Centre Number

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Candidate Number

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**ADVANCED SUBSIDIARY (AS)**
**General Certificate of Education**

# Mathematics

## Assessment Unit AS 1

*assessing*

Pure Mathematics

## [SMT11]

## PRACTICE PAPER

**TIME**

1 hours 45 minutes

**INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all nine** questions in the spaces provided.
**Do not write outside the boxed area on each page or on blank pages or tracing paper**.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.
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**INFORMATION FOR CANDIDATES**

The total mark for this paper is 100
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is ln z where it is noted that $\ln z \equiv \log_e z$

1. Solve the simultaneous equations

$$x + 3y - z = 14$$
$$2x - y + z = 5$$
$$3x + 4y - 2z = 29$$

[7]

2. **Fig. 1** below shows a sketch of the graph of the function $y = \mathrm{f}(x)$

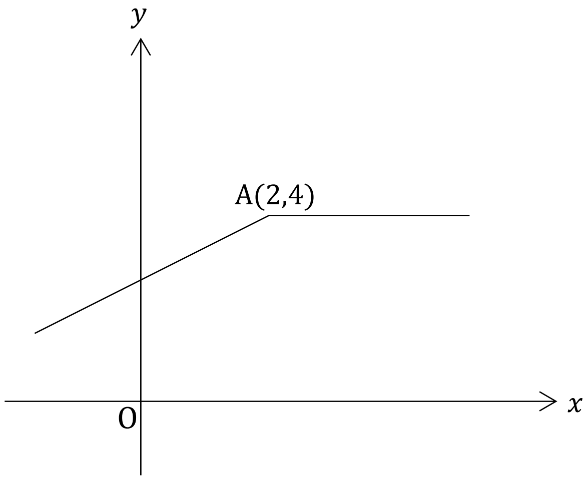


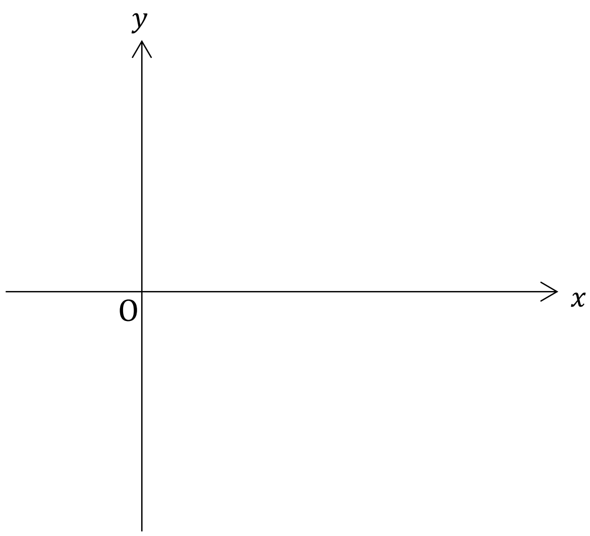

**Fig. 1**

Point A has coordinates (2, 4)

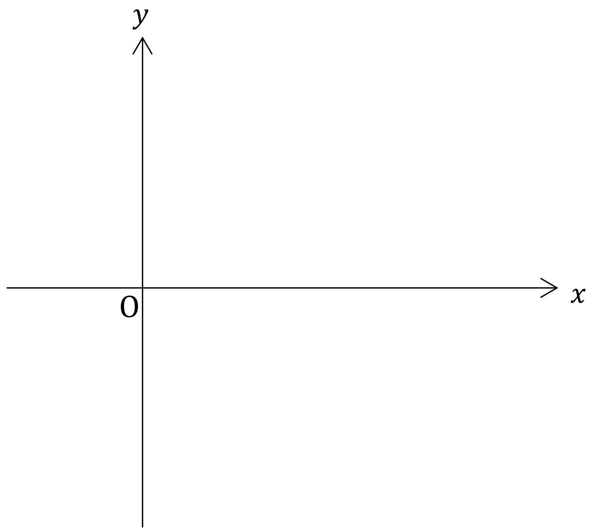
Sketch the graphs of the following functions, clearly labelling the image of the point A.

