GCE

Rewarding Learning

CCEA GCE Specification in

Further Mathematics

For first teaching from September 2018
For first award of AS level in Summer 2019
For first award of A level in Summer 2020
Subject Code: 2330

Contents

Subject Code	2330
QAN AS Level	603/1763/2
QAN A Level	603/1762/0

This specification is available online at www.ccea.org.uk

1 Introduction

This specification sets out the content and assessment details for our Advanced Subsidiary (AS) and Advanced (A level) GCE courses in Further Mathematics. First teaching is from September 2018.

Students can take:

- the AS course as a final qualification; or
- the AS units plus the A2 units for a full GCE A level qualification.

We assess the AS units at a standard appropriate for students who have completed the first part of the full course. A2 units have an element of synoptic assessment (to assess students' understanding of the subject as a whole), as well as more emphasis on assessment objectives that reflect higher order thinking skills.

The full Advanced GCE award is based on students' marks from the AS (40 percent) and the A2 (60 percent). The guided learning hours for this specification, as for all GCEs, are:

- 180 hours for the Advanced Subsidiary level award; and
- 360 hours for the Advanced level award.

We will make the first AS awards for the specification in 2019 and the first A level awards in 2020. The specification builds on the broad objectives of the Northern Ireland Curriculum.

If there are any major changes to this specification, we will notify centres in writing. The online version of the specification will always be the most up to date; to view and download this please go to www.ccea.org.uk

1.1 Aims

This specification aims to encourage students to:

- understand mathematics and mathematical processes in a way that promotes confidence, fosters enjoyment and provides a strong foundation for progress to further study;
- extend their range of mathematical skills and techniques;
- understand coherence and progression in mathematics and how different areas of mathematics are connected;
- apply mathematics in other fields of study and be aware of the relevance of mathematics to the world of work and to situations in society in general;
- use their mathematical knowledge to make logical and reasoned decisions in solving problems both within pure mathematics and in a variety of contexts, and communicate the mathematical rationale for these decisions clearly;
- reason logically and recognise incorrect reasoning;
- generalise mathematically;
- construct mathematical proofs;
- use their mathematical skills and techniques to solve challenging problems that require them to decide on the solution strategy;
- recognise when they can use mathematics to analyse and solve a problem in context;
- represent situations mathematically and understand the relationship between problems in context and mathematical models that they may apply to solve these;
- draw diagrams and sketch graphs to help explore mathematical situations and interpret solutions;
- make deductions and inferences and draw conclusions by using mathematical reasoning;
- interpret solutions and communicate their interpretation effectively in the context of the problem;
- read and comprehend mathematical arguments, including justifications of methods and formulae, and communicate their understanding;
- read and comprehend articles concerning applications of mathematics and communicate their understanding;
- use technology such as calculators and computers effectively, and recognise when such use may be inappropriate; and
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

1.2 Key features

The following are important features of this specification.

- It includes four externally assessed assessment units.
- Assessment at A2 includes more demanding question types and synoptic assessment that encourages students to develop their understanding of the subject as a whole.
- It gives students a sound basis for progression to higher education and to employment.
- A range of support is available, including specimen assessment materials.

1.3 Prior attainment

This specification assumes knowledge of GCSE Mathematics (Higher Tier) and GCE Mathematics.

1.4 Classification codes and subject combinations

Every specification has a national classification code that indicates its subject area. The classification code for this qualification is 2330.

Please note that if a student takes two qualifications with the same classification code, schools and colleges that they apply to may take the view that they have achieved only one of the two GCEs. The same may occur with any two GCE qualifications that have a significant overlap in content, even if the classification codes are different. Because of this, students who have any doubts about their subject combinations should check with the universities and colleges that they would like to attend before beginning their studies.

2 Specification at a Glance

The table below summarises the structure of the AS and A level courses:

Content	Assessment	Weightings
AS 1: **Pure Mathematics**	External written examination 1 hour 30 mins Students answer all questions.	50% of AS 20% of A level
AS 2: **Applied Mathematics**	External written examination 1 hour 30 mins Students answer all questions from their chosen sections.	50% of AS 20% of A level
A2 1: **Pure Mathematics**	External written examination 2 hours 15 mins Students answer all questions.	30% of A level
A2 2: **Applied Mathematics**	External written examination 2 hours 15 mins Students answer all questions from their chosen sections.	30% of A level

3 Subject Content

We have divided this course into four units: two units at AS level and two units at A2. This section sets out the content and learning outcomes for each unit.

The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the teaching of the units in this specification.

Calculators used must include:

- an iterative function;
- the ability to perform calculations with matrices up to at least order 3 × 3; and
- the ability to compute summary statistics and access probabilities from standard statistical distributions.

Students must not have access to technology with a computer algebra system function during examinations.

3.1 Overarching themes in GCE Further Mathematics

This GCE Further Mathematics specification gives students opportunities to demonstrate the following knowledge and skills. They must apply these, along with associated mathematical thinking and understanding, across the whole content of the AS and A2 units set out below.

