

What is an average?

An average summarises a set of data into a single value which is typical or central and can therefore represent the whole set of data. Averages can be used to make general statements and draw conclusions about the data.

Types of average

We commonly use three types of average:

1. **Mean** the sum of all values \div the number of values
2. **Median** the midpoint of data when it is arranged in order
3. **Mode** the most frequently occurring value or item



Rewarding Learning

Mean

Advantages

The mean represents all the data, as every piece of data is used to calculate it.

Disadvantages

It is affected by outliers (extreme values).

Median

Advantages

The median lies at the centre of the data and is unaffected by outliers (extreme values).

Disadvantages

Not all the values are used.

Since the data must be arranged in order of size, it can be a long process to identify the median in a large data set.

Mode

Advantages

The mode is easy to understand and identify.

It can be used for qualitative (non-numerical) and quantitative (numerical) data.

Disadvantages

Not all the values are used.

A data set can have more than one mode or no mode.

Level 5

Understand, calculate and use the mean.

Level 6

Work out the median and mode.

Level 7

Choose the most appropriate average – mean, median or mode.

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How to calculate the mean of a set of values

Mean is the sum of all values \div the number of values.

1. Find the total by adding all the values.
2. Find how many values there are.
3. Divide the total by the number of values.

Example

Find the **mean** of the following numbers:

13, 18, 13, 14, 13, 16, 14, 21, 13

1. $13 + 18 + 13 + 14 + 13 + 16 + 14 + 21 + 13 = 135$
2. 9 values
3. $135 \div 9 = 15$

Mean = 15

The mean can be written using mathematical notation.

The sum of all the values can be represented as Σx
 Σ represents 'sum of' and x represents all of the values.

Furthermore, the sum of all the values divided by the number of values can be represented as $\frac{\Sigma x}{n}$

This means, for a set of values, the mean can be written as $\frac{\Sigma x}{n}$

Another way of writing the mean is \bar{x} (x-bar).

How to calculate the median of a set of values

Median is the midpoint of data when it is arranged in order.

1. Arrange the numbers in order of size, for example smallest to largest.
2. Find the middle value by adding 1 to the number of values and dividing by 2

Example

Find the **median** of the following numbers:

13, 18, 13, 14, 13, 16, 14, 21, 13

1. Arrange in order of size:
13, 13, 13, 13, 14, 14, 16, 18, 21
2. Find the middle number.
There are nine numbers in the list; $9 + 1 = 10$, $10 \div 2 = 5$ so the fifth value is in the middle:
13, 13, 13, 13, **14**, 14, 16, 18, 21

Median = 14

An even number of values, for example 20 values, will have two numbers in the middle. To help find these, add 1 and divide by 2

Example

$20 + 1 = 21$ $21 \div 2 = 10.5$

Now, to calculate the median, find the **10th and 11th** values, **add** them together, then **divide** by 2

Example

1, 1, 1, 2, 2, 3, 3, 3, 4, **5**, **6**, 6, 7, 7, 7, 8, 8, 9, 9, 9

$5 + 6 = 11$ $11 \div 2 = 5.5$

Median = 5.5

How to find the mode of a set of values

Mode is the most frequently occurring value in the data.

Example

Find the **mode** of the following numbers:

13, 18, 13, 14, 13, 16, 14, 21, 13

Mode = 13, as it appears the most often.

There are times when there is more than one mode.

Example

Find the mode of the following numbers:

11, 12, 14, 15, 15, 17, 18, 19, 19

This time there are two modes, 15 and 19

There are also times when there is no mode at all.

Example

Find the mode of the following numbers:

17, 19, 20, 22, 24, 26, 29, 31, 35

As each number only appears once, there is no mode.

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Work out the median and mode.

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Identifying the mean, median and mode from a frequency table

Mean

The **mean** is:
the total of all the values \div total frequency.

