



Centre Number

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Candidate Number

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ADVANCED SUBSIDIARY (AS)
General Certificate of Education

Mathematics

Assessment Unit A2 2

Assessing

Applied Mathematics

[AMT21]

PRACTICE PAPER

TIME

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.

You must answer the questions in the spaces provided.

Do not write outside the boxed area on each page or on blank pages.

Complete in black ink only. **Do not write with a gel pen.**

Questions which require drawing or sketching should be completed using an HB pencil.

Candidates must answer **all** questions from sections A and B.

Equal time should be spent on each section.

Show clearly the full development of your answers. **Answers without working may not gain full credit.**

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100. The total available mark for each section of this paper is 50.

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8\text{ms}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all questions

SECTION A

Mechanics

1. Two smooth spheres A and B of equal radii rest on a smooth horizontal surface as shown in **Fig. 1** below.

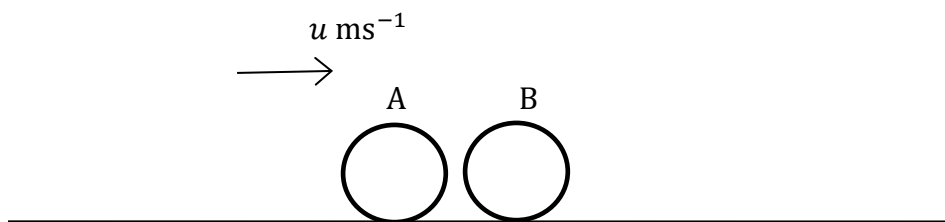


Fig. 1

A and B have masses m_1 kg and m_2 kg respectively.

A is projected towards B with a speed of $u \text{ ms}^{-1}$ and collides directly with B.

After the collision A has a speed of $\frac{1}{2}u \text{ ms}^{-1}$ in its original direction.

- (i) Find the velocity of B after the collision in terms of u , m_1 and m_2

[4]

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(ii) Show that $m_1 \geq m_2$

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2. An athlete throws a shot put with a speed of 15 ms^{-1} at 38° to the horizontal

The shot put is thrown from shoulder height which is 1.6 m above the ground.

Model the shot put as a particle.

(i) Find the time taken to reach the ground.

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It has been reported that the athlete has broken the world record of 23.37m.

(ii) Find the horizontal distance by which the shot putter beats the world record. [3]

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In the 1970s sports analysts were puzzled that the best shotput distances were not occurring with a projection angle of 45° which was well known to be the optimum projection angle of a projectile.

(iii) Suggest what they were not taking into consideration. [1]

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3. A uniform beam AB, of mass $8m$ kg and length 2 metres, is smoothly hinged at end A to a fixed point on a wall as shown in **Fig. 2** below.

The beam is maintained in a horizontal position by a light inelastic string attached to a point D on the wall and a point C on the beam. The string is inclined to the beam at an angle β .

$$AD = 1.2\text{m} \quad AC = 1.6 \quad CD = 2\text{m}$$

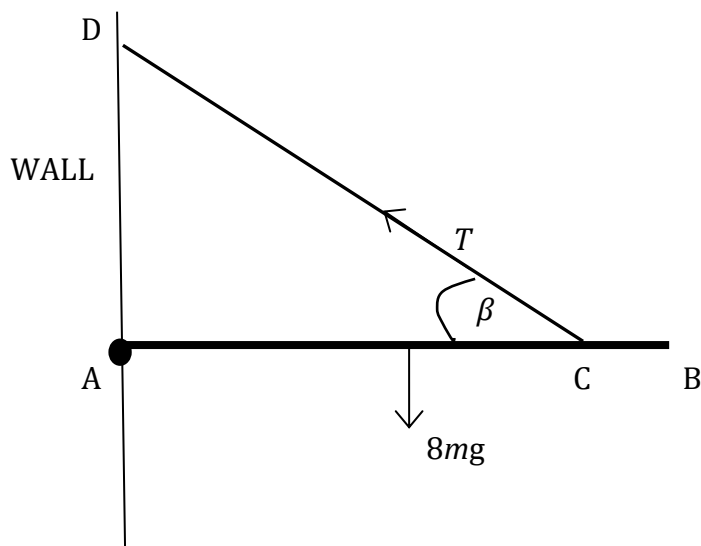


Fig. 2

(i) By taking moments about A find the tension T in the string, in terms of m and g . [4]

