GCE

Rewarding Learning

Revised GCE

Mathematics

Assessment Unit AS 1

assessing

Pure Mathematics

Practice Paper and Mark Scheme

For first teaching from September 2018
For first award of AS Level in Summer 2019
For first award of A Level in Summer 2019

Centre Number

Candidate Number

ADVANCED SUBSIDIARY (AS)
General Certificate of Education

Mathematics

Assessment Unit AS 1
assessing
Pure Mathematics

[SMT11]

Practice Paper

TIME
1 hour 45 minutes

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all ten** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.
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INFORMATION FOR CANDIDATES
The total mark for this paper is 100
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

1. The temperature, $C°$ centigrade, of a cup of tea t minutes after it has been made is given by the formula

$$C = 80 - 10t + \frac{1}{2}t^2$$

which is valid for $0 \leq t \leq 10$

(i) Find the initial temperature of the cup of tea. [1]

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(ii) Find the rate of change of C when $t = 3$ [4]

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2. (a) Integrate

$$3 - \frac{2}{x^4} + \sqrt{x}$$

[4]

(b) Points P and Q have position vectors

$$\overrightarrow{OP} = 4\mathbf{i} - 7\mathbf{j}$$

and $\overrightarrow{OQ} = 9\mathbf{i} + 5\mathbf{j}$

Find a unit vector in the direction $\overrightarrow{PQ}$ [4]

3. In a triangle ABC

AB = 6 cm, AC = 9cm and angle A = 39°

Find:

(i) the length of BC. [3]

(ii) the angle B. [4]

4. (a) Solve the equation

$$\cos(2x + 30°) = 0.5 \quad \text{for } 0° \leq x \leq 180°$$ [5]

(b) (i) Prove the identity

$$\tan\theta \sin\theta \equiv \frac{1 - \cos^2\theta}{\cos\theta}$$ [4]

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(ii) Hence or otherwise solve the equation

$$8 = 3\tan\theta\sin\theta$$

for $0° \leq \theta \leq 360°$ [6]

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5. The function

$$f(x) = 2x^3 - 3x^2 - px + 6$$

has a turning point at $x = 2$

(i) Find the value of p. [3]

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(ii) Find the range of values of x for which $f(x)$ is an increasing function. [4]

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6. (a) Express

$$\log_a 10x - \log_a 15 + 2\log_a 3x$$

as a single logarithm. [5]

(b) Solve the following equation for x

$$2^{2x+1} - 5(2^x) - 3 = 0$$ [7]

7. A circle has equation

$$x^2 + y^2 - 16x + ay + 20 = 0$$

and radius $\sqrt{45}$

(i) Show that $a = 2$ or $a = -2$ [3]

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When $a = -2$ the circle has equation

$$x^2 + y^2 - 16x - 2y + 20 = 0$$

(ii) The line $y = 2x$ is a tangent to this circle and touches it at the point $(b, 2b)$.

Find the value of b. [5]

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8. **A solution to this question by scale drawing will not be accepted.**

The points A, B, C and D form a rhombus.

Point A has coordinates (1, 4).

Point C has coordinates (2, 1).

(i) Find the equation of the line BD. [6]

The equation of the line AD is $4y - 3x = 13$

(ii) Find the coordinates of the point D. [3]

9. (a) In the binomial expansion of $(1 + nx)^{10}$, the coefficient of x^2 is three times the coefficient of x .

Find the value of n, where $n \neq 0$ [6]

(b) Solve the simultaneous equations

$$4x - 3y = 11$$

and

$$27^x \times 9^{y+3} = 3\sqrt{3}$$ [8]

10. **Fig. 1** below shows the curve $y = 4x - 3x^2$ and the line $y = mx$

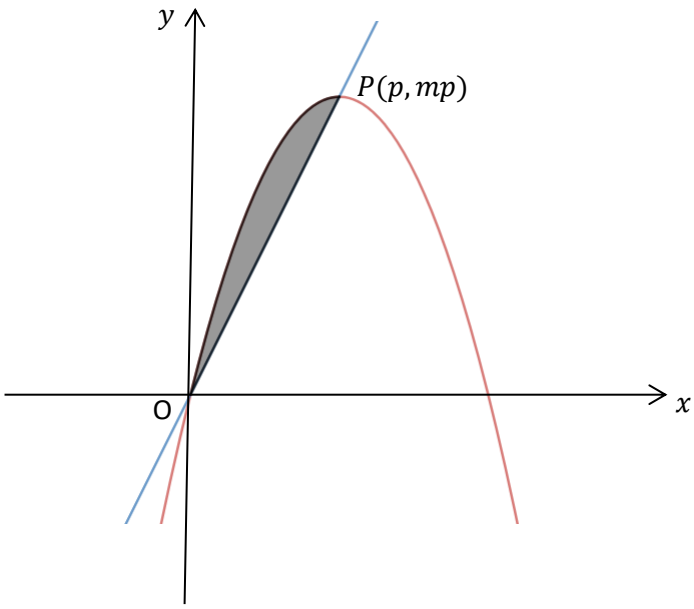

Fig. 1

The line and the curve intersect at the origin and the point P(p, mp).

