. Pupils are also able to analyse to what extent they feel they have answered the questions given successfully and whether any corrections are needed; for example, using rounding throughout the calculations may result in an answer different from that of a pupil who rounded at the end.
• **use mathematical understanding and language to ask and answer questions, talk about and discuss ideas and explain ways of working;**	• **use appropriate mathematical language/notation to communicate and explain their work for a wider audience;** Pupils are able to communicate why they used/needed conversion rates. They explain the outcomes of their results. For example, in Activity 2 they can show and explain which plane is more fuel efficient for the journey.	• **use appropriate mathematical language/notation to explain and justify their findings or solutions;** Pupils can explain why they have chosen to take their selected approach and can give reasons why it was the most effective, for example the use of Pythagoras' Theorem in Activity 5. Pupils can also effectively explain what limitations and problems there could be with their answers based on the (given) formula they used, and suggest further variables.

Knowledge & Understanding		
Number and Algebra	• **add, subtract, multiply and divide decimals;** Throughout Activities 1–4, pupils are able to convert and calculate with decimal numbers when required. • **round to a given number of decimal places;** Pupils round to an appropriate number of decimal places that is given to them, for example 2 d.p. for cost, 1 d.p. for distance in miles, and the nearest second for time. • **use appropriate formulae;** Pupils identify the formula s = d/t and use t = d/s in Activity 1. They also use the given formulae in Activities 3 and 4. Throughout the task they use conversion rates when needed. • **use conventional notation in algebra;** Pupils clearly relate the variables in each formula to their given numerical value	• **round to an appropriate number of decimal places and significant figures;** Pupils choose their own degree of accuracy when rounding, and the answers given should be suitable for the question at hand. • **manipulate simple algebraic expressions, equations and formulae;** Pupils are able to solve Activities 4.3 and 5.1 by rearranging the relevant formulae to make the required variable the subject.
Shape, Space and Measures	• **use, convert and calculate measures involving metric and, where appropriate, imperial units;** Pupils convert when necessary using given conversion rates, for example between miles and km, lbs and kg.	• **understand and apply Pythagoras' Theorem;** Pupils identify the need for Pythagoras' Theorem in Activity 5 and use it to solve 5.1, which will help them to solve 5.2.