(i) $y = \mathrm{f}(x) + 1$

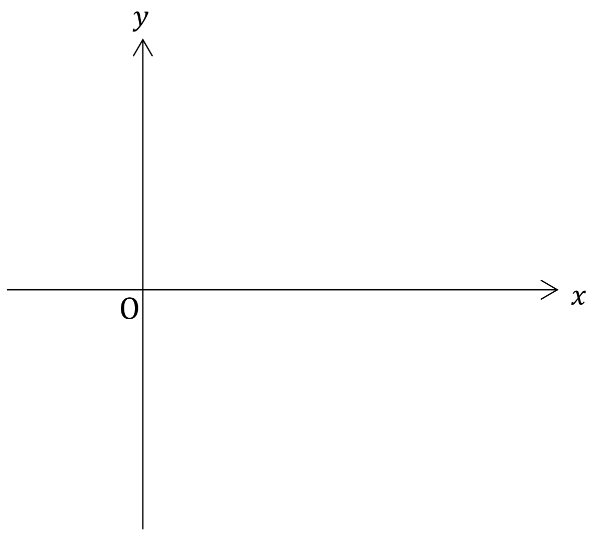
[2]



(ii) $y = -f(x)$ [2]




(iii) $y = f(2x)$ [2]



3. (a) $\overrightarrow{OB} = 5\mathbf{i} + 4\mathbf{j}$

$\overrightarrow{AB} = 2\mathbf{i} - 6\mathbf{j}$

Find the angle $\overrightarrow{OA}$ makes with the positive $x$-axis. [6]

(b) Find the first four terms, in ascending powers of $x$, in the binomial expansion of

$$(3-2x)^8$$

[6]

(b) Find the first four terms, in ascending powers of $x$, in the binomial expansion of

4. A circle is given by the equation

$$x^2 + y^2 - 4x - 8y - 5 = 0$$

(i) Find the coordinates of C, the centre of the circle. [3]

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The point A(5, 8) lies on the circumference of the circle.

(ii) Find the equation of the tangent to the circle at A.
Leave your answer in the form $ax + by + c = 0$, where $a, b$ and $c$ are integers to be found. [6]

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5. (a) Solve

$$3\log_4 2x - \log_4 x = 2$$

[6]

(b) Solve

$$e^{3x} - 7e^{x} = 0$$

[5]

6. (a) The route map for a yacht race is shown in **Fig. 2** below.

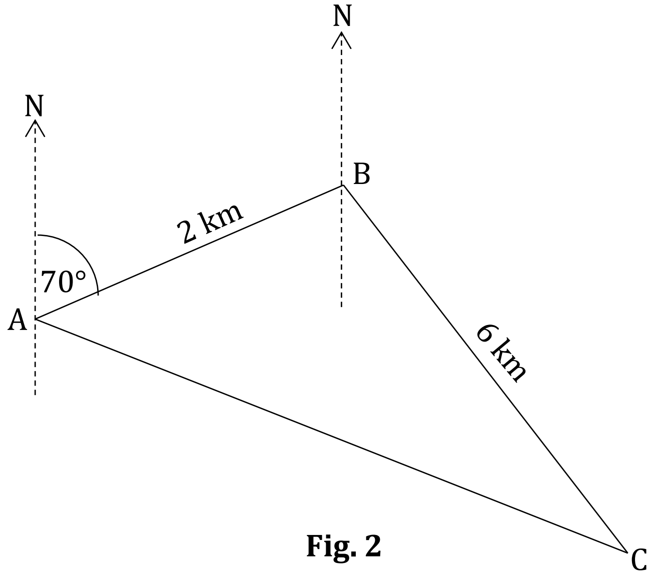


**Fig. 2**

The yachts start at A and move to B on a bearing of 070°
AB = 2 km

The yachts then move from B to C on a bearing of 150°
BC = 6 km

The yachts then return to A.

(i) Find the distance AC. [4]

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(ii) Find the angle $A\hat{C}B$ and hence the bearing of A from C. [5]

(b) Solve

$$1 - \cos^2 \theta = 2 - \sin \theta \qquad 0° \leq \theta \leq 360°$$

[7]

(b) Solve

7. (a) Integrate

$$4x + \frac{2}{3x^5} - 8\sqrt{x}$$ [4]

(b) **Fig. 3** below shows a sketch of a quadratic curve.

