Summer 2021

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Extra Assessment Resources: GCE Maths
AS1 Supplementary Questions and Mark Scheme

Version 1.0

Supplementary Questions by Grade

Mathematics Unit AS1

Notes

1. A question designated as e.g. E grade indicates that a typical E grade candidate should achieve all/almost all marks in this question.

2. Since these questions have been selected from Legacy examination papers, the mark allocations may show some slight variation from the current specification. This should not have any effect on the grade designation.

Grade E

1. Find $\frac{dy}{dx}$ when

$$y = 2x^3 - 3x^2 + 4x - 6 \qquad [4]$$

2. Find the equation of the tangent to the curve

$$y = 6 - x - x^2$$

at the point where $x = 2$ [6]

3. The points A and B have coordinates (3, 2) and (9, 5) respectively.

(i) Find the gradient of AB. [2]

(ii) Find the equation of the line through A perpendicular to AB. [3]

4. Solve the simultaneous equations

$$2x + y + 2z = 6$$
$$4x - y + 2z = 13$$
$$2x - 2y - z = 3 \qquad [7]$$

5. The diagram in **Fig. 1** below shows the graph of the function $y = f(x)$

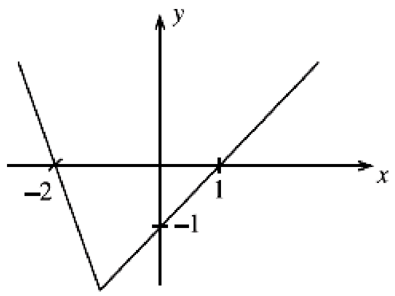

Fig. 1

Sketch, on separate diagrams, the graphs of:

(i) $y = -f(x)$; [2]

(ii) $y = f(x - 2)$, [2]

6. Find the centre and radius of the circle whose equation is

$$x^2 + y^2 - 4x + 6y = 12$$ [3]

7. Express

$$\log_3 6 - \log_3 14 + 2 \log_3 7$$

as a single logarithm. [6]

Grade C

1. **(i)** Use the factor theorem to show that $(x - 1)$ is a factor of

$$9x^3 - 9x^2 - x + 1$$ [2]

(ii) Hence fully factorise

$$9x^3 - 9x^2 - x + 1$$ [3]

2. Find the range of values of k for which the quadratic equation

$$kx^2 + 4x + (k - 3) = 0$$

has no real roots. [6]

3. **(i)** Find how many real roots the equation $x^2 + 5 = 4x$ has. [3]

(ii) Express $x^2 - 4x + 5$ in the form $(x - a)^2 + b$ [3]

(iii) **Hence write down** the minimum value of $x^2 - 4x + 5$ and the value of x at which it occurs. [2]

4. Find the coordinates of the points of intersection of the curve $y = x^2 - 5x$ and the line $x + y = 12$ [6]

5. **Fig. 2** below shows the curve $y = 4x - 2x^2$ and the line $y = -x + 2$

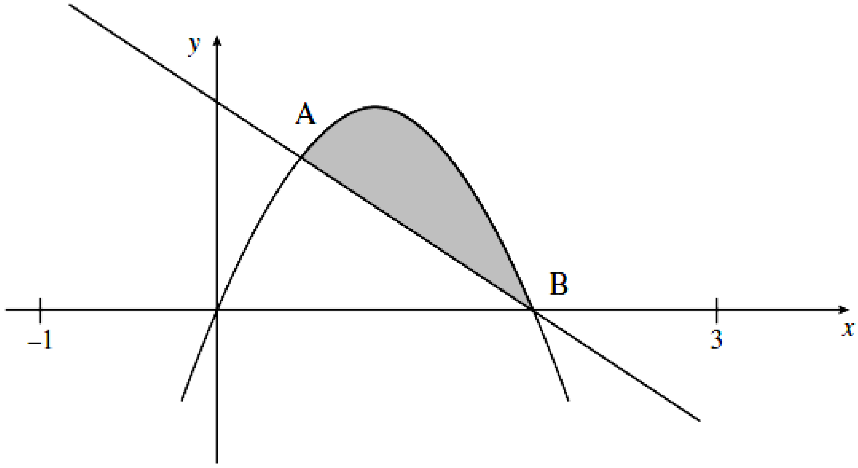

Fig. 2

The curve and line intersect at points A and B.

(i) Verify that the x coordinates of A and B are $\frac{1}{2}$ and 2 [3]

(ii) Find the shaded area enclosed between this curve and line. [10]

Grade A

1. A container for tennis balls is to be made. It consists of an open cylinder with a circular base and a lid in the shape of a hemisphere, as shown in **Fig. 4** below.

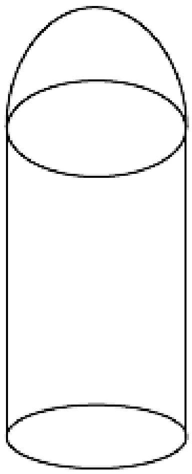

Fig. 4

Sphere	Volume = $\frac{4}{3}\pi r^3$	Area = $4\pi r^2$
Cylinder	Volume = $\pi r^2 h$	C.S. Area = $2\pi rh$

The cylinder has radius r cm and height h cm.
The hemisphere has radius r cm.
The volume of the container must be $\frac{320\pi}{3}$ cm^3

(i) Express h in terms of r [4]

(ii) Hence show that the total surface area of the container is

$$\frac{5}{3}\pi r^2 + \frac{640}{3}\pi r^{-1}$$ [4]

(iii) Find the value of r that makes the total surface area of the container minimum. [7]

2. Solve the following equations for x and y

$$3^x \times 3^{2y} = 27$$

and

$$\frac{8^x}{4^{2y}} = 16$$

[8]

3. A curve has equation

$$y = 2x^2 + 5$$

A straight line has equation

$$y = mx - 3$$

The line intersects the curve at two points.

Find the range of possible values of m [8]

4. **(a)** Prove the identity

$$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}$$ [5]

(b) Solve the equation

$$\sin^2 x = \frac{1}{4}$$

for $-90° < x \leqslant 90°$ [4]

5. The network coverage of a mobile phone mast M may be modelled as a circle as shown in **Fig. 2** below.

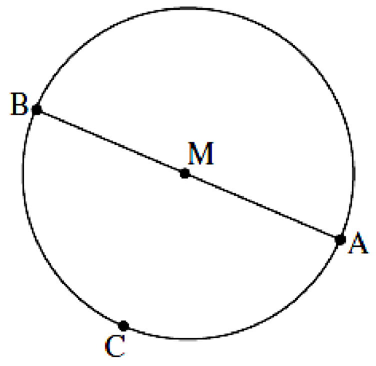

Fig. 2

Points A (2, 1), B(k, k + 5) and C (–1, –1) lie on the circumference of the circle, centre M. AB is a diameter of the circle.

(i) Find the gradient of AC. [2]

(ii) Hence, write down the gradient of BC and **prove** that $k = -3$ [4]

(iii) Find the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ [5]

Supplementary Questions by Grade

Mathematics Unit AS1

Markscheme

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Grade E

1. $\frac{dy}{dx} = 6x^2 - 6x + 4$ MW4

Total = 4

2. $\frac{dy}{dx} = -1 - 2x$ M1W2

$x = 2 \Rightarrow m = -5$ MW1

$x = 2 \Rightarrow y = 0$

$(y - 0) = -5(x - 2)$ M1W1

$y = -5x + 10$

Total = 6

3. **(i)** $m = \frac{5-2}{9-3} = \frac{1}{2}$ M1W1

(ii) $m_{\perp} = -2$ MW1

$(y - 2) = -2(x - 3)$ M1W1

$y + 2x = 8$

Total = 5

4.
$$\begin{array}{r} 4x - y + 2z = 13 \\ 2x + y + 2z = 6 \\ \hline 2x - 2y = 7 \end{array}$$
M1W1

$$\begin{array}{r} 4x - y + 2z = 13 \\ 4x - 4y - 2z = 6 \\ \hline 8x - 5y = 19 \end{array}$$
MW1

$$\begin{array}{r} 8x - 8y = 28 \\ 8x - 5y = 19 \\ \hline -3y = 9 \end{array}$$
M1