(i) Show that $m = 4 - 3p$ [3]

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The area of the shaded region enclosed between this curve and line is $\frac{4}{27}$ square units.

(ii) Find the value of p.

[12]

THIS IS THE END OF THE QUESTION PAPER

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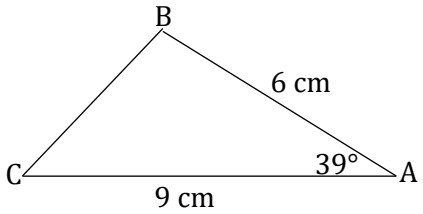
PRACTICE PAPER

MARK SCHEME

			AVAILABLE MARKS
1.	(i) $t = 0 \Longrightarrow C = 80$		
	Initial temperature = 80° centigrade	MW1	
	(ii) $\frac{dC}{dt} = -10 + t$	M1 W1	
	$t = 3 \Longrightarrow \frac{dC}{dt} = -7$	M1	
	Rate of change is $-7°$ centigrade per minute	W1	5
2.	(a) $3 - 2x^{-4} + x^{\frac{1}{2}}$		
	Integrating gives		
	$3x + \frac{2}{3x^3} + \frac{2x^{\frac{3}{2}}}{3} + c$	MW4	
	(b) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$	MW1	
	$= 5\mathbf{i} + 12\mathbf{j}$		
	$\left\|\overrightarrow{PQ}\right\| = \sqrt{5^2 + 12^2}$	M1	
	$= 13$	W1	
	Unit vector = $\frac{1}{13}(5\mathbf{i} + 12\mathbf{j})$	MW1	8
3.			
	(i) $BC^2 = 9^2 + 6^2 - 2(9)(6)\cos 39°$	M1 W1	
	$= 33.068\ldots$		
	BC $= 5.75$ cm (3sf)	MW1	
	(ii) Angle B is largest angle and therefore may be obtuse. Find angle C first.	M1	
	$\frac{\sin 39°}{5.75\ldots} = \frac{\sin C}{6}$	M1	
	$\sin C = \frac{6 \sin 39°}{5.75\ldots}$		
	C = 41.0… °	W1	
	Angle B = $180° - (41.0\ldots + 39)°$		
	B = 100° (3sf)	MW1	7

			AVAILABLE MARKS
4.	(a) $\cos(2x + 30°) = 0.5 \qquad 30° \leq 2x + 30° \leq 390°$		
	$\cos^{-1}0.5 = 60°$	MW1	
	$2x + 30° = 60°, 300°$	M1 W2	
	$2x = 30°, 270°$		
	$x = 15°, 135°$	W1	
	(b) (i) $\tan\theta \sin\theta \equiv \frac{\sin\theta}{\cos\theta} \times \sin\theta$	M1	
	$\equiv \frac{\sin^2\theta}{\cos\theta}$	W1	
	$\equiv \frac{1 - \cos^2\theta}{\cos\theta}$	M1W1	
	(ii) $8 = 3\tan\theta \sin\theta$		
	$8 = \frac{3(1 - \cos^2\theta)}{\cos\theta}$	MW1	
	$8\cos\theta = 3 - 3\cos^2\theta$	W1	
	$3\cos^2\theta + 8\cos\theta - 3 = 0$		
	$(3\cos\theta - 1)(\cos\theta + 3) = 0$		
	$\cos\theta = \frac{1}{3}, -3$	MW2	
	$\theta = 70.5°, 289°$; no solution for $\cos\theta = -3$	W2	15
5.	(i) $f(x) = 2x^3 - 3x^2 - px + 6$		
	$f'(x) = 6x^2 - 6x - p$	MW1	
	$x = 2, f'(x) = 0$		
	$6(2)^2 - 6(2) - p = 0$		
	$p = 12$	M1	
	(ii) Increasing function	W1	
	$\Rightarrow f'(x) > 0$		
	$6x^2 - 6x - 12 > 0$		
	$6(x - 2)(x + 1) > 0$		
	Critical values are $2, -1$		
	$x < -1, x > 2$	M1	
		W1	
		MW2	7

			AVAILABLE MARKS
6.	(a) $\log_a 10x - \log_a 15 + 2\log_a 3x$		
	$= \log_a 10x - \log_a 15 + \log_a(9x^2)$	M1 W1	
	$= \log_a\left(\frac{10x \times 9x^2}{15}\right)$	M1 W1	
	$= \log_a\left(\frac{90x^3}{15}\right)$		
	$= \log_a(6x^3)$	MW1	
	(b) $2^{2x+1} = 2(2^{2x})$		
	Therefore equation becomes		
	$2(2^{2x}) - 5(2^x) - 3 = 0$	MW1	
	Let $y = 2^x$	M1	
	$\Longrightarrow 2y^2 - 5y - 3 = 0$	W1	
	$(2y + 1)(y - 3) = 0$		
	$y = -\frac{1}{2}, y = 3$	W2	
	$2^x = -\frac{1}{2}$ $\quad$ $2^x = 3$		
	x not feasible $\quad$ $x = \log_2 3 = 1.58$ (3sf)	W2	12
7.	(i) $x^2 + y^2 - 16x + ay + 20 = 0$		
	$r = \sqrt{g^2 + f^2 - c}$		
	$\sqrt{45} = \sqrt{\left(\frac{a}{2}\right)^2 + (-8)^2 - 20}$	M1 W1	
	$\frac{a^2}{4} = 45 - 64 + 20$		
	$a^2 = 4$		
	$a = \pm 2$	W1	
	(ii) Centre of circle is (8, 1)	MW1	
	$m_{\text{tangent}} = 2$	MW1	
	$m_{\text{radius}} = -\frac{1}{2}$	MW1	
	$\frac{1 - 2b}{8 - b} = -\frac{1}{2}$	M1	
	$2 - 4b = -8 + b$		
	$b = 2$	W1	8