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4. A particle A has a displacement \mathbf{x} metres, from a fixed point O, at time t seconds, where \mathbf{x} is given by

$$\mathbf{x} = e^{-2t}\mathbf{i} + t\mathbf{j}$$

- (i) Find the particle's displacement from O at $t = 3$ [2]

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When $t = T$ the speed of the particle is $\sqrt{3} \text{ ms}^{-1}$

- (ii) Find T . [6]

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(iii) Find the acceleration of the particle at time $t = T$.

[3]

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5. Three forces act on a particle of mass 0.1kg at time t seconds.

$$F_1 = 0.3t^2 \text{ newtons}$$

$$F_2 = -1.4t \text{ newtons}$$

$$F_3 = 1 \text{ newtons}$$

(i) Find the acceleration of the particle in terms of t .

[3]

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When $t = 0$ the particle is at rest.

(ii) Find the times when the particle is instantaneously at rest for $t > 0$

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SECTION B

Statistics

6. **Table 1** below shows the melting points and boiling points of six alkali metals.

Table 1

Atomic Number	Alkali Metal	Melting Point (C)	Boiling Point (C)
3	Lithium	180.5	1347
11	Sodium	97.8	883
19	Potassium	63.28	759
37	Rubidium	39	688
55	Caesium	28	671
87	Francium	27	677

John is a chemistry student, and he calculates the product moment correlation coefficient between the melting points and boiling points of these Alkali Metals to be 0.8993

He decides to carry out a one tailed hypothesis test at the 0.5% level of significance to test if there is a correlation between the melting point and boiling point of the metals.

- (i) Identify the null and alternative hypothesis he should use.

[2]

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(ii) Complete the hypothesis test.

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John is surprised at his result in (ii)

(iii) Explain to John what change he could have made to his test to have achieved a different outcome.

[1]

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7. A bus company runs a non-stop bus route which travels from the local bus station to another town.

The journey time for this route has been measured over the year with a mean time of 30 minutes and standard deviation of 4.2 minutes.

(i) Find the percentage of journeys in the year which took over 35 mins. [3]

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The bus timetable states the bus journey takes 30 mins. In the interest of good public relations, the bus company would like 90% of the journeys to take no more than 35 mins.

(ii) Assuming the mean stays the same find the maximum standard deviation of the journey times the company would like to see. [4]

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A new manager in the bus company decides to alter the route slightly. This new route has a mean journey time of 28 mins and standard deviation of 4.5 minutes.

(iii) What journey time should the manager document in the bus timetable for the new route to guarantee to the commuters that 90% of its journeys take no more than the time stated in the timetable. Give your answer to the nearest minute. [3]

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8. The company who makes Crunchy Crisps states that the mass of the packets of crisps are normally distributed with a mean of 25kg and a variance of 0.36kg.

Patrick buys two packets of crisps.

- (i) Find the probability that each of these packets has a mass greater than 24.5 kg. [4]

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Trevor buys these crisps and starts to suspect that the mean mass of the packets of crisps is less than 25kg.

He decides to test his hypothesis using a significance level of 5%

He takes a random sample of 35 packets of crisps and calculates the mean of the sample to be 24.8kg.

- (ii) Complete the hypothesis test. [9]

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9. The probability that it rains on any day in April in Northern Ireland is thought to be 0.4
A meteorologist now suspects that climate change has affected this probability.
To test his suspicion, he carries out a hypothesis test.
He takes a random sample of ten days in April.

(i) State the null hypothesis and alternative hypothesis he should use. [3]

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He decides to use at a 10% significance level.

(ii) Find the critical region for this test. [7]

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The meteorologists find that it has rained on seven of the ten days.

(iii) What conclusion can the meteorologist draw from his test?