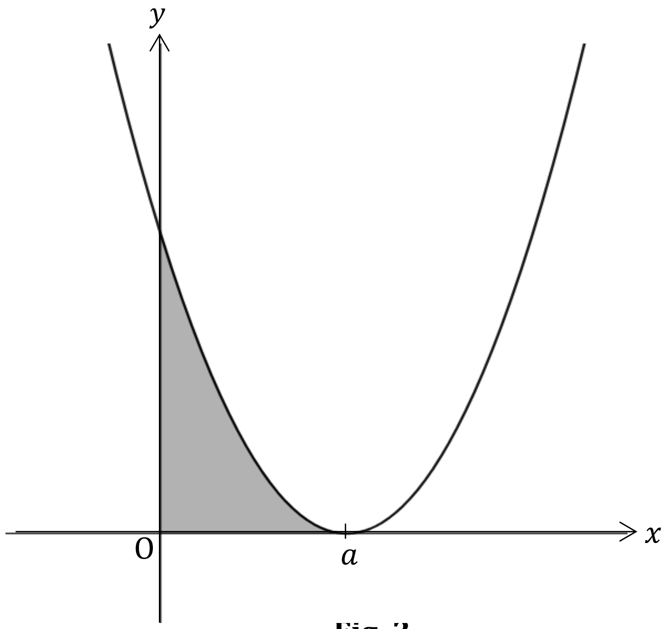


**Fig. 3**

The curve is a single transformation of the curve $y = x^2$

The curve touches the $x$-axis at the point where $x = a$

Find, in terms of $a$, the value of the shaded area. [7]

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8. A concrete pillar is shown in **Fig. 4** below.

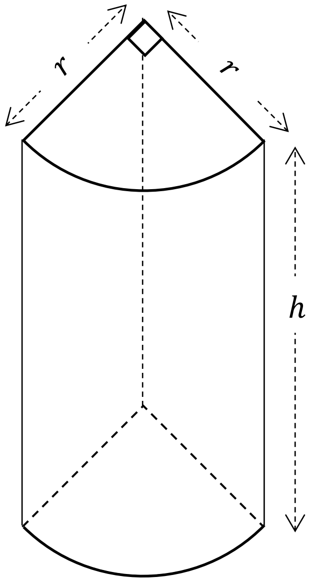


**Fig. 4**

The pillar is a prism with cross sectional area in the shape of a quarter circle of radius $r$ metres.

The pillar has height $h$ metres.

The pillar has volumes 2π cubic metres.

(i) Show that the total surface area A of the pillar can be expressed as

$$A = \frac{1}{2}\pi r^2 + \frac{16}{r} + \frac{4\pi}{r}$$

[8]

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(ii) Hence find the exact value of $r$ for which the total surface area is a minimum. [8]

9. (a) Find the values of $p$ for which the equation

$$x^2 + px + p = 1$$

has distinct real roots. [7]

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(b) Find an expression for the difference in the roots of the following equation

$$x^2 - 2ax + (a^2 - b^2) = 0$$ [5]

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**THIS IS THE END OF THE QUESTION PAPER**

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**General Certificate of Education**

# Mathematics

## Assessment Unit AS 1

*assessing*

Pure Mathematics

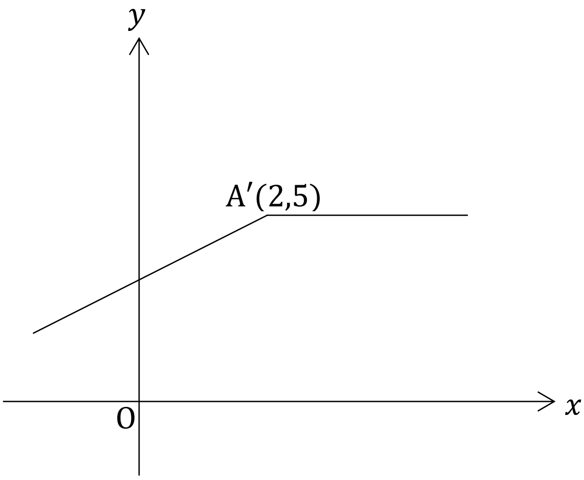
**[SMT11]**

**PRACTICE PAPER**

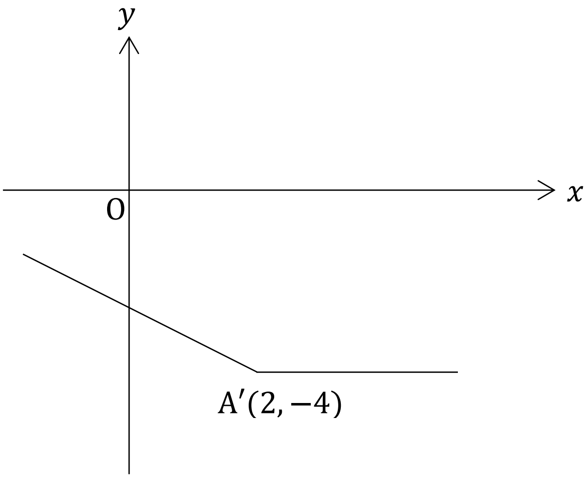
# MARK
# SCHEME

| | | AVAILABLE MARKS |
|---|---|---|
| 1. ① + ② $\Longrightarrow$ $3x + 2y = 19$ | M1W1 | |
| 2 ② + ③ $\Longrightarrow$ $\underline{7x + 2y = 39}$ | MW1 | |
| $4x = 20$ | M1 | |
| $x = 5$ | | |
| $y = 2$ | | |
| $z = -3$ | W3 | 7 |
| 2. (i) | M1W1 | |
| (ii) | M1W1 | |
| (iii) | M1W1 | 6 |