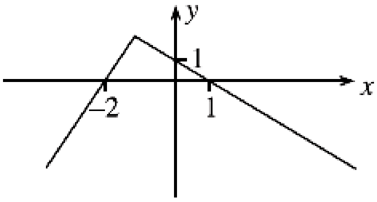
$y = -3$ W1

$x = \frac{1}{2}$ MW1

$z = 4$ MW1

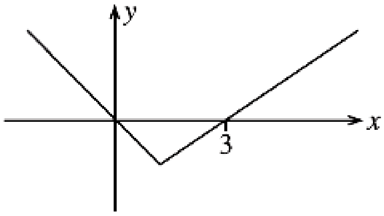
Total = 7

5. (i)



M1W1

(ii)



M1W1

<u>Total = 4</u>

6. $x^2 + y^2 - 4x + 6y = 12$

$(x - 2)^2 - 4 + (y + 3)^2 - 9 = 12$ M1

$(x - 2)^2 + (y + 3)^2 = 25$

Centre $(2, -3)$ Radius $= 5$ W2

<u>Total = 3</u>

7. $\log_3 6 - \log_3 14 + 2\log_3 7$

$= \log_3 \dfrac{6}{14} + 2\log_3 7$ M1W1

$= \log_3 \dfrac{6}{14} + \log_3 7^2$ M1W1

$= \log_3 \dfrac{6 \times 49}{14}$ M1

$= \log_3 21$ W1

<u>Total = 6</u>

Grade C

1. **(i)** $f(1) = 9 - 9 - 1 + 1 = 0 \quad \therefore (x - 1)$ is a factor — M1W1

(ii)

$$\begin{array}{r} 9x^2 - 1 \\ x - 1 \overline{)\, 9x^3 - 9x^2 - x + 1} \\ \underline{9x^3 - 9x^2} \quad\quad\quad \\ -x + 1 \\ \underline{-x + 1} \\ 0 \end{array}$$

M1W1

$(x - 1)(9x^2 - 1) = (x - 1)(3x - 1)(3x + 1)$ — MW1

<u>Total = 5</u>

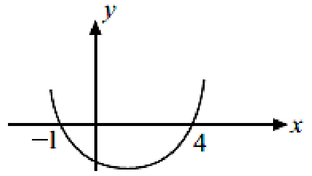
2. $16 - 4k(k - 3) < 0$ — M1W1

$16 - 4k^2 + 12k < 0$ — MW1

$k^2 - 3k - 4 > 0$ — MW1

$(k + 1)(k - 4)$

$k = -1$ or $k = 4$ — W1



$k < -1$ or $k > 4$ — MW1

<u>Total = 6</u>

3. **(i)** $x^2 - 4x + 5 = 0$ — MW1

$b^2 - 4ac = 16 - 20 = -4 \therefore$ no real solns — M1W1

(ii) $x^2 - 4x + 5 = (x - 2)^2 - 4 + 5 = (x - 2)^2 + 1$ — M1W2

(iii) Min value = 1, when $x = 2$ — MW2

<u>Total = 8</u>

4. $x + x^2 - 5x = 12$ — M1

$x^2 - 4x - 12 = 0$ — W1

$(x - 6)(x + 2) = 0$

$x = 6 \quad x = -2$ — W2

$y = 6 \quad y = 14$

$(6, 6) \quad (-2, 14)$W2

<u>Total = 6</u>

5. **(i)** Filling into both line and curve for one of the values of x given — M1

If $x = \frac{1}{2}$, $y = \frac{3}{2}$ for both line and curve — W1

If $x = 2$, $y = 0$ for both line and curve — W1

Alternative

$y = 4x - 2x^2$ and $y = -x + 2$

$4x - 2x^2 = -x + 2$ — M1

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = \frac{1}{2}, 2$ — W2

(ii) Area $= \int_{0.5}^{2} 4x - 2x^2 \, dx$ — M1W1W1

$= \left[2x^2 - \frac{2}{3}x^3\right]_{0.5}^{2}$ — W2

$= \left(2 \times 2^2 - \frac{2}{3} \times 2^3\right) - \left(2 \times 0.5^2 - \frac{2}{3} \times 0.5^3\right)$

