Problem Solving in Mathematics
Number
Crack the Code

Teacher Notes

Introduction

It is important that pupils have covered binary numbers before attempting this problem. They will also have experience of converting between binary and decimal.

Pupils can work on this problem individually or with others.

- They can discuss possibilities for the initial code using the given information.
- They can discuss how each number is converted into a letter.
- It may be useful to prompt a discussion on different number systems.
- They can discuss how to change from binary to decimal and vice versa.
- They can compare their approach, adapting their own strategy if needed.

This problem deals with a pupil's ability to:

- read through information;
- logically make a decision on how letters can be represented by numbers;
- recognise another number system; and
- use their number skills to convert back and forth between binary and decimal.

What I know (think)

The pupils should know the following from the given problem:

- Stephen has created a code.
- The coded message he sends to Matthew has five numbers and they represent five letters. The five letters spell HELLO.
- Matthew replies to Stephen using his own code.
- The coded message he sends to Stephen in his reply also has five numbers.
- Stephen cracks Matthew's code and sends the word MATHS back in his reply using Matthew's code.

What I need to know (identify)

Pupils need to identify:

- how each decimal number is changed into a letter to get the word HELLO;
- how Matthew's coded message can have the numbers 10 010 and 1011 even though the alphabet only has 26 letters;
- what number system only uses ones and zeros;
- what number system Matthew uses for his code;
- how to convert Matthew's message into letters;
- the word Matthew's coded message spells; and
- how to convert MATHS into a coded message using Matthew's code.
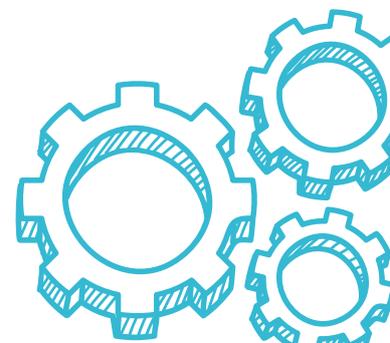
What I need to do (employ)

Pupils should use the information they have been given and come up with appropriate steps to help them solve the problem. There are different ways in which pupils can determine how to crack Matthew's code and convert MATHS into a coded message:

- When working out that 8, 5, 12, 12, and 15 represent HELLO, pupils could spot that H is the eighth letter of the alphabet and that E is the fifth, and so on. Alternatively, they could list the alphabet and identify the numerical position of each letter.
- When cracking Matthew's code, pupils may be able to use mental methods to convert each binary number to decimal. Alternatively, they could list the powers of two from right to left and place each binary digit underneath, then add up the corresponding powers of two to get the decimal number, like so:

8	4	2	1	$1011_2 = 8 + 2 + 1 = 11$
1	0	1	1	

- They then need to identify what letter the decimal number represents using its numerical position in the alphabet, as before.
- Once each binary number has been converted to decimal and each letter identified, pupils should conclude that the coded message Matthew sent to Stephen was the word **CRACK**.
- Pupils then need to identify which five decimal numbers would represent MATHS using the numerical position of each letter in the alphabet.
- They would then covert these decimal numbers into binary using a suitable method. One way of converting is as follows:

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2 | 13

2 | 6    1  ↑

2 | 3    0

2 | 1    1

    0    1
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- Once each letter has been given a decimal number and each decimal number has been converted into binary, pupils should put together the code Stephen should send back to Matthew
(1101, 1, 10 100, 1000, 10 011).
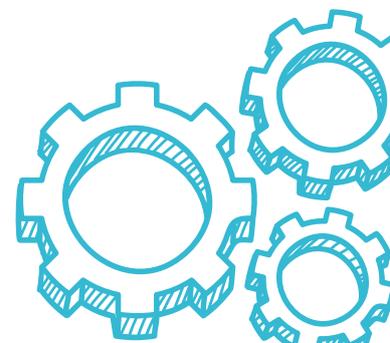
Crack the Code (Continued)

What I did (review)

Pupils will use self-assessment, peer assessment or teacher feedback to decide whether they have approached the problem as intended.

- How did they identify which letter each number represented in the alphabet?
- Did they identify that Matthew's code used binary numbers?
- Did they convert the binary numbers to decimal correctly?
- Did they correctly identify the word CRACK?
- Did they correctly identify the decimal numbers for the word MATHS?
- Did they convert the decimal numbers to binary correctly?
- Did they produce Stephen's coded message correctly?
- Did they check any of their answers by using a reverse calculation?

Curriculum Objectives
This problem should enable pupils to demonstrate their knowledge, understanding and skills through:

Developing pupils as Individuals	Work collaboratively in problem-solving, taking account of others' viewpoints to reach consensus:
	• Pupils take part in any initial group discussions on the code, giving their own opinions and listening to the opinions of others.
	Demonstrate an ability and willingness to develop logical arguments:
	• Pupils explain how they initially changed the decimal numbers into letters. They explain how they decided that the new code was binary, and they can convert between binary and decimal to change binary numbers into letters, and vice versa.
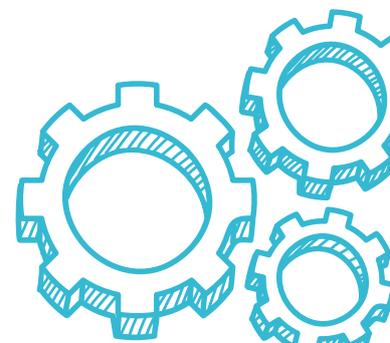
Crack the Code (Continued)

Thinking Skills and Personal Capabilities

This problem can provide an opportunity for pupils to demonstrate a variety of the following Thinking Skills and Personal Capabilities:

Managing Information	• Ask focused questions • Plan and set goals and break task into sub-tasks
Thinking, Problem-Solving and Decision Making	• Sequence, order, classify and make comparisons • Justify methods, opinions and conclusions • Generate possible solutions, try out alternative approaches and evaluate outcomes
Being Creative	• Experiment with ideas and questions • Make new connections between ideas/information • Learn from and value other people's ideas
Working with Others	• Listen actively and share opinions • Suggest ways of improving their approach and working collaboratively
Self-Management	• Seek advice when necessary • Compare their own approach with others' and in different contexts • Organise and plan how to go about a task • Focus, sustain attention and persist with tasks

Cross-Curricular Skills

This problem should enable pupils to demonstrate a variety of the following Cross-Curriculum Skills:

Using Mathematics