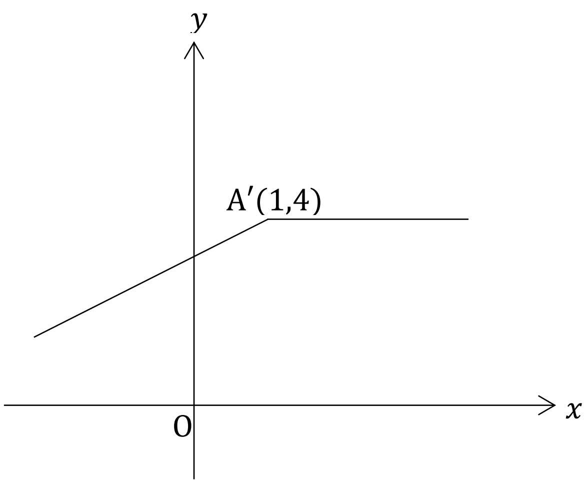
2. (i)




(ii)




(iii)



| | | | AVAILABLE MARKS |
|---|---|---|---|
| 3. | (a) $\overrightarrow{OA} = \overrightarrow{OB} + \overrightarrow{BA}$ | M1W1 | |
| | $= (5\mathbf{i} + 4\mathbf{j}) + (-2\mathbf{i} + 6\mathbf{j})$ | W1 | |
| | $= 3\mathbf{i} + 10\mathbf{j}$ | W1 | |
| | $\tan\theta = \dfrac{10}{3}$ | M1 | |
| | $\theta = 73.3°$ | W1 | |
| | (b) $(3-2x)^8 = \binom{8}{0}3^8(-2x)^0 + \binom{8}{1}3^7(-2x)^1 + \binom{8}{2}3^6(-2x)^2 +$ | M1 | |
| | $\binom{8}{3}3^5(-2x)^3 + \cdots$ | W3 | |
| | $= 6561 - 34\,992x + 81\,648x^2 - 108\,864x^3 + \cdots$ | MW2 | 12 |
| 4. | (i) $x^2 - 4x + y^2 - 8y - 5 = 0$ | M1 | |
| | $(x-2)^2 + (y-4)^2 - 4 - 16 - 5 = 0$ | | |
| | $(x-2)^2 + (y-4)^2 = 25$ | W1 | |
| | Centre $= (2, 4)$ | W1 | |
| | (ii) $\text{Grad}_{AC} = \dfrac{4-8}{2-5} = \dfrac{-4}{-3} = \dfrac{4}{3}$ | M1W1 | |
| | Gradient of tangent $= -\dfrac{3}{4}$ | MW1 | |
| | $y = mx + c$ | M1 | |
| | $8 = -\dfrac{3}{4}(5) + c$ | | |
| | $c = \dfrac{47}{4}$ | W1 | |
| | $y = -\dfrac{3}{4}x + \dfrac{47}{4}$ | | |
| | $4y = -3x + 47$ | | |
| | $3x + 4y - 47 = 0$ | MW1 | 9 |

| | | | AVAILABLE MARKS |
|---|---|---|---|
| 5. | (a) $\log_4(2x)^3 - \log_4 x = 2$ | M1 | |
| | $\log_4 8x^3 - \log_4 x = 2$ | W1 | |
| | $\log_4\left(\frac{8x^3}{x}\right) = 2$ | M1 | |
| | $\log_4 8x^2 = 2$ | W1 | |
| | $8x^2 = 16$ | M1 | |
| | $x^2 = 2$ | | |
| | $x = \sqrt{2}$ | W1 | |
| | (b) $e^x(e^{2x} - 7) = 0$ | M1W1 | |
| | $e^x = 0$     $e^{2x} - 7 = 0$ | MW1 | |
| | [no solutions]     $e^{2x} = 7$ | | |
| | $2x = \ln 7$ | M1 | |
| | $x = \frac{1}{2}\ln 7$ | W1 | 11 |
| 6. | (a) (i) $A\hat{B}C = 100°$ | MW1 | |
| | $AC^2 = 2^2 + 6^2 - 2(2)(6)\cos 100°$ | M1W1 | |
| | $AC = 6.65$ km | W1 | |
| | (ii) $\frac{2}{\sin A\hat{C}B} = \frac{6.65}{\sin 100°}$ | M1W1 | |
| | $A\hat{C}B = 17.2°$ | W1 | |
| | Bearing of A from C $= 360° - (30° + 17.2°)$ | M1 | |
| | $= 313°$ [3sf] | W1 | |
| | (b) $1 - (1 - \sin^2\theta) = 2 - \sin\theta$ | M1W1 | |
| | $\sin^2\theta + \sin\theta - 2 = 0$ | MW1 | |
| | $(\sin\theta + 2)(\sin\theta - 1) = 0$ | M1W1 | |
| | $\sin\theta + 2 = 0$     $\sin\theta - 1 = 0$ | | |
| | $\sin\theta = -2$     $\sin\theta = 1$ | | |
| | [no solutions]     $\theta = 90°$ | MW2 | 16 |

| | | | AVAILABLE MARKS |
|---|---|---|---|
| 7. | (a) $2x^2 - \frac{1}{6x^4} - \frac{16x^{\frac{3}{2}}}{3} + c$ | MW4 | |
| | (b) $y = (x-a)^2$ | M1W1 | |
| | $y = x^2 - 2ax + a^2$ | | |
| | Area $= \int_0^a (x^2 - 2ax + a^2)\mathrm{d}x$ | M1 | |
| | $= \left[\frac{x^3}{3} - ax^2 + a^2x\right]_0^a$ | MW2 | |
| | $= \left[\frac{a^3}{3} - a^3 + a^3\right] - 0$ | M1 | |
| | $= \frac{a^3}{3}$ unit$^2$ | W1 | 11 |
| 8. | (i) $V = \frac{1}{4}\pi r^2 h$ | MW1 | |
| | $\frac{1}{4}\pi r^2 h = 2\pi$ | M1 | |
| | $r^2 h = 8$ | | |
| | $h = \frac{8}{r^2}$ | W1 | |
| | $A = \frac{1}{2}\pi r^2 + 2rh + \frac{1}{2}\pi rh$ | M1W2 | |
| | $A = \frac{1}{2}\pi r^2 + 2r\left(\frac{8}{r^2}\right) + \frac{1}{2}\pi r\left(\frac{8}{r^2}\right)$ | M1 | |
| | $A = \frac{1}{2}\pi r^2 + \frac{16}{r} + \frac{4\pi}{r}$ | W1 | |
| | (ii) $\frac{\mathrm{d}A}{\mathrm{d}r} = \pi \mathrm{r} - \frac{16}{r^2} - \frac{4\pi}{r^2}$ | M1W3 | |
| | $0 = \pi \mathrm{r} - \frac{16}{r^2} - \frac{4\pi}{r^2}$ | M1 | |
| | $\frac{16 + 4\pi}{r^2} = \pi r$ | | |
| | $\pi r^3 = 16 + 4\pi$ | | |
| | $r^3 = \frac{16 + 4\pi}{\pi}$ | | |
| | $r = \sqrt[3]{\frac{16 + 4\pi}{\pi}}$ | W1 | |
| | $\frac{\mathrm{d}^2 A}{\mathrm{d}r^2} = \pi + \frac{32}{r^3} + \frac{8\pi}{r^3}$ | M1 | |
| | $> 0$ | | |
| | $\Longrightarrow$ Minimum | W1 | 16 |

| | | | AVAILABLE MARKS |
|---|---|---|---|
| 9. | (a) $x^2 + px + (p - 1) = 0$ | MW1 | |
| | $b^2 - 4ac = p^2 - 4(1)(p - 1)$ | M1W1 | |
| | $= p^2 - 4p + 4$ | W1 | |
| | $= (p - 2)^2$ | W1 | |
| | Distinct real roots $\Rightarrow b^2 - 4ac > 0$ | M1 | |
| | $\Rightarrow p \neq 2$ | W1 | |
| | (b) $x = \dfrac{2a \pm \sqrt{4a^2 - 4(1)(a^2 - b^2)}}{2}$ | M1W1 | |
| | $x = \dfrac{2a \pm 2b}{2}$ | | |
| | $x = a \pm b$ | W1 | |
| | Difference in roots $= (a + b) - (a - b)$ | M1 | |
| | $= 2b$ | W1 | 12 |
| | | TOTAL | 100 |