GCE

Revised GCE

Mathematics

Assessment Unit A2 1

assessing

Pure Mathematics

Practice Paper and Mark Scheme

For first teaching from September 2018
For first award of AS Level in Summer 2019
For first award of A Level in Summer 2019

Centre Number

Candidate Number

ADVANCED
General Certificate of Education

Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

[AMT11]

Practice Paper

TIME
2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all twelve** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.
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INFORMATION FOR CANDIDATES
The total mark for this paper is 150
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is ln z where it is noted that $\ln z \equiv \log_e z$

1. A bowl is formed by rotating the arc of the curve

$$y = \sqrt{5x}$$

between $x = 0$ and $x = a$, where a is a positive constant, through 2π radians about the x-axis.

The bowl is full of water.

Find, in terms of a, the volume of water in the bowl. [6]

2. A sector of a circle of radius r metres contains an angle of θ radians at its centre.

The sector has perimeter 2.4 m and area 0.36 m^2

Find r and θ. [11]

3. (i) Show that the equation $e^x = x^2 + 2$ has a solution between $x = 1$ and $x = 2$ [4]

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(ii) By taking $x = 1$ as a first approximation and using the Newton-Raphson method twice, find a better approximation to the solution of the equation [6]

$$e^x = x^2 + 2$$

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4. **Fig. 1** below shows the curves $y = ax - x^2$ and $y = x^2$, where a is a positive constant.

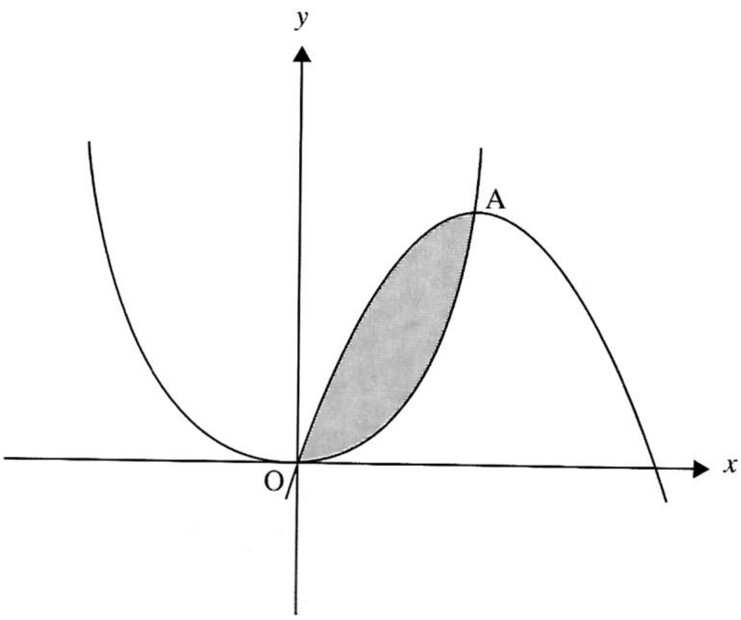

Fig. 1

The curves intersect at the origin and at the point A.

(i) Show that the x coordinate of A is $\frac{a}{2}$ [3]

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(ii) Hence, find the shaded area bounded by the curves [7]

$$y = ax - x^2 \text{ and } y = x^2$$

5. (a) A sequence is defined by

$$u_{n+1} = 2bu_n \quad u_1 = 6 \quad (n = 1, 2, 3, 4 \ldots)$$

(i) Find u_2 and u_3 in terms of b. [3]

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(ii) Find the range of values of b for which the series [3]

$u_1 + u_2 + u_3 + u_4 + \ldots\ldots$ converges.