AS and A level students should be able to:

- construct and present mathematical arguments through appropriate use of diagrams, sketching graphs, logical deduction, precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term and variable;
- understand and use mathematical language and syntax including equals, identically equals, therefore, because, implies, is implied by, necessary, sufficient, $\therefore$, $=$, $\equiv$, $\neq$, $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$;
- understand and use Venn diagrams, language and symbols associated with set theory, including complement, $\varnothing$, $\cap$, $\cup$, $\in$, $\notin$ and $\mathscr{E}$;
- comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics;
- recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to solve problems;
- construct extended arguments to solve problems presented in an unstructured form, including problems in context;
- interpret and communicate solutions in the context of the original problem;
- understand the concept of a problem-solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle;
- understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems;

- translate a situation in context into a mathematical model, making simplifying assumptions;
- use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student);
- interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student);
- understand that a mathematical model can be refined by considering its outputs and simplifying assumptions;
- evaluate whether a mathematical model is appropriate; and
- understand and use modelling assumptions.

A level students should also be able to understand and use the definition of a function, including domain and range of a function.

3.2 Unit AS 1: Pure Mathematics

This unit, which assumes knowledge of Unit AS 1 of GCE Mathematics, covers the pure content of AS Further Mathematics. It is compulsory for both AS and A level Further Mathematics. The unit is assessed by a 1 hour 30 minute external examination, with 6–10 questions worth 100 raw marks.

Content	Learning Outcomes
Further algebra and functions	Students should be able to: • demonstrate understanding of and use the relationship between roots and coefficients of quadratic equations; • form a quadratic equation whose roots are related to the roots of a given quadratic equation;
Complex numbers	• solve any quadratic equation with real coefficients; • solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics); • add, subtract, multiply and divide complex numbers in the form $x + \mathrm{i}y$ with x and y real; • demonstrate understanding of and use the terms 'real part' and 'imaginary part'; • demonstrate understanding of and use the complex conjugate; • demonstrate understanding of and use the fact that non-real roots of polynomial equations with real coefficients occur in conjugate pairs; • use and interpret Argand diagrams; • demonstrate understanding of and use radian measure for angles; and • demonstrate understanding of and use the modulus-argument form of a complex number.

Content	Learning Outcomes
Complex numbers (cont.)	Students should be able to: • convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed); • multiply and divide complex numbers in modulus-argument form; • construct and interpret simple loci in the Argand diagram such as $\|z-a\|=r$, $\|z-a\|=\|z-b\|$ and $\arg(z-a)=\theta$
Matrices	• add, subtract and multiply conformable matrices; • multiply a matrix by a scalar; • demonstrate understanding of and use zero and identity matrices; • use matrices to represent linear transformations in 2D; • demonstrate understanding of and use matrices to represent successive transformations; • find invariant points and lines for a linear transformation in 2D; • calculate determinants of 2 × 2 and 3 × 3 matrices; • interpret the determinant of a 2 × 2 matrix as the scale factor of a linear transformation; • demonstrate understanding of the implication of the zero value of the determinant of a simple 2 × 2 transformation matrix; • demonstrate understanding of and use singular and non-singular matrices; and • demonstrate understanding of and use the properties of inverse matrices.

Content	Learning Outcomes
Matrices (cont.)	Students should be able to: • calculate and use the inverse of a non-singular 2 × 2 or 3 × 3 matrix; • solve systems of simultaneous linear equations in two or three variables, including use of the inverse of the coefficient matrix to solve a system of equations with a unique solution and determination of the solution set when the solution exists but is not unique; • demonstrate understanding of the significance of the zero value of the determinant of the coefficient matrix of a system of simultaneous linear equations; • interpret geometrically the solution and failure of solution of three simultaneous linear equations;
Vectors	• use vectors in three dimensions (including **i**, **j** and **k** unit vectors); • demonstrate understanding of and use the vector and Cartesian forms of an equation of a straight line in 3D; • demonstrate understanding of and use the vector and Cartesian forms of the equation of a plane; • calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane; • check whether vectors are perpendicular by using the scalar product; • find the intersection of two lines or a line and a plane; • demonstrate understanding of and work with skew lines; • calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane; and • find the equation of the line of intersection of two planes.

Content	**Learning Outcomes**
Vectors (cont.)	Students should be able to: • calculate the vector product of two vectors, including link to a 3 × 3 determinant (vector triple product is excluded); • demonstrate understanding of and use the properties of the vector product; and • interpret $\lvert\mathbf{a}\times\mathbf{b}\rvert$ as an area and $\mathbf{a}.\,\mathbf{b}\times\mathbf{c}$ as a volume.

3.3 Unit AS 2: Applied Mathematics

This unit, which assumes knowledge of Units AS 1 and AS 2 of GCE Mathematics and Unit AS 1 of CCEA GCE Further Mathematics, covers the applied content of AS Further Mathematics and is compulsory for both AS and A level Further Mathematics. The unit is assessed by a 1 hour 30 minute external examination, with 6–10 questions worth 100 raw marks. The unit has four sections and students must answer questions from two of the four sections (A and B **or** A and C **or** A and D **or** C and D):

- Section A: Mechanics 1;
- Section B: Mechanics 2 (also assumes knowledge of Section A of this unit);
- Section C: Statistics; and
- Section D: Discrete and Decision Mathematics.

Each section is worth 50 percent to the assessment of the unit.

Section A: Mechanics 1

Content	Learning Outcomes
Hooke's law	Students should be able to: • