GCE

Rewarding Learning

Revised GCE

Elaboration Document

Further Mathematics

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1.1 Unit AS 1: Pure Mathematics

This unit, which assumes knowledge of Unit AS 1 of GCE Mathematics, covers the pure content of AS Further Mathematics. It is compulsory for both AS and A level Further Mathematics. The unit is assessed by a 1 hour 30 minute external examination, with 6–10 questions worth 100 raw marks.

Content and Learning Outcomes	Guidance
Further algebra and functions Students should be able to: • demonstrate understanding of and use the relationship between roots and coefficients of quadratic equations; • form a quadratic equation whose roots are related to the roots of a given quadratic equation.	e.g. Given a quadratic polynomial with roots α and β students should be able to evaluate expressions such as : (i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$ (iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (iv) $(2+\alpha)(2+\beta)$
Complex numbers Students should be able to: • solve any quadratic equation with real coefficients; • solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics); • add, subtract, multiply and divide complex numbers in the form $x + \mathrm{i}y$ with x and y real;	
• demonstrate understanding of and use the terms 'real part' and 'imaginary part'; • demonstrate understanding of and use the complex conjugate;	Students should understand and be able to use the terms 'modulus' and 'argument'.
• demonstrate understanding of and use the fact that non-real roots of polynomial equations with real coefficients occur in conjugate pairs;	Students should know that if z_1 is a root of $f(z) = 0$ then $z_1{}^*$ is also a root.
• use and interpret Argand diagrams; • demonstrate understanding of and use radian measure for angles; and • demonstrate understanding of and use the modulus-argument form of a complex number.	Students should be able to represent the sum or difference of two complex numbers on an Argand diagram.

Content and Learning Outcomes	Guidance
Complex numbers (continued) Students should be able to: • convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed); • multiply and divide complex numbers in modulus-argument form; • construct and interpret simple loci in the Argand diagram such as $\lvert z-a\rvert=r$, $\lvert z-a\rvert=\lvert z-b\rvert$ and $\arg\ (z-a)\ =\theta$	May include simple regions of the form $\lvert z-a\rvert< r$ etc.
Matrices Students should be able to: • add, subtract and multiply conformable matrices; • multiply a matrix by a scalar; • demonstrate understanding of and use zero and identity matrices; • use matrices to represent linear transformations in 2D; • demonstrate understanding of and use matrices to represent successive transformations; • find invariant points and lines for a linear transformation in 2D; • calculate determinants of 2 × 2 and 3 × 3 matrices; • interpret the determinant of a 2 × 2 matrix as the scale factor of a linear transformation; • demonstrate understanding of the implication of the zero value of the determinant of a simple 2 × 2 transformation matrix; • demonstrate understanding of and use singular and non-singular matrices; and • demonstrate understanding of and use the properties of inverse matrices.	

Content and Learning Outcomes	Guidance
Matrices (continued) Students should be able to: • calculate and use the inverse of a non-singular 2 × 2 or 3 × 3 matrix; • solve systems of simultaneous linear equations in two or three variables, including use of the inverse of the coefficient matrix to solve a system of equations with a unique solution and determination of the solution set when the solution exists but is not unique; • demonstrate understanding of the significance of the zero value of the determinant of the coefficient matrix of a system of simultaneous linear equations; • interpret geometrically the solution and failure of solution of three simultaneous linear equations. **Vectors** Students should be able to: • use vectors in three dimensions (including **i**, **j** and **k** unit vectors); • demonstrate understanding of and use the vector and Cartesian forms of an equation of a straight line in 3D; • demonstrate understanding of and use the vector and Cartesian forms of the equation of a plane; • calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane; • check whether vectors are perpendicular by using the scalar product; • find the intersection of two lines or a line and a plane; • demonstrate understanding of and work with skew lines; • calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane; and • find the equation of the line of intersection of two planes.	