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(b) (i) Prove that the sum of n terms of a geometric progression with first term a and constant ratio r is

$$S_n = \frac{a(1-r^n)}{1-r}$$ [6]

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The first four terms of a geometric series are

$$0.43, \quad 0.0043, \quad 0.000043, \quad 0.00000043, \ldots$$

(ii) By finding the sum to infinity of the series, express the recurring decimal

$$0.43434343\ldots$$

as a fraction in its simplest form. [5]

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6. (a) Use the binomial theorem to expand

$$(1 + x + x^2)^{-1}$$

in ascending powers of x up to and including the term in x^3 [7]

(a) Use the binomial theorem to expand

(b) Solve the equation

$$3\cos 2\theta = \sin(2\theta + 30°)$$

where $0° \leq \theta \leq 360°$ [9]

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7. The graph of a function $y = \mathrm{f}(x)$ is sketched in **Fig. 2** below.

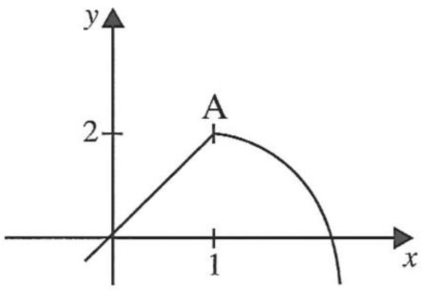
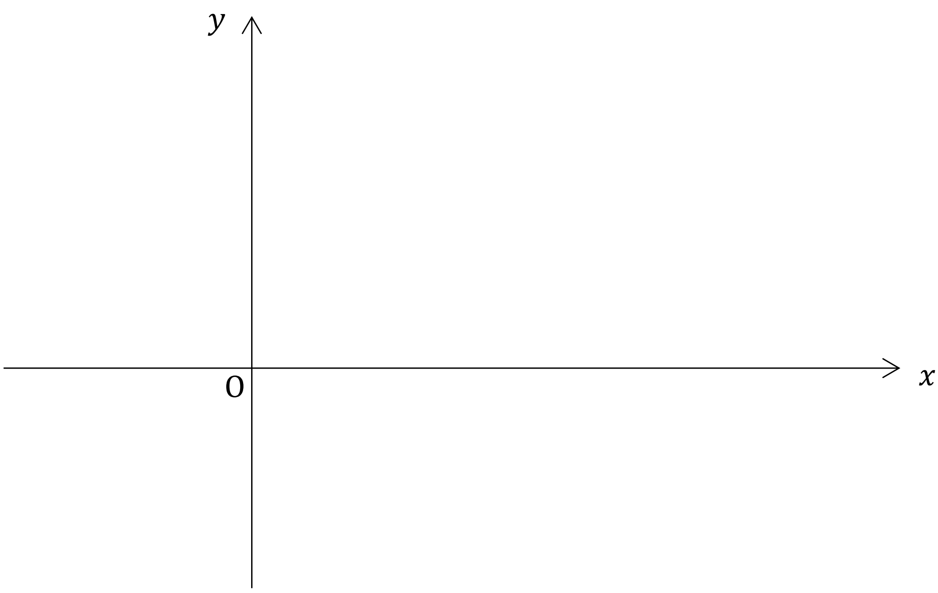


Fig. 2

(i) On the axes below sketch the graph of

$$y = 3\mathrm{f}\left(\frac{1}{2}x\right)$$

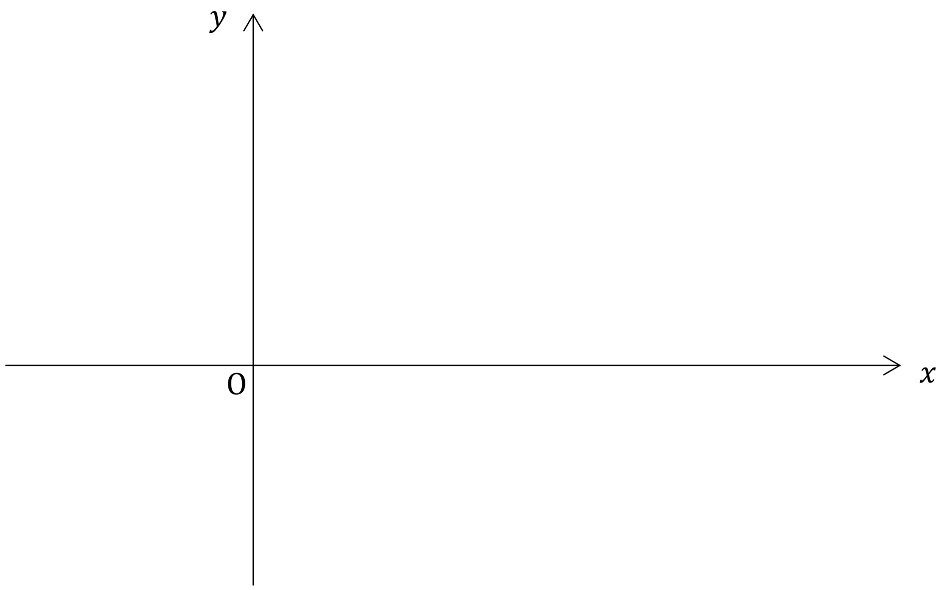
and clearly label the image of A. [2]