[3]

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10. In a school during the summer term the 100 Year 10 students can take part in athletics (A) and cricket (C).

Some students take part in both cricket and athletics, some students take part in either cricket or athletics and some take part in neither.

$$P(A|C) = \frac{3}{8} \quad P(A' \cap C) = \frac{1}{4} \quad P(A \cap C) = x$$

(i) Show that $x = 0.15$

[5]

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(ii) The table below represents the 100 students' participation in athletics and cricket. Complete this table.

	C	C'	Total
A			
A'	25	36	
Total			

[2]

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THIS IS THE END OF THE PAPER

ADVANCED

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Assessment Unit A2 2

assessing

Applied Mathematics

[AMT21]

PRACTICE PAPER

**MARK
SCHEME**

Section A
Mechanics

1 (i)

$$m_1 u + m_2 \times 0 = \frac{1}{2} m_1 u + m_2 v_2$$

$$m_2 v_2 = \frac{1}{2} m_1 u$$

$$v_2 = \frac{m_1}{2m_2} u$$

M1W1W1

W1

(ii)

$$v_2 \geq v_1$$

$$\frac{m_1}{2m_2} u \geq \frac{1}{2} u$$

$$m_1 \geq m_2$$

M1

W1

6

2(i)

$$-1.6 = (15 \sin 38^\circ) t - \frac{1}{2} \times 9.8 t^2$$

$$4.9 t^2 - 9.2349 t - 1.6 = 0$$

$$t = -0.16, 2.0444$$

$$t = 2.044 \text{ s}$$

M1W2

W1

MW1

(ii) $s = 15 \cos 38^\circ \times 2.0444$

$s = 24.1651$ (24.17 m)

Athlete beats record by $24.17 - 23.37 = 0.795$ m (3sf)

M1

W1

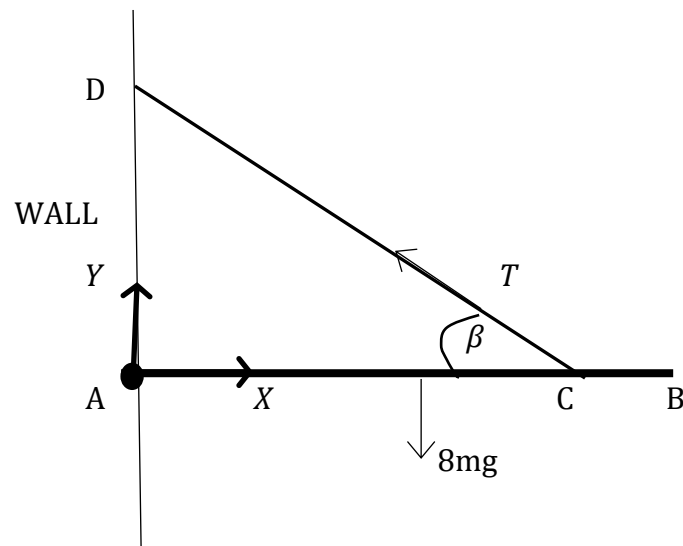
MW1

(iii) The analyst did not take into consideration the height of projection above the ground.

MW1

9

3.



(i) M(A)

$$T \sin \beta \times 1.6 = 8mg \times 1$$

M1M1W1

$$T \times \frac{3}{5} \times 1.6 = 8mg$$

$$T = 8mg \times \frac{5}{4.8}$$

$$= \frac{25}{3} mg \text{ N}$$

W1

(ii) Resolve horizontally

$$X = T \cos \beta$$

M1

$$= \frac{25}{3} mg \times \frac{4}{5}$$

$$= \frac{20}{3} mg$$

W1

Resolve vertically $Y + \frac{25}{3} mg \times \frac{3}{5} = 8mg$

M1

$$Y + 5mg = 8mg$$

$$Y = 3mg$$

W1

$$\text{Reaction} = \sqrt{(3mg)^2 + \left(\frac{20}{3}mg\right)^2} = 7.31mg$$

MW1 9

4. (i) $\mathbf{x} = e^{-2t}\mathbf{i} + t\mathbf{j}$
 $t = 3$ $\mathbf{x} = e^{-6}\mathbf{i} + 6\mathbf{j}$ M1W1