Content and Learning Outcomes	Guidance
Vectors (continued) Students should be able to: • calculate the vector product of two vectors, including link to a 3 × 3 determinant (vector triple product is excluded); • demonstrate understanding of and use the properties of the vector product; and • interpret $\lvert \mathbf{a}\times\mathbf{b} \rvert$ as an area and $\mathbf{a}.\ \mathbf{b}\times\mathbf{c}$ as a volume.	

1.2 Unit AS 2: Applied Mathematics

This unit, which assumes knowledge of Units AS 1 and AS 2 of GCE Mathematics and Unit AS 1 of CCEA GCE Further Mathematics, covers the applied content of AS Further Mathematics and is compulsory for both AS and A level Further Mathematics. The unit is assessed by a 1 hour 30 minute external examination, with 6–10 questions worth 100 raw marks. The unit has four sections and students must answer questions from two of the four sections (A and B **or** A and C **or** A and D **or** C and D):

- Section A: Mechanics 1;
- Section B: Mechanics 2 (also assumes knowledge of Section A of this unit);
- Section C: Statistics; and
- Section D: Discrete and Decision Mathematics.

Each section is worth 50 percent to the assessment of the unit.

Section A: Mechanics 1

Content and Learning Outcomes	Guidance
Hooke's law Students should be able to: • use Hooke's law as a model, relating the force in an elastic string or spring to the extension or compression, and understand the term 'modulus of elasticity'; • demonstrate understanding of and use the modelling assumptions in problems involving the application of Hooke's law, including familiarity with the idea of elastic limits.	
Work and energy Students should be able to: • calculate work done by a force when its point of application undergoes a displacement (including use of the scalar product);	
• calculate the work done by a variable force, where the force is given as a simple function of displacement: Work $=\int_a^b F\mathrm{d}x$; and	$F(x)$ will be a simple function within the confines of the Mathematics Unit AS 1 content.
• demonstrate understanding of the concepts of kinetic energy, gravitational potential energy and elastic potential energy, and use the formulae to calculate these.	

Content and Learning Outcomes	Guidance
Work and energy (continued) Students should be able to: • demonstrate understanding of and use the relationship between the change in energy of a system and the work done by the external forces, and use the Principle of Conservation of Mechanical Energy in appropriate cases. **Power** Students should be able to: • use the definition of power as the rate at which a force does work (leading to $P = Fv$) and the rate of increase of energy; • solve problems involving power, including vehicles in motion and pumps raising and ejecting water; **Circular motion** Students should be able to: • demonstrate understanding of the concept of angular speed for a particle moving in a circle, and use the relation $v = r\omega$ • demonstrate understanding that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae: $a = r\omega^2$ and $a = \frac{v^2}{r}$ • solve problems that can be modelled by the motion of a particle moving in a horizontal circle with constant speed, including the conical pendulum and banked corners (but excluding sliding or overturning problems).	To include Work – Energy principle.

Section B: Mechanics 2

Content and Learning Outcomes	Guidance
Further particle equilibrium Students should be able to: • solve more complex problems involving particle equilibrium, including those involving a particle attached to elastic strings or springs on a rough plane.	

Content and Learning Outcomes	Guidance
Resultant and relative velocity Students should be able to: • solve problems involving resultant velocity (using graphical or vector component method); • solve problems involving relative velocity, including problems involving minimum distance or interception, but not involving course for closest approach. **Further circular motion** Students should be able to: • solve problems involving motion in a vertical circle, including proofs of standard results. **Gravitation** Students should be able to: • demonstrate understanding of and use the universal law of gravitation; • solve problems involving satellite motion. **Dimensions** Students should be able to: • check expressions and equations for dimensional consistency; and • derive equations connecting physical quantities where a product relationship is assumed.	