(ii) On the axes below sketch the graph of

$$y = 4 - \mathrm{f}(x)$$ [2]

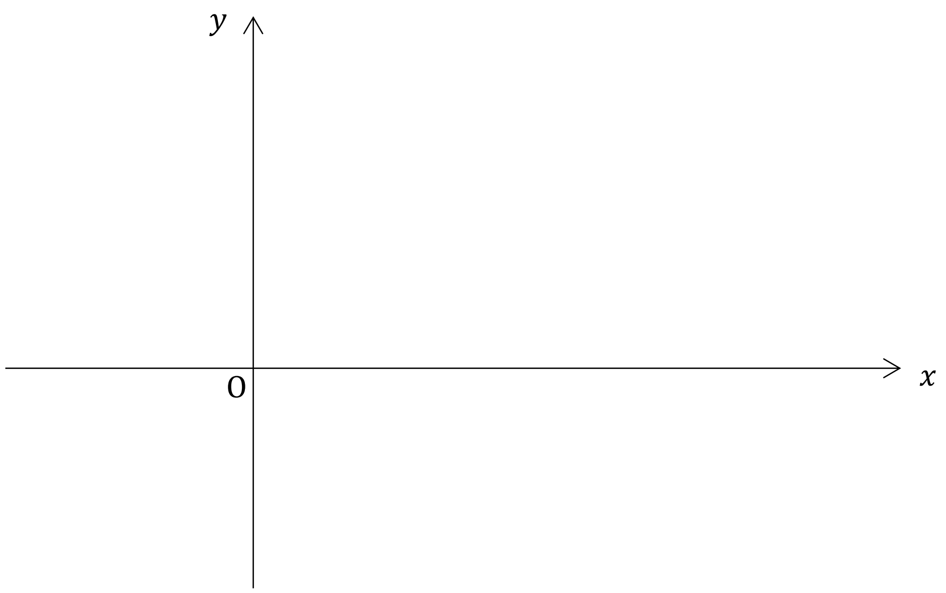
and clearly label the image of A.



(iii) On the axes below sketch the graph of

$$y = |\mathrm{f}(x) - 1|$$

and clearly label the image of A. [3]



8. (a) Differentiate and simplify each of the following:

(i) $5x \ln(x^2 - 2)$ [5]

(ii) $\dfrac{\sin x}{\cos 3x}$ [5]

(b) A curve is defined by

$$3x^2 + xy - 2y^2 = 0$$

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y + 6x}{4y - x}$$ [7]

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9. (a) Using the substitution $u = 2 + x$ find

$$\int x(2+x)^{10}\mathrm{d}x$$

[8]

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(b) Find the exact value of

$$\int_0^{\frac{\pi}{4}} 8x \cos 2x \, \mathrm{d}x$$

[7]

(b) Find the exact value of

10. Given the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3y(x+1)^2$$

and that $x = -1$ when $y = 16$, express y in terms of x.