(ii) $\mathbf{x} = e^{-2t}\mathbf{i} + t\mathbf{j}$
 $\mathbf{v} = -2e^{-2t}\mathbf{i} + \mathbf{j}$ M1W1

Speed of the particle squared

$$= (-2e^{-2T})^2 + 1^2$$

$$= 4e^{-4T} + 1$$

M1W1

If speed is $\sqrt{3} \text{ ms}^{-1}$

$$4e^{-4T} + 1 = 3$$

MW1

$$4e^{-4T} = 2$$

$$e^{-4T} = \frac{1}{2}$$

$$T = \frac{1}{4} \ln(2) = 0.173 \text{ (3sf)}$$

W1

(iii) $\mathbf{v} = -2e^{-2t}\mathbf{i} + \mathbf{j}$
 $\mathbf{a} = 4e^{-2T}\mathbf{i}$ M1W1

At time $t = T$ $e^{-4T} = \frac{1}{2}$

Hence $\mathbf{a} = 2\sqrt{2}\mathbf{i} = 2.83\mathbf{i} \text{ ms}^{-2}$ MW1 11

5.(i)

$$F = ma$$

M1

$$F_1 + F_2 + F_3 = ma$$

$$0.3t^2 - 1.4t + 1 = 0.1a$$

W1

$$a = 3t^2 - 14t + 10 \text{ ms}^{-2}$$

W1

(ii)

$$a = 3t^2 - 14t + 10$$

$$v = \int a \, dt = t^3 - 7t^2 + 10t + c$$

M1W1

$$\text{If } t = 0 \quad v = 0 \Rightarrow c = 0$$

MW1

$$0 = t(t^2 - 7t + 10)$$

M1

$$0 = t(t - 2)(t - 5)$$

$$t = 2, 5 \quad (t > 0)$$

W1

(iii)

$$v = t^3 - 7t^2 + 10t$$

$$s = \frac{1}{4}t^4 - \frac{7}{3}t^3 + 5t^2 + c$$

M1W1

$$\text{If } t = 0 \quad s = 0 \Rightarrow c = 0$$

$$s = \frac{1}{4}t^4 - \frac{7}{3}t^3 + 5t^2$$

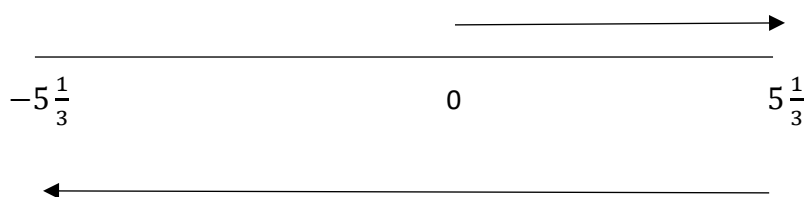
MW1

$$s(2) = 4 - \frac{56}{3} + 20 = 5\frac{1}{3}$$

MW1

$$s(4) = 64 - \frac{448}{3} + 80 = -5\frac{1}{3}$$

MW1



$$\text{Distance} = 5\frac{1}{3} + 5\frac{1}{3} + 5\frac{1}{3} = 16 \text{ m}$$

M1W1

15

SECTION B
STATISTICS

6. (i) ρ is the product moment correlation coefficient

$$H_0: \rho = 0$$

M1

$$H_1: \rho > 0 \quad (\text{one-tail})$$

MW1

(ii) $n = 6$, $r = 0.8993$

MW1

To be significant at the 0.5% level of significance $r \geq 0.9172$

$$0.8930 < 0.9172$$

MW1

Hence there is insufficient evidence to reject the null hypothesis and accept there is a correlation between the boiling and melting points of the six Alkali metals at the 0.5% level of significance.

MW2

(ii) A significance level is the probability of incorrectly rejecting the null hypothesis. In this case the significance level is very small. If he uses a 1% level of significance, there is sufficient evidence to reject the null hypothesis and accept there is a correlation between the boiling and melting points of the six Alkali metals.