Section C: Statistics

Content and Learning Outcomes	Guidance
Sampling Students should be able to: • demonstrate understanding of the techniques of: – simple random sampling; – stratified sampling; – opportunity sampling; – quota sampling; and – cluster sampling; • select and critique sampling techniques in the context of solving statistical problems, including describing the advantages and disadvantages associated with the different techniques. **Probability** Students should be able to: • demonstrate understanding of and enumerate the arrangements of r objects from n objects, including the use of the formula: $nP_r = \frac{n!}{(n-r)!}$	

Content and Learning Outcomes	Guidance
Probability (continued) Students should be able to: • demonstrate understanding of and enumerate the combinations of r objects from n objects, including the use of the binomial coefficients $n\text{C}r$ • demonstrate understanding of and enumerate arrangements and combinations with repetitions and/or restrictions; • evaluate probabilities in simple cases using permutations and combinations.	
Statistical distributions Students should be able to: • demonstrate understanding of and use the geometric distribution as a model, including the calculation of probabilities using the geometric distribution; • demonstrate understanding of and use discrete probability distributions, including: – probability functions; – mean; – variance; and – standard deviation; • calculate probabilities such as $\text{P}\ (a \le X \le b)$, $\text{E}\ (X)$ and $\text{Var}\ (X)$ for simple cases of a discrete random variable X • understand and use continuous probability distributions, including: – probability density functions; – mean; – variance; and – standard deviation;	
• calculate probabilities such as $\text{P}(a < X < b)$, $\text{E}(X)$ and $\text{Var}(X)$ for a continuous random variable X, where the probability density function of X is given as a simple function of x; and • understand and use the expressions for $\text{E}\ (aX + b)$ and $\text{Var}(aX + b)$, where X is a discrete or continuous random variable.	The probability density function will be a simple function within the confines of the Mathematics Unit AS 1 content.

Content and Learning Outcomes	Guidance
Statistical distributions (continued) Students should be able to: • demonstrate understanding of and use the Poisson distribution as a model, including the calculation of probabilities using the Poisson distribution; • use the expressions for the mean and variance of the binomial, geometric and Poisson distributions;	Including examples where the parameter λ changes e.g. if X = the number of events per day and $X \sim \mathrm{Po}(\lambda)$, then the number of events per week $\sim \mathrm{Po}(7\lambda)$
Bivariate distributions Students should be able to: • calculate the product-moment correlation coefficient and understand its use, interpretation and limitations; • demonstrate understanding of explanatory (independent) and response (dependent) variables; • calculate the equation of a regression line using the method of least squares; • use the equation of the regression line to make predictions within the range of the explanatory variable; and • demonstrate understanding of the dangers of extrapolation.	

Section D: Discrete and Decision Mathematics

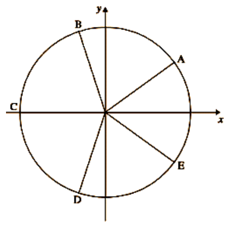
Content and Learning Outcomes	Guidance
Group theory Students should be able to: • recall that a group consists of a set of elements together with a binary operation which is closed and associative, for which an identity exists in the set and for which every element has an inverse in the set; • use the basic group properties to show that a given structure is, or is not, a group (questions may be set on, for example, symmetry groups, permutation groups, groups of 2 × 2 matrices or the group of residue classes mod m); • recall the meaning of the term 'order of a group'; • determine the period of elements in a given group; • demonstrate understanding of the idea of a subgroup of a group, find subgroups in simple cases and show that given subsets are, or are not, (proper) subgroups; • recall and apply Lagrange's theorem concerning the order of a subgroup of a finite group (the proof of the theorem is not required); • demonstrate understanding of the meaning of the term 'cyclic' as applied to groups; • use the term 'generator' in relation to cyclic groups; • demonstrate understanding of the idea of isomorphism between groups and determine whether given groups are, or are not, isomorphic.	
Graph theory Students should be able to: • demonstrate understanding of and use the basic concepts of graph theory, including vertex, edge, degree, planarity and subgraph; • demonstrate understanding of certain basic graphs, including the complete graph on n vertices (K_n), the complete bipartite graph ($K_{m,n}$) and the star on n vertices (S_n).	A graph consists of points (vertices or nodes) which are connected by lines (edges or arcs).