[10]

11. (a) Find the **exact** solutions of the equation

$$\tan^2 \theta + 2 \sec^2 \theta = 3$$

in the range $-\pi < \theta < \pi$ [6]

(b) Prove the identity

$$\frac{\sec\theta - \cos\theta}{\operatorname{cosec}\theta - \sin\theta} \equiv \tan^3\theta$$

[9]

12. The expression $2\sin x + 2\sqrt{3}\cos x$ can be written in the form $r\sin(x+\alpha)$, where r is an integer and $0 \leq \alpha \leq \frac{\pi}{2}$

(i) Find the values of r and α.

[6]

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The function f is defined as

$$\mathrm{f} : x \to 2\sin x + 2\sqrt{3}\cos x \qquad 0 \leq x \leq 2\pi$$

(ii) State the range of f. [1]

(iii) Using a sketch of f, or otherwise, explain why f does not have an inverse. [2]

The function g is defined as

$$g : x \to 2\sin x + 2\sqrt{3}\cos x \qquad a \le x \le b$$

where $0 \le a < b \le 2\pi$

The range of g is the same as the range of f and the inverse function g^{-1} exists. [2]

(iv) Write down the values of a and b.

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(v) Find the inverse function g^{-1} [5]

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THIS IS THE END OF THE QUESTION PAPER

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PRACTICE PAPER

MARK SCHEME

		AVAILABLE MARKS
1. $V = \pi \int y^2 \mathrm{d}x$	M1	
$= \pi \int_0^a 5x \mathrm{d}x$	M1 W2	
$= \pi \left[\frac{5x^2}{2}\right]_0^a$	W1	
$= \frac{5\pi a^2}{2}$	W1	6
2. $2r + r\theta = 2.4$	M2 W1	
$\frac{1}{2}r^2\theta = 0.36$	M1 W1	
Substituting for θ		
$2r + r \times \frac{0.36 \times 2}{r^2} = 2.4$	M1 W1	
$2r^2 - 2.4r + 0.72 = 0$	W1	
$25r^2 - 30r + 9 = 0$		
$(5r - 3)(5r - 3) = 0$		
$r = 0.6$ m	W1	
$\theta = \frac{0.36 \times 2}{r^2}$	M1	
$\theta = 2$	W1	11
3. (i) $x^2 + 2 - \mathrm{e}^x = 0$	M1	
$x = 1 \quad 1^2 + 2 - \mathrm{e}^1 = 0.282$	MW1	
$x = 2 \quad 2^2 + 2 - \mathrm{e}^2 = -1.39$	MW1	
Since the curve is continuous between $x = 1$ and $x = 2$ and there is a change of sign, therefore there is a root between $x = 1$ and $x = 2$	MW1	
(ii) $\mathrm{f}(x) = x^2 + 2 - \mathrm{e}^x$		
$\mathrm{f}'(x) = 2x - \mathrm{e}^x$	MW2	
$x_0 = 1$		
$x_1 = x_0 - \frac{\mathrm{f}(x)}{\mathrm{f}'(x)}$	M1	
$x_1 = 1 - \frac{1 + 2 - \mathrm{e}^1}{2 - \mathrm{e}^1}$		
$= 1.392211191 \ldots$	W1	
$x_2 = 1.392 \ldots - \frac{(1.392 \ldots)^2 + 2 - \mathrm{e}^{1.392\ldots}}{2(1.392 \ldots) - \mathrm{e}^{1.392\ldots}}$	M1	
$= 1.32$ (3sf)	W1	10