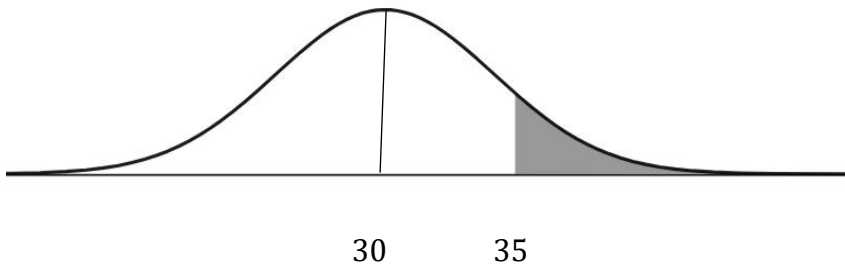
MW1

7

Solution1: Using calculator functions

7. (i) $X \sim N(30, 4.2^2)$

M1



Calculator use: $\mu = 30, \sigma = 4.2$

M1

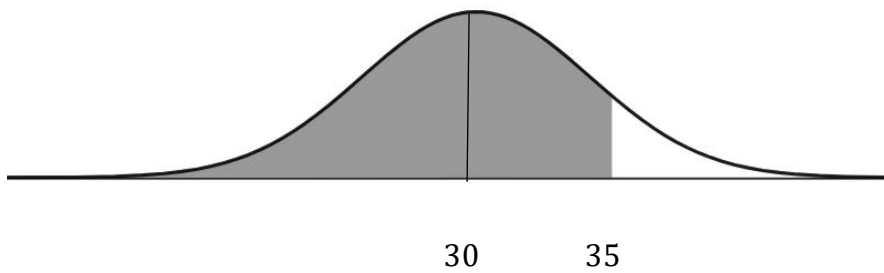
$P(X \leq 35) = 0.88307$

Percentage more than 35 is 11.7%

W1

(ii) $X \sim N(30, \sigma^2)$

M1



$\Phi^{-1}(0.9) = 1.28155$

MW1

$\frac{35-30}{\sigma} = 1.28155$

M1

$\sigma = 3.90$ mins

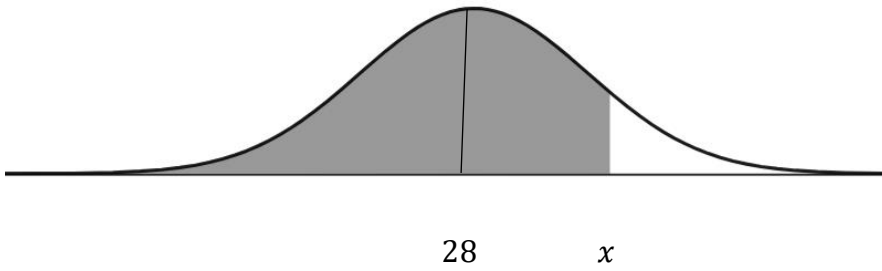
W1

(iii)

$$X \sim N(28, 4.5^2)$$

M1

AVAILABLE
MARKS



Calculator use: $\mu = 28, \sigma = 4.5, p = 0.9$

$$x = 33.767... \text{ mins}$$

MW1

Place a journey time of 34 mins in the timetable.

MW1

10

Solution2: Using tables.

(i) $P\left(Z > \frac{35-30}{4.2}\right)$
 $= P(Z > 1.1905)$
 $= 1 - \Phi(1.1905)$
 $= 1 - 0.8830751$
 $= 11.7\%$