Content and Learning Outcomes	Guidance
Graph theory (continued) Students should be able to: • demonstrate understanding of and use the traversability of graphs including the terms 'circuits', 'Eulerian circuits' and 'Hamiltonian paths', and basic conditions necessary for their existence; • demonstrate understanding of and deal with weighted edges and digraphs; • demonstrate understanding of and use the basic concepts associated with trees: root, connectedness, binary tree and spanning tree.	A walk is a sequence of vertices and edges, arranged alternatively. A walk is closed if it starts and ends at the same vertex. A path is a walk with no repeated vertices. A cycle is a closed path. A Hamiltonian cycle is a cycle in which every vertex is included. A trail is a walk with no repeated edges. A circuit is a closed trail. An Eulerian circuit is a circuit in which every edge is included i.e. a closed walk that uses every edge exactly once.
Algorithms on graphs Students should be able to: • demonstrate understanding of the definition of an algorithm, including the term 'greedy algorithm'; • solve problems involving critical path analysis, including: – a precedence table for an activity network; – event times and float times; and – an algorithm for finding the critical path; • recall and use Prim's algorithm to find a minimal spanning tree for a connected weighted graph; • recall binary trees and traversing them using breadth first search and depth first search; • recall and use Dijkstra's algorithm to find a shortest path.	Students should be familiar with the terms early event time, late event time, float time and critical activities.
Recurrence Relationships Students should be able to: • demonstrate understanding of and apply the basic structure of recurrence models, namely a recurrence relation together with initial conditions; • solve homogenous, constant coefficient and linear recurrence relations, including Fibonacci-type relations.	Students should be familiar with the terms auxiliary (characteristic) equation, complementary function and particular solution.
Boolean algebra Students should be able to: • use truth tables to prove the equivalence of propositional statements (involving no more than three variables).	Students should be familiar with set notation. The notation $\sim A$ will be used to represent "not A"

1.3 Unit A2 1: Pure Mathematics

This unit, which assumes knowledge of GCE Mathematics Units AS 1 and A2 7 and Unit AS 1 of GCE Further Mathematics, covers the pure content of A2 Further Mathematics and is compulsory for A level Further Mathematics. The unit is assessed by a 2 hour 15 minute external examination, with 7–12 questions worth 150 raw marks.

Content and Learning Outcomes	Guidance
Proof Students should be able to: • construct proofs using mathematical induction (contexts may, for example, include sums of series, divisibility and powers of matrices).	
Further algebra and functions Students should be able to: • decompose rational functions into partial fractions (including denominators with quadratic factors); • demonstrate understanding of and use formulae for the sums of integers, squares and cubes and use these to sum other series; • demonstrate understanding of and use the method of differences for summation of series including use of partial fractions; • find the Maclaurin series of a function, including the general term; • recognise and use the Maclaurin series for e^x, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$ and be aware of the range of values of x for which they are valid; • derive the series expansions of simple compound functions; • demonstrate understanding of and use the standard small angle approximations of sine, cosine and tangent $\sin x \approx x$ $\cos x \approx 1 - \frac{x^2}{2}$ $\tan x \approx x$ where x is in radians.	e.g. Use standard series to find $\sum_{r=1}^{r=n} r(r^2+5)$ Use partial fractions to find $\sum_{r=1}^{r=n} \frac{1}{r(r+3)}$ Use differences to find $\sum_{r=1}^{r=n} \frac{2r+1}{r^2(r+1)^2}$

Content and Learning Outcomes	**Guidance**
Complex numbers Students should be able to: • demonstrate understanding of De Moivre's theorem and use it to find multiple angle formulae and sums of series; • demonstrate understanding of and use the definition $e^{i\theta} = \cos\theta + \mathrm{i}\sin\theta$ and the form $z = re^{i\theta}$ • find the n^{th} roots of $re^{i\theta}$ for $r \neq 0$ and demonstrate knowledge that they form the vertices of a regular *n*-gon in the Argand diagram; • use complex roots of unity to solve geometric problems.	To include use of $z + \frac{1}{z} = 2\cos\theta$ and $z - \frac{1}{z} = 2\mathrm{i}\sin\theta$ in finding $\cos n\theta$, $\sin n\theta, \tan n\theta$ in terms of powers of sin, cos, tan and also in finding powers of sin, cos, tan in terms of multiple angles. e.g. (i) Show that $\sin 5\theta \equiv 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ (ii) Find a, b, c, d where $\cos^6\theta \equiv a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d$ (iii) Find, in the form $r\,e^{i\theta}$, the values of the 5 roots of the equation $z^5 + 32 = 0$, which are shown in Fig. 1 below  Fig. 1
Further calculus • evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity; • integrate using partial fractions (extend to include quadratic factors in the denominator); • differentiate inverse trigonometric functions; • integrate functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 + x^2)^{-1}$ and choose trigonometric substitutions to integrate associated functions;	e.g. $\int_0^9 \frac{1}{\sqrt{x}}\,\mathrm{d}x$, $\int_2^\infty \frac{1}{x^2}\,\mathrm{d}x$

Content and Learning Outcomes	Guidance
Further calculus (continued)	
• use repeated integration by parts; • demonstrate understanding of and use simple reduction formulae in integration.	e.g. Given that $I_n = \int_0^{\frac{\pi}{2}} x^n \cos x \, dx$ show that for $n \geq 2$ $I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$
Polar co-ordinates	
Students should be able to:	
• demonstrate understanding of and use polar co-ordinates and convert between polar and Cartesian co-ordinates;	
• sketch curves with r given as a function of θ (including use of trigonometric functions);	
• find the area enclosed by a polar curve;	To include use of the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$
Hyperbolic functions	
Students should be able to:	
• demonstrate understanding of the definitions of hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$, including their domains and ranges, and sketch their graphs;	
• differentiate and integrate hyperbolic functions;	e.g. differentiate $\sinh 5x$, $x^2 \cosh^2 x$, $\frac{\tanh 2x}{x+3}$
• demonstrate understanding of and use the definitions of the inverse hyperbolic functions and their domains and ranges;	
• derive and use the logarithmic forms of the inverse hyperbolic functions;	
• integrate functions of the form $(x^2 + a^2)^{-\frac{1}{2}}$ and $(x^2 - a^2)^{-\frac{1}{2}}$ and choose substitutions to integrate associated functions;	To include derivatives of $\sinh^{-1}x$, $\cosh^{-1}x$ and $\tanh^{-1}x$

Content and Learning Outcomes	Guidance
Differential equations Students should be able to: • find and use an integrating factor to solve differential equations of the form $\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{P}\,(x)\,\mathrm{y} = \mathrm{Q}\,(x)$ and recognise when it is appropriate to do so; • find both general and particular solutions to differential equations; • use differential equations in modelling. • solve differential equations of the form $y'' + ay' + by = 0$ where a and b are constants, by using the auxiliary equation; • solve differential equations of the form $y'' + ay' + by = f(x)$ where a and b are constants, by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $\mathrm{f}(x)$ is a polynomial, exponential or trigonometric function, but not a solution of the corresponding homogeneous equation); • demonstrate understanding of and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of the solution of the differential equation.	The integrating factor $e^{\int P\mathrm{d}x}$ may be used without proof.