$= \frac{8}{3} - \frac{5}{12} = \frac{27}{12} = \frac{9}{4} = 2.25$ — W1

Area of triangle $= \frac{1}{2} \times 1.5 \times 1.5 = 1.125$ — MW1MW1

Area subtracted $= 2.25 - 1.125 = 1.125$

$= 1.13$ — M1W1

<u>Total = 13</u>

Grade A

1.	(i)	$V = \pi r^2 h + \frac{2}{3}\pi r^3 = \frac{320\pi}{3}$	M1W2
		$3r^2 h = 320 - 2r^3$	
		$h = \frac{320 - 2r^3}{3r^2}$	MW1
	(ii)	$SA = 2\pi r^2 + 2\pi r h + \pi r^2$	MW1
		$SA = 3\pi r^2 + \frac{2\pi r(320 - 2r^3)}{3r^2}$	M1W1
		$SA = \frac{5}{3}\pi r^2 + \frac{640}{3}\pi r^{-1}$	MW1
	(iii)	$\frac{dA}{dr} = \frac{10}{3}\pi r - \frac{640}{3}\pi r^{-2}$	M1W2
		$\frac{10}{3}r = \frac{640}{3}r^{-2}$	M1
		$r^3 = \frac{640}{10} = 64$	
		$r = 4$	
			W1
		$\frac{d^2A}{dr^2} = \frac{10\pi}{3} + \frac{1280}{3}\pi r^{-3} \Rightarrow +\text{ve} \Rightarrow \text{min}$	M1
			MW1

Total = 15

2.	$3^{x+2y} = 3^3$	M1 W1
	Equating indices $x + 2y = 3$ ①	MW1
	$\frac{2^{3x}}{2^{4y}} = 2^4$	MW1
	Equating indices $3x - 4y = 4$ ②	MW1
	$2 \times$ ① $2x + 4y = 6$	M1
	$5x = 10$	
	$x = 2$	W1
	$y = \frac{1}{2}$	W1

Total = 8

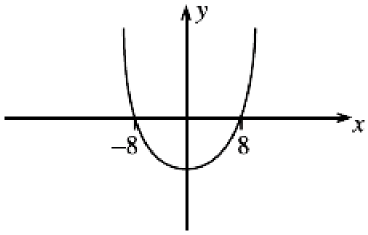
3. $mx - 3 = 2x^2 + 5$ MW1

$2x^2 - mx + 8 = 0$ MW1

$b^2 - 4ac > 0$ M1MW1

$m^2 - 64 > 0$ W1

$(m - 8)(m + 8) > 0$ W1



M1

$m > 8$ or $m < -8$ W1

Total = 8

4. **(a)** $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$

L.H.S. $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ M1W1

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$ MW1M1

$= \dfrac{1}{\sin\theta\ \cos\theta}$ W1

(b) $\sin^2 x = \frac{1}{4}$

$\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{2}$ MW2

$x = 30°$ or $x = -30°$ MW2

Total = 9

5. **(i)** Gradient AC $= \frac{1-(-1)}{2-(-1)} = \frac{2}{3}$ M1W1

(ii) Gradient BC $= -\frac{3}{2}$ MW1

Gradient BC $= \frac{(k+5)-(-1)}{k-(-1)} = \frac{k+6}{k+1} = \frac{-3}{2}$ M1W1

$$2(k+6) = -3(k+1)$$
$$2k+12 = -3k-3$$
$$5k = -15$$
$$k = -3$$ MW1

(iii) Coordinates of B = (–3, 2)

Centre $\frac{(2+(-3))}{2}, \frac{(1+2)}{2} = (-0.5, 1.5)$ MW2

Radius $= \sqrt{(2--0.5)^2 + (1-1.5)^2} = \sqrt{(2.5)^2 + (-0.5)^2} = \sqrt{6.5}$ M1W1

Equation $(x+0.5)^2 + (y-1.5)^2 = 6.5$ MW1

<u>Total = 11</u>

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