			AVAILABLE MARKS
8.	(i) BD is perpendicular to AC and passes through its midpoint		
	$m_{AC} = \frac{4-1}{1-2} = -3$	M1 W1	
	$m_{BD} = \frac{1}{3}$	MW1	
	Midpoint of AC		
	$\Rightarrow \left(\frac{3}{2}, \frac{5}{2}\right)$	MW1	
	Equation of BD		
	$y - \frac{5}{2} = \frac{1}{3}\left(x - \frac{3}{2}\right)$	M1	
	$3y = x + 6$	W1	
	(ii) $3y = x + 6$ ①		
	$4y - 3x = 13$ ②		
	$3 \times$① $9y - 3x = 18$	M1	
	② $4y - 3x = 13$		
	$5y = 5$		
	$y = 1$	W1	
	$x = -3$		
	D$(-3, 1)$	W1	9
9.	(a) $(1 + nx)^{10}$		
	$= 1 + 10nx + \frac{10.9n^2x^2}{2.1}$	M1 W1	
	$= 1 + 10nx + 45n^2x^2$	W1	
	Hence $45n^2 = 3(10n)$	M1 W1	
	$45n^2 - 30n = 0$		
	$15n(3n - 2) = 0$		
	$n = \frac{2}{3} \quad n \neq 0$	W1	
	(b) $27^x \times 9^{y+3} = 3\sqrt{3}$		
	$(3^3)^x \times (3^2)^{y+3} = 3 \times 3^{\frac{1}{2}}$	M1 W1	
	$3^{3x+2y+6} = 3^{\frac{3}{2}}$	MW1	
	$3x + 2y = -\frac{9}{2}$ ①	M1 W1	
	$4x - 3y = 11$ ②		
	$4 \times$① $12x + 8y = -18$	M1	
	$3 \times$② $12x - 9y = 33$		
	$17y = -51$	W1	
	$y = -3$		
	$x = \frac{1}{2}$	W1	14

	AVAILABLE MARKS

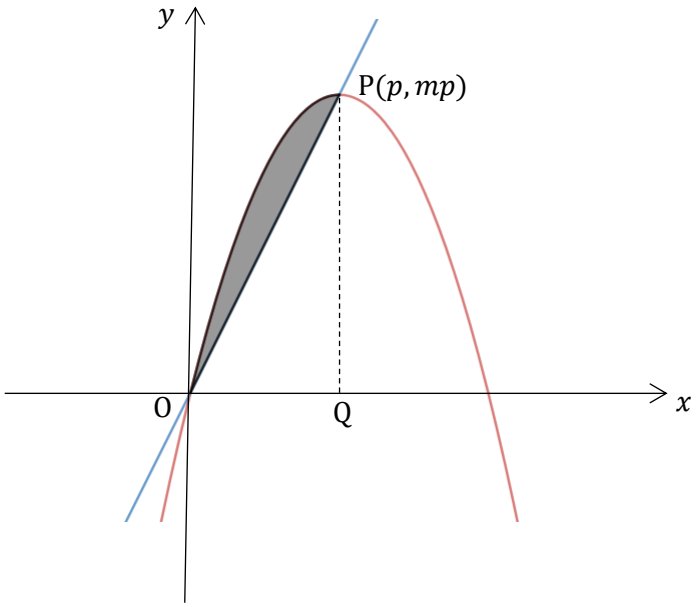
10. (i) (p, mp) lies on the curve.

$mp = 4p - 3p^2$ M1 W1

$p(m - 4 + 3p) = 0$

$m = 4 - 3p$ or $p = 0$ (but this is the origin) W1

(ii)



Area under curve

$\int_0^p (4x - 3x^2)\mathrm{d}x$ M1 W1

$= [2x^2 - x^3]_0^p$ MW2

$= 2p^2 - p^3$ W1

Area of triangle OPQ

$\frac{1}{2}p(mp)$ MW1

$= \frac{1}{2}mp^2$

$= \frac{1}{2}(4 - 3p)p^2$ M1

$= 2p^2 - \frac{3}{2}p^3$ W1

Shaded region

$2p^2 - p^3 - \left(2p^2 - \frac{3}{2}p^3\right) = \frac{4}{27}$ M2 W1

$\frac{1}{2}p^3 = \frac{4}{27}$

$p^3 = \frac{8}{27}$

$p = \frac{2}{3}$ MW1

15

Total 100

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