1. To find the total of all the values:
Add a new column, **fx**
Find **fx** by multiplying the **x** value (numerical data) by its frequency, **f**
 Σfx , the total of all the values, is the sum of the **fx** column.
2. Find the total frequency, **Σf** (sum of all frequencies).
3. Divide the total of all the values by the total frequency,
 $\Sigma fx \div \Sigma f$

Example

Hours of sport, x	Frequency, f	Cumulative Frequency
1	4	$(1 \times 4) = 4$
2	7	$(2 \times 7) = 14$
3	9	$(3 \times 9) = 27$
4	8	$(4 \times 8) = 32$
5	2	$(5 \times 2) = 10$
	$\Sigma f = 30$	$\Sigma fx = 87$

$$\Sigma fx = 4 + 14 + 27 + 32 + 10 = 87$$

$$\Sigma f = 4 + 7 + 9 + 8 + 2 = 30$$

$$\begin{aligned} \text{The mean} &= \Sigma fx \div \Sigma f \\ &= 87 \div 30 \\ &= 2.9 \text{ hours of sport.} \end{aligned}$$

Median

The **median** is the value of the middle number.

1. Find the middle number by calculating the total frequency, **Σf** , adding 1, then dividing by 2
2. Use cumulative frequency to find this value.

Example

Hours of sport, x	Frequency, f	Cumulative Frequency
1	4	4
2	7	11
3	9	20
4	8	
5	2	
	$\Sigma f = 30$	

$$\Sigma f = 30$$

$$30 + 1 = 31$$

$$31 \div 2 = 15.5$$

The median will be the **15th** and **16th** values. These both occur when **x = 3**, which contains the 12th value to the 20th value.

The median is 3 hours of sport.

Mode

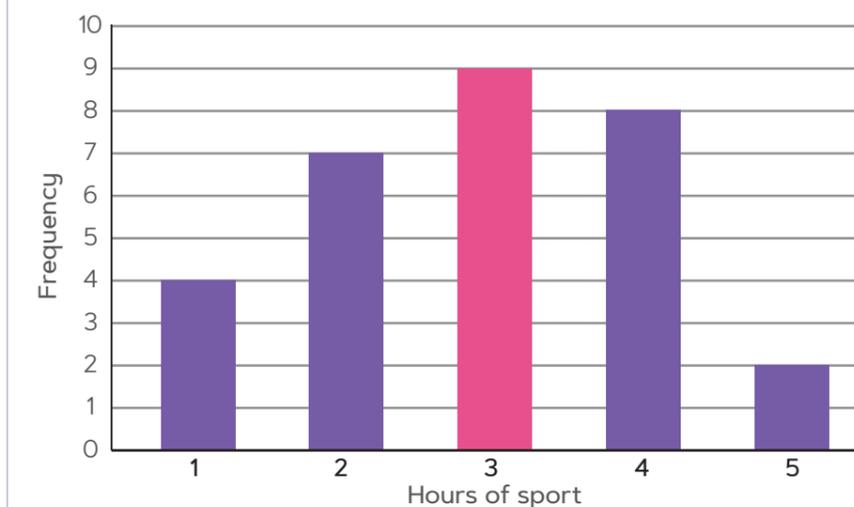
The **mode** is easily identified as the value with the **highest frequency.**

Example

Hours of sport	Frequency
1	4
2	7
3	9
4	8
5	2

The mode is 3 hours of sport.

Identifying the mode from a bar chart



Level 6

Work out the mean, median and mode of a frequency distribution.

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Estimating a mean, and identifying a median and modal group from a grouped frequency table

Mean

The **mean** can be **estimated** from a grouped frequency table.

1. Record the midpoint of each group in a new column.
2. Estimate the total of all the values in another new column, **fx**
Find **fx** by multiplying the midpoint, **x**, value by its frequency, **f**
The estimated total of the values, **Σfx** , is the total of the **fx** column.
3. Find the total frequency, **Σf**
4. Divide the estimated total of all the values by the total frequency:
 $\Sigma fx \div \Sigma f$ = the estimate of the mean.