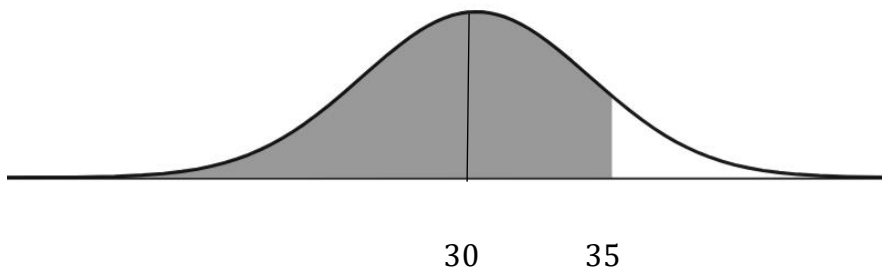
MW1

M1

W1

(ii) $X \sim N(30, \sigma^2)$

M1



$\phi^{-1}(0.9) = 1.28155$

MW1

$\frac{35-30}{\sigma} = 1.28155$

M1

$\sigma = 3.90$ mins

W1

(iii) $\Phi^{-1}(0.9) = 1.281551$

$\frac{x-28}{4.5} < 1.281551$

M1W1

$x - 28 < 5.767$

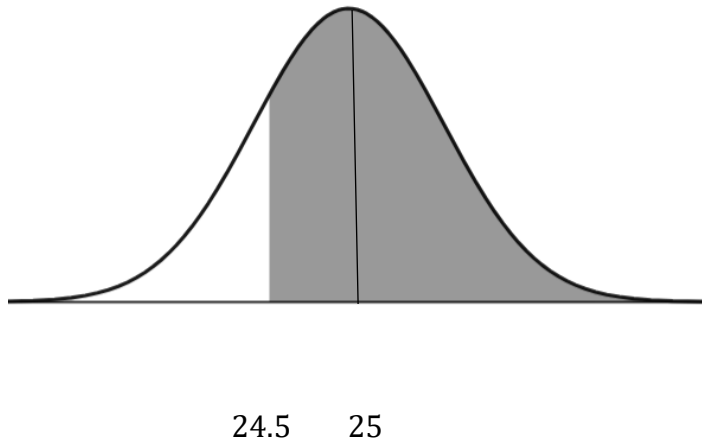
$x < 33.77$ mins

Place a journey time of 34 mins in the timetable.

MW1

Solution1: Using calculator functions

8. (i) $X \sim N(25, 0.6^2)$



$$\sigma = \text{SD} = \sqrt{0.36} = 0.6$$

$$\mu = 25\text{kg}$$

MW1

$$P(W > 24.5)$$

M1

$$\text{Probability} = 0.79767$$

W1

Probability that both packets of crisp is greater than 24.5kg

$$= (0.79767)^2 = 0.636$$

MW1

AVAILABLE
MARKS

(ii)

$$H_0: \mu = 25$$

M1

$$H_1: \mu < 25 \text{ (one-tail)}$$

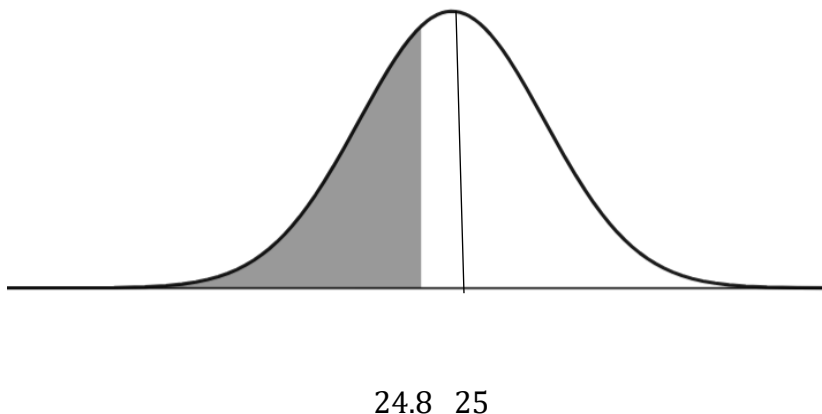
MW1

Reject H_0 if $p < 0.05$

M1

Weight is distributed normally with mean $\mu = 25$ and $\sigma = 0.6$

$$P(W < 24.8)$$



$$\sigma = \frac{0.6}{\sqrt{35}}$$

MW1

$$\text{Probability } p = 0.5 - 0.475697 = 0.0243$$

M1W1

Since $0.0243 < 0.05$

There is sufficient evidence at the 5% significance level to reject the null hypothesis that the mean weight of the crisps is 25kg and accept that the bags mean weight is less than 25kg.

