



PROBLEM SOLVING IN MATHEMATICS

Introduction

1. Active Problem Solving

The Organisation for Economic Co-operation and Development (OECD) states that there are three processes in which students will engage as active problem solvers.

These are:

- formulating;
- employing; and
- interpreting.

The OECD defines each of these processes as such:

- **Formulating** mathematics involves pupils identifying opportunities to apply and use mathematics. When they formulate, they are seeing that mathematics can be applied to understand or resolve a real-life problem or challenge. It means that pupils are able to take a situation as presented and transform it into a form amenable to mathematical treatment by:
 - providing mathematical structure and representations; and
 - identifying variables and making simplifying assumptions to help solve the problem or meet the challenge.
- **Employing** mathematics involves pupils applying mathematical reasoning and using concepts, procedures, facts and tools to find a mathematical solution.

This includes:

- performing calculations;
 - manipulating algebraic expressions and equations or other mathematical models;
 - analysing information mathematically from diagrams and graphs;
 - developing mathematical descriptions and explanations; and
 - using mathematical tools to solve the problems.
- **Interpreting** mathematics involves pupils reflecting on mathematical solutions or results, and interpreting them in the context of a real-life problem or challenge.

This includes evaluating mathematical solutions or reasoning in relation to the problem to determine whether the results are reasonable, and make sense in the situation.

2. Problem-Solving Opportunities

When problem solving, pupils should be provided with a question or situation in which it may not be immediately clear to them how they would go about finding a solution. The mathematics needed to approach the problem may not be obvious and they may, at face value, lack the information they need to directly solve it.

They should be presented with situations that may require them to test out ideas, develop their own methods and strategies, take risks, and to consider different approaches if they are unsuccessful.

The problems may require the pupils to take multiple steps, and to make connections between different strands of knowledge and understanding. There may be more than one way to find a solution. At times a solution may not even be forthcoming, but the process involved in working through the problem will allow pupils to demonstrate their mathematics and review their approach.

Working collaboratively is important when problem solving. The ability to work together to share ideas and discuss ways of working is an important skill that can be developed and progressed through solving problems.

Teachers should give pupils opportunities to reflect upon and review their approach, whether they are working alone or in a group. This allows them to identify the strategies that work or do not work, as well as discussing with each other or with the teacher what they have done and what gaps in knowledge may exist.

3. Resources

CCEA has designed an online resource for problem solving in mathematics that supports the teaching and learning of the minimum statutory content for Mathematics and Numeracy at Key Stage 3. This resource helps promote problem solving at Key Stage 3 in order to better prepare pupils for the demands of Key Stage 4.

The online resource contains problems that should allow pupils to demonstrate their knowledge and understanding of:

- Number
- Algebra
- Shape, Space and Measures
- Handling Data.

These problems should also allow pupils to demonstrate their knowledge and understanding through the three curriculum objectives:

- Developing as individuals
- Developing as contributors to society
- Developing as contributors to the economy and environment.

The online resource supports the Northern Ireland Curriculum. (For more guidance, see the [Statutory Curriculum at Key Stage 3: Rationale and Detail](#) and the [Statutory Requirements for Mathematics with Financial Capability](#), both available at www.ccea.org.uk).

The aim of the online resource is to provide suggested problem-solving activities that allow pupils to demonstrate their mathematics in real-life and work situations. The range of activities provides opportunities for pupils to:

- solve problems by identifying what mathematics is needed and how it should be used;
- make connections between the different strands of mathematical knowledge and understanding to solve a problem;
- work individually or collaboratively in order to find a solution; and
- reflect on what methods and strategies they used, and whether they found a solution.

Each of the **problems** available online come with **solutions** as well as **teacher notes**. Pupils can work on these problems either individually or collaboratively – this will be at the teacher's discretion.

Problems

- Teachers can access them online and print them off for pupils, or simply display them on an interactive white board.
- There are no levels associated with the problems, but they do vary in difficulty.
- The difficulty level is indicated by the following symbols:
 - 1 circle = low difficulty. ○
 - 2 circles = medium difficulty. ○○
 - 3 circles = high difficulty. ○○○
 - 4 circles = highest difficulty. ○○○○
- There is no time limit or suggested duration for the problems. This is because the purpose of the problems is to see what strategies and methods the pupils use when solving them, rather than finding out the speed at which they can.



Solutions

- The teacher can access these online and print them off, or share them with the pupils after they have attempted the problem.
- The solutions provide the expected and/or possible responses to the problems.
- The pupils may approach a problem in a way not outlined in the accompanying solution but the solutions are there to provide likely steps and expected answers.

Teacher Notes

- There is a suggestion as to how a pupil might approach each of the given problems in the teacher notes.
- The notes will contain an overview of the problem, outlining what is expected, and any prior learning required before attempting the problem.
- They will also contain a 'tiered' approach to the problem. This outlines the processes that a pupil may engage in as part of active problem solving – similar to the OCED processes:
 - **What I know (Think):** This is the first **tier** of engaging with the problem. Pupils read the problem and think about what they are being asked. This provides them with what they know about the problem.
 - **What I need to know (Identify):** This is the second **tier**. Pupils should identify what mathematics they will need to approach the problem. They should consider whether they have all the information they need and how they will proceed, recognising the steps needed.
 - **What I need to do (Employ):** The third **tier** is where pupils use their mathematics to solve the problem. This can involve pupils using mathematics in conventional methods, or it can involve them developing their own methods and strategies. They may find limits to their approach, and can adapt it if need be.
 - **What I did (Review):** The fourth **tier** is where pupils review the extent to which they have solved the problem. Was their approach successful? Did they find a solution? What issues did they encounter, and did they overcome them? What approach did others take?
- The Teacher Notes are for guidance purposes, they are not a set list of how pupils are required to work. Each bulleted point does not have to be demonstrated or evidenced; they are just signposts for consideration.

- The Teacher Notes will also state which Curriculum Objectives the problems will provide learning opportunities for pupils to develop as:
 - Individuals
 - Contributors to Society
 - Contributors to Economy and Environment.
- They will also refer to the Thinking Skills and Personal Capabilities, outlining which can be progressed as a result of solving the problems:
 - Managing Information
 - Thinking, Problem-Solving and Decision-Making
 - Being Creative
 - Working with Others
 - Self-Management.
- They will refer to the Cross-Curricular Skills, suggesting which skills can be demonstrated by working through the problems such as:
 - Communication
 - Using ICT
 - Using Mathematics.

4. Summary

These problems are not prescribed for all pupils. They should only be given to pupils who have acquired and developed the relevant knowledge, understanding and skills that they need to solve the problems.

Teachers can also use the ideas in the problems to devise and develop their own problem-solving activities for their pupils.

We hope that you find this online resource helpful and can see how promoting problem solving within mathematics helps to give our pupils opportunities to engage in mathematical approaches. These approaches enhance their application of their knowledge and understanding as well as developing the skills they will need, not only for mathematics beyond Key Stage 3, but for life in general.