Distance in km to work, d	Frequency, f	Midpoint, x	fx
0 < d ≤ 5	6 ×	2.5	= 15
5 < d ≤ 10	15 ×	7.5	= 112.5
10 < d ≤ 15	24 ×	12.5	= 300
15 < d ≤ 20	29 ×	17.5	= 507.5
20 < d ≤ 25	26 ×	22.5	= 585
	$\Sigma f = 100$		$\Sigma fx = 1520$

Estimate of the mean = $\Sigma fx \div \Sigma f = 1520 \div 100 = 15.2$

Median

The **median group** is the group that contains the middle number(s).

Find the middle number by calculating the total frequency, **Σf** , adding 1 then dividing by 2
Then use cumulative frequency to find this value.

Distance in km to work, d	Frequency, f	Cumulative Frequency
0 < d ≤ 5	6	6
5 < d ≤ 10	15	21
10 < d ≤ 15	24	45
15 < d ≤ 20	29	74
20 < d ≤ 25	26	
	$\Sigma f = 100$	

$\Sigma f = 100$
 $100 + 1 = 101$
 $101 \div 2 = 50.5$
 The median will be the **50th** and **51st** values.

These both occur in the group **15 < d ≤ 20** which contains the 46th to 74th values.

The median distance to work is in the group 15 < d ≤ 20

Level 7

From a set of grouped data, estimate the mean and identify the limits of the median and modal groups.

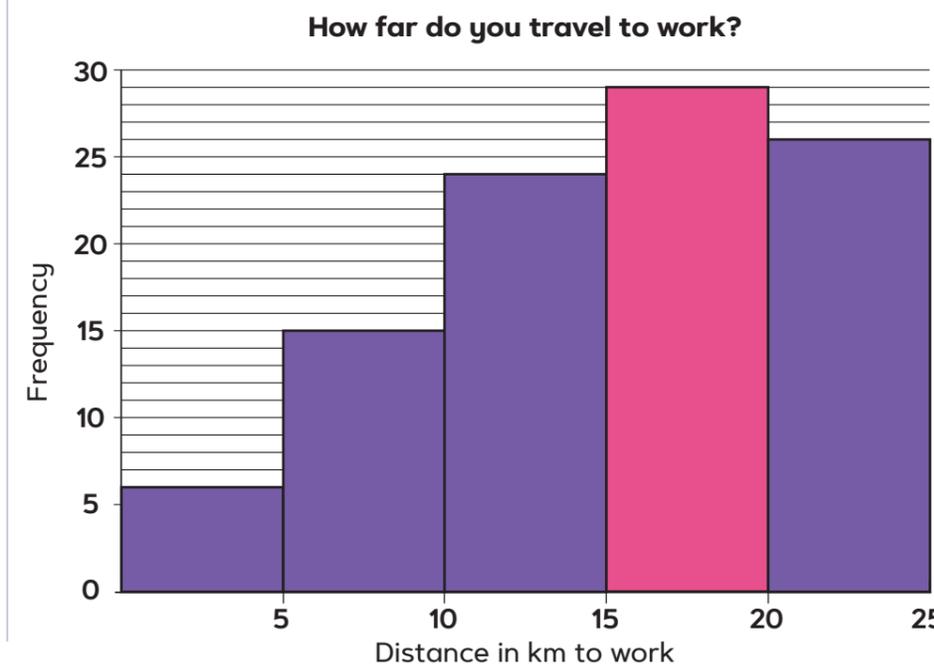
Mode

The **modal group** is easily identified as the group with the **highest frequency**.

Distance in km to work, d	Frequency, f
0 < d ≤ 5	6
5 < d ≤ 10	15
10 < d ≤ 15	24
15 < d ≤ 20	29
20 < d ≤ 25	26
	$\Sigma f = 100$

The modal group is 15 < d ≤ 20

Identifying the modal group from a frequency diagram



Note This value is an estimate of the mean. An exact value cannot be found, as accuracy has been lost by grouping the data.