MW3

13

Solution2: Using tables.

(i) $P\left(Z > \frac{24.5-25}{0.6}\right)$

$= P(Z > -0.83333)$

$= \Phi(0.8333)$

$= 0.798$

Probability that both packets of crisp is greater than 24.5kg

$= (0.79767)^2 = 0.636$

MW1

M1

W1

MW1

(ii)

$H_0: \mu = 25$

$H_1: \mu < 25$ (one-tail)

Reject H_0 if $z < -1.645$

M1

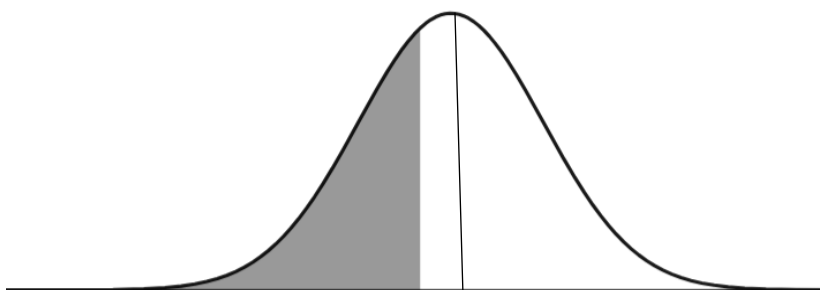
MW1

M1

Weight is distributed normally with mean

$\mu = 25$ and $\sigma = 0.6$

$P(W < 24.8)$



24.8 25

$$\bar{z} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\bar{z} = \frac{24.8 - 25}{\frac{0.6}{\sqrt{35}}}$$

M1MW1

$$= -1.972..$$

W1

Since $-1.972 < -1.645$

There is sufficient evidence at the 5% level to reject the null hypothesis that the mean weight of the crisps is 25kg and accept that the bag's mean weight is less than 25kg.

MW3

- 9(i) p is the probability that it rains in any day in April in Northern Ireland.
 $X \sim \text{Bin}(10, 0.4)$ MW1
 $H_0: p = 0.4$ MW1
 $H_1: p \neq 0.4$ (Two tailed) MW1

- (ii) This is a two tailed test hence we consider 5% significance level at top and bottom.

Using Calculator:

Top

x	6	7	8	9
$P(X \leq x)$	0.9452	0.9877	0.9983	0.9998

consider $P(X \geq 7) = 1 - P(X \leq 6) = 0.0548 = 5.48\%$ M1W1
 $P(X \geq 8) = 1 - P(X \leq 7) = 0.0123 = 1.23\%$ W1

Bottom

x	0	1	2	3
$P(X \leq x)$	0.006	0.0463	0.1672	0.3822

consider $P(X \leq 0) = 0.006 = 0.6\%$
 $P(X \leq 1) = 0.0463 = 4.63\%$
 $P(X \leq 2) = 0.1672 = 16.72\%$ MW2

Critical region 0, 1, 8, 9, 10 MW2

- (iii) Seven is not in the critical region. There is insufficient evidence at the 10% significance level to reject the null hypothesis that the probability is 0.4 that it will rain in any day in April. MW3

13

10 (i) $P(A|C) = \frac{3}{8}$ $P(A' \cap C) = \frac{1}{4}$ $P(A \cap C) = x$

$$P(A|C) = \frac{P(A \cap C)}{P(C)}$$

$$\frac{3}{8} = \frac{x}{P(C)}$$

$$P(C) = P(A' \cap C) + P(A \cap C) = \frac{1}{4} + x$$

Hence

$$\frac{3}{8} = \frac{x}{x + \frac{1}{4}}$$

$$3\left(x + \frac{1}{4}\right) = 8x$$

$$x = 0.15$$

(ii)

	<i>C</i>	<i>C'</i>	Total
<i>A</i>	15	24	39
<i>A'</i>	25	36	61
Total	40	60	100

AVAILABLE
MARKS

M1

W1

M1

M1

W1

MW2

7

Total 100