			AVAILABLE MARKS
4.	(i) $ax - x^2 = x^2$	M1	
	$2x^2 - ax = 0$		
	$x(2x - a) = 0$	MW1	
	$x = 0, \quad x = \frac{a}{2}$		
	A has x coordinate $\frac{a}{2}$	MW1	
	(ii) Area $= \int_0^{\frac{a}{2}} (ax - x^2 - x^2)\mathrm{d}x$	M2 W2	
	$= \int_0^{\frac{a}{2}} (ax - 2x^2)\mathrm{d}x$		
	$= \left[\frac{ax^2}{2} - \frac{2x^3}{3}\right]_0^{\frac{a}{2}}$	MW2	
	$= \frac{a^3}{8} - \frac{a^3}{12}$		
	$= \frac{a^3}{24}$	W1	10
5.	(a) (i) $u_2 = 2bu_1 = 12b$	M1 W1	
	$u_3 = 2bu_2 = 24b^2$	MW1	
	(ii) $r = 2b$	MW1	
	$\lvert 2b \rvert < 1$	M1	
	$\lvert b \rvert < \frac{1}{2}$		
	$-\frac{1}{2} < b < \frac{1}{2}$	W1	
	(b) (i) $S_n = a + ar + ar^2 + ar^3 + \cdots ar^{n-2} + ar^{n-1}$	MW1	
	$rS_n = ar + ar^2 + ar^3 + ar^4 + \cdots ar^{n-1} + ar^n$	M1 W1	
	$S_n - rS_n = a - ar^n$	M1 W1	
	$(1 - r)S_n = a(1 - r^n)$		
	$S_n = \frac{a(1 - r^n)}{1 - r}$	MW1	
	(ii) $a = 0.43 \quad r = 0.01$		
	$S_\infty = \frac{a}{1 - r} = \frac{0.43}{1 - 0.01}$	MW2 M1 W1	
	$= \frac{0.43}{0.99}$		
	$= \frac{43}{99}$	W1	17

AVAILABLE MARKS

6. (a) $(1 + x + x^2)^{-1}$

$= \{1 + (x + x^2)\}^{-1}$ M1 W1

$= 1 - (x + x^2) + \frac{(-1)(-2)}{2!}(x + x^2)^2 + \frac{(-1)(-2)(-3)}{3!}(x + x^2)^3 + \cdots$ MW3

$= 1 - x - x^2 + x^2 + 2x^3 + x^4 - x^3 + \cdots$

$= 1 - x + x^3 + \cdots$ W2

(b) $3\cos 2\theta = \sin(2\theta + 30°)$

$3\cos 2\theta = \sin 2\theta \cos 30° + \cos 2\theta \sin 30°$ M1 W1

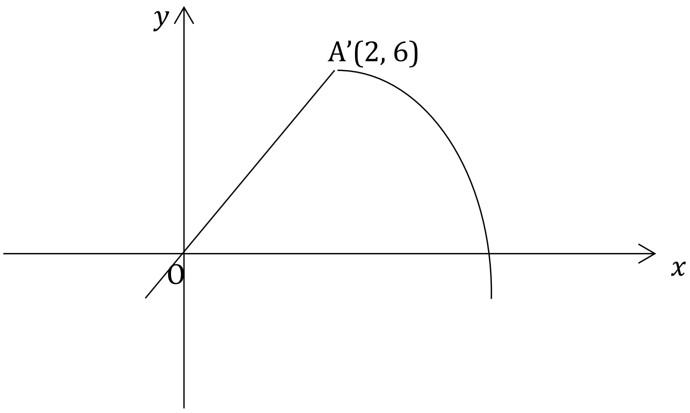
$\cos 2\theta\,(3 - \sin 30°) = \sin 2\theta \cos 30°$ MW1

$\tan 2\theta = \frac{3 - \sin 30°}{\cos 30°} = \frac{5\sqrt{3}}{3}$ M1 W1

$2\theta = 70.89°, 250.89°, 430.89°, 610.89°$ M1 W1

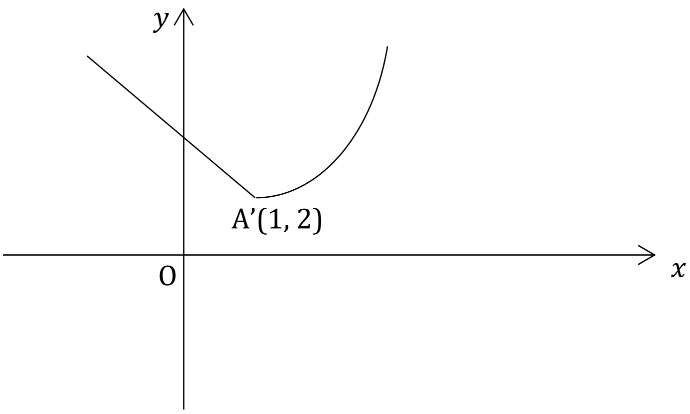
$\theta = 35.4°, 125°, 215°, 305°$ MW2 16