1.4 Unit A2 2: Applied Mathematics

This unit, which assumes knowledge of all GCE Mathematics units and Units AS 1 and A2 1 of GCE Further Mathematics, covers the applied content of A2 Further Mathematics and is compulsory for A level Further Mathematics. The unit is assessed by a 2 hour 15 minute external examination, with 7–12 questions worth 150 raw marks. The unit has four sections and students must answer questions from two of the sections (A and B **or** A and C **or** A and D **or** C and D):

- Section A: Mechanics 1 (also assumes knowledge of Section A of Unit AS 2);
- Section B: Mechanics 2 (also assumes knowledge of Section A of Unit AS 2 and Section A of this unit);
- Section C: Statistics (also assumes knowledge of Section C of Unit AS 2); and
- Section D: Discrete and Decision Mathematics (also assumes knowledge of Section D of Unit AS 2).

Each section is worth 50 percent to the assessment of the unit.

Section A: Mechanics 1

Content and Learning Outcomes	Guidance
Simple harmonic motion Students should be able to: • use the definition of and standard results for simple harmonic motion; • solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and relate the solution to the motion; • solve problems involving the simple pendulum, including a seconds pendulum; • solve problems involving oscillations of a particle attached to the end of an elastic spring or string (oscillations will be in the direction of the spring or string).	
Damped oscillations Students should be able to: • model damped oscillations using 2nd order differential equations and interpret their solutions.	Students should be familiar with damping constant (coefficient), dashpot, critical damping, over-damping and under-damping.
Centre of mass Students should be able to: • demonstrate understanding of the concept of centre of mass; • find the centre of mass of systems of particles at fixed points and rods (including use of symmetry but excluding use of calculus or variable density problems).	

Content and Learning Outcomes	Guidance
Centre of mass (continued) Students should be able to: • find the centre of mass of rectangular, triangular and circular laminae; • find the centre of mass of a composite lamina; • solve problems involving suspended laminae. **Frameworks** Students should be able to: • solve problems involving light pin-jointed frameworks (use of Bow's notation is optional; questions may involve identifying forces as tension or thrust as well as calculating their magnitude. **Further circular motion** Students should be able to: • solve more complex problems involving motion on banked corners, including questions on sliding or overturning.	

Section B: Mechanics 2

Content and Learning Outcomes	Guidance
Further kinematics Students should be able to: • solve problems involving kinematics in three dimensions, including use of calculus and **i**, **j** and **k** unit vectors; • solve problems involving variable acceleration along a straight line, where acceleration is given as a function of time, velocity or displacement (including examples involving constant power). **Further centre of mass** Students should be able to: • find the centre of mass of laminae and solids, including the use of calculus (proof of standard results for solid cone and solid hemisphere only may be required; table of standard results may be used); • find the centre of mass of composite bodies; • solve problems involving suspended bodies; • solve sliding/toppling problems. • solve sliding/toppling problems.	The use of calculus may include any of that contained in AS & A2 Mathematics Units 1 and AS & A2 Further Mathematics Units 1

Content and Learning Outcomes	Guidance
Force systems in two dimensions • find the general resultant of a system of coplanar forces; • solve problems involving the replacement of a force system by a single force, by a couple or by a single force acting at a specific point together with a couple.	
Restitution Students should be able to: • demonstrate understanding of and use Newton's law of restitution; • solve problems involving direct elastic collisions between smooth spheres or between a smooth sphere and a fixed plane.	Questions involving impulsive tensions in strings will not be set.