7. (i)



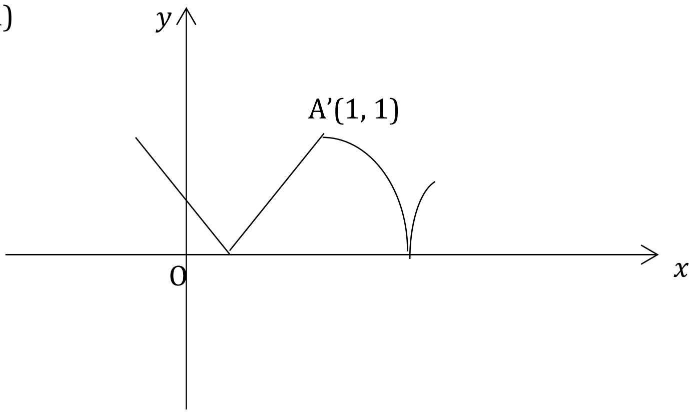
MW2

(ii)



MW2

(iii)



MW3

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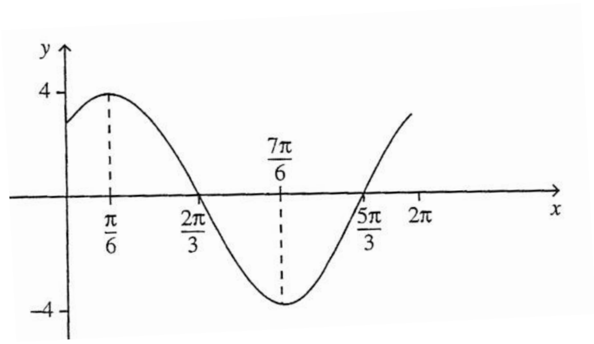
		AVAILABLE MARKS
8. (a) (i) $y = 5x\ln(x^2-2)$		
$u = 5x \qquad v = \ln(x^2-2)$		
$\frac{\mathrm{d}u}{\mathrm{d}x} = 5 \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{2x}{x^2-2}$	MW2	
$\frac{\mathrm{d}y}{\mathrm{d}x} = 5x\left(\frac{2x}{x^2-2}\right) + 5\ln(x^2-2)$	M1 W1	
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{10x^2}{x^2-2} + 5\ln(x^2-2)$	W1	
(ii) $y = \frac{\sin x}{\cos 3x}$		
$u = \sin x \qquad v = \cos 3x$		
$\frac{\mathrm{d}u}{\mathrm{d}x} = \cos x \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = -3\sin 3x$	MW2	
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos 3x\,(\cos x) - \sin x\,(-3\sin 3x)}{\cos^2 3x}$	M1 W1	
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos 3x\cos x + 3\sin x\sin 3x}{\cos^2 3x}$	W1	
(b) $3x^2 + xy - 2y^2 = 0$		
$6x + y + x\frac{\mathrm{d}y}{\mathrm{d}x} - 4y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1MW4	
$(4y - x)\frac{\mathrm{d}y}{\mathrm{d}x} = y + 6x$	M1	
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y+6x}{4y-x}$	W1	17