Section C: Statistics

Content and Learning Outcomes	Guidance
Linear combinations of independent variables Students should be able to: • demonstrate understanding of and use the expressions for $\mathrm{E}(aX + bY)$ and $\mathrm{Var}(aX + bY)$, where X and Y are independent random variables; • solve problems involving linear combinations of independent normally distributed variables, including the expressions for the mean and variance of the sum of a number of independent observations from a given population; • demonstrate understanding of and use the distribution of a multiple of a single observation from a given population.	Proofs will not be required. Students are expected to know that if X and Y are independent variables such that $X \sim \mathrm{N}(\mu_x, {\sigma_x}^2)$ and $Y \sim \mathrm{N}(\mu_y, {\sigma_y}^2)$, then $\mathrm{a}X \pm \mathrm{b}Y \sim \mathrm{N}\left(\mathrm{a}\mu_x \pm \mathrm{b}\mu_y, \mathrm{a}^2{\sigma_x}^2 + \mathrm{b}^2{\sigma_y}^2\right)$
Sampling and estimation Students should be able to: • demonstrate understanding of and use the central limit theorem for samples of 30 or more observations; • calculate point estimates of the population mean and variance, including use of $S^2 = \frac{\sum(X_i - \bar{X})^2}{n-1}$ as an unbiased estimator of σ^2	

Content and Learning Outcomes	Guidance
Sampling and estimation **(continued)** Students should be able to: • demonstrate understanding of and use the standard error of the mean; • calculate confidence intervals for the population mean. **The *t*-distribution** Students should be able to: • demonstrate understanding of when it is appropriate to use the *t*-distribution; • carry out a hypothesis test for the population mean using a small sample drawn from a normally distributed variable; • formulate a hypothesis and carry out either a two-sample or paired-sample *t*-test (as appropriate) for the difference of the sample means and demonstrate understanding of the conditions for these tests to be valid.	Tables of percentage points for the *t* – distribution are included in the formula booklet. The formula for pooled variance will be given.
χ^2 tests Students should be able to: • fit a theoretical distribution, as prescribed by a given hypothesis, to given data (questions set will not involve lengthy calculations); • use a χ^2 test with the appropriate number of degrees of freedom to carry out the corresponding goodness of fit test (classes should be combined so that each expected frequency is at least 5); • use a χ^2 test with the appropriate number of degrees of freedom to test for independence in a contingency table (rows or columns, as appropriate, should be combined so that each expected frequency is at least 5, and Yates' correction should be used in the special case of a 2 × 2 table).	Tables of percentage points for the χ^2 – distribution are included in the formula booklet.

Section D: Discrete and Decision Mathematics

Content and Learning Outcomes	Guidance
Counting Students should be able to: • demonstrate understanding of and use the Principle of Inclusion and Exclusion (PIE), including the connection to Venn diagrams and the problem of derangements; • enumerate restricted positions, including the use of Rook polynomials. **Graph theory** Students should be able to: • demonstrate understanding of the concept of vertex colouring and edge colouring of graphs; • demonstrate understanding of cutsets and use the max-flow min-cut theorem; • demonstrate understanding of and work with bipartite graphs, including matchings and complete matchings; • demonstrate understanding of and use Hall's marriage theorem. **Algorithms on graphs** Students should be able to: • recall and use the nearest neighbour algorithm to construct a Hamiltonian cycle; • solve problems involving program evaluation and review technique (PERT), including: – precedence table for an activity network; – algorithm for finding the critical path; and – calculating the overall probability of completing a project by a certain time; • use the simplex algorithm and tableau to solve two variable linear programming problems. **Generating functions** Students should be able to: • demonstrate understanding of the meaning of a generating function; • formulate a generating function to solve simple summation problems; • use combinatorial arguments and elementary generating functions to prove simple formulae involving, for example, binomial coefficients.	

Content and Learning Outcomes	Guidance
Group theory Students should be able to: • demonstrate understanding of and work with symmetry groups, including the cyclic group (C_n), the dihedral group (D_{2n}) and the symmetry groups of the cube, octahedron and tetrahedron; • understand the concept of a cycle index $P_G(\times 1, \times 2, ..., \times n)$ • use a table of cycle indices for simple symmetry groups; • use Polya's Enumeration Theorem to enumerate colourings in simple symmetrical structures; • demonstrate understanding of the use of the pattern inventory $P_G((b+w), (b^2+w^2), ..., (b^n+w^n))$ for 2 colours and the similar result for 3 colours.	