		AVAILABLE MARKS
9. (a) $u = 2 + x$		
$\frac{\mathrm{d}u}{\mathrm{d}x} = 1$	M1 W1	
$\int x\,(2+x)^{10}\mathrm{d}x = \int (u-2)\,u^{10}.1.\mathrm{d}u$	M1 W1	
$= \int (u^{11} - 2u^{10})\mathrm{d}u$	M1	
$= \frac{u^{12}}{12} - \frac{2u^{11}}{11} + c$	W2	
$= \frac{(2+x)^{12}}{12} - \frac{2(2+x)^{11}}{11} + c$	MW1	
(b) $u = 8x$ $\frac{\mathrm{d}v}{\mathrm{d}x} = \cos 2x$	M1	
$\frac{\mathrm{d}u}{\mathrm{d}x} = 8$ $v = \frac{1}{2}\sin 2x$	MW2	
$\int_0^{\frac{\pi}{4}} 8x\cos 2x\,\mathrm{d}x = \left[8x\left(\frac{1}{2}\sin 2x\right)\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 8\left(\frac{1}{2}\sin 2x\right)\mathrm{d}x$	M1 W1	
$= [4x\sin 2x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 4\sin 2x\,\mathrm{d}x$		
$= [4x\sin 2x]_0^{\frac{\pi}{4}} - [-2\cos 2x]_0^{\frac{\pi}{4}}$	MW1	
$= \pi - 2$	W1	15
10. $\frac{\mathrm{d}y}{\mathrm{d}x} = 3y(x+1)^2$		
$\int \frac{\mathrm{d}y}{y} = \int 3(x+1)^2\mathrm{d}x$	M1 MW2	
$\ln y = (x+1)^3 + c$	MW2	
$x = -1, y = 16$		
$\ln 16 = (-1+1)^3 + c$	M1	
$c = \ln 16$	W1	
$\ln y = (x+1)^3 + \ln 16$		
$\ln\left(\frac{y}{16}\right) = (x+1)^3$	M1 W1	
$y = 16\mathrm{e}^{(x+1)^3}$	MW1	10

		AVAILABLE MARKS
11. (a) $\tan^2\theta + 2(\tan^2\theta + 1) = 3$	M1 W1	
$3\tan^2\theta = 1$		
$\tan\theta = \pm\frac{1}{\sqrt{3}}$	M1 W1	
$\theta = \pm\frac{\pi}{6}, \pm\frac{5\pi}{6}$	MW2	
(b) $\frac{\sec\theta - \cos\theta}{\operatorname{cosec}\theta - \sin\theta} \equiv \tan^3\theta$		
Starting with LHS		
$\equiv \frac{\frac{1}{\cos\theta} - \cos\theta}{\frac{1}{\sin\theta} - \sin\theta} \times \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$	MW2 M1	
$\equiv \frac{\sin\theta - \sin\theta\cos^2\theta}{\cos\theta - \sin^2\theta\cos\theta}$	W1	
$\equiv \frac{\sin\theta\,(1 - \cos^2\theta)}{\cos\theta\,(1 - \sin^2\theta)}$	M1 W1	
$\equiv \frac{\sin\theta\sin^2\theta}{\cos\theta\cos^2\theta}$	MW1	
$\equiv \frac{\sin^3\theta}{\cos^3\theta}$	W1	
$\equiv \tan^3\theta$	MW1	15

		AVAILABLE MARKS
12. (i) $2\sin x + 2\sqrt{3}\cos x \equiv r(\sin x\cos\alpha + \cos x\sin\alpha)$	MW1	
$2 = r\cos\alpha \qquad 2\sqrt{3} = r\sin\alpha$	M1	
$\tan\alpha = \dfrac{2\sqrt{3}}{2}$	M1	
$\alpha = \dfrac{\pi}{3}$	W1	
$r = \sqrt{2^2 + (2\sqrt{3})^2}$	M1	
$r = 4$	W1	
(ii) $-4 \le f(x) \le 4$	MW1	
(iii) 	W1	
Since f is a many-to-one function, then the inverse of f does not exist.	MW1	
(iv) $a = \frac{\pi}{6}$, $b = \frac{7\pi}{6}$	MW2	
(v) $y = 4\sin\left(x + \frac{\pi}{3}\right)$	M1	
$\dfrac{y}{4} = \sin\left(x + \dfrac{\pi}{3}\right)$	W1	
$x + \dfrac{\pi}{3} = \sin^{-1}\left(\dfrac{y}{4}\right)$	MW1	
$x = \sin^{-1}\left(\dfrac{y}{4}\right) - \dfrac{\pi}{3}$	W1	
$g^{-1}: x \to \sin^{-1}\left(\dfrac{x}{4}\right) - \dfrac{\pi}{3} \qquad -4 \le x \le 4$	W1	16
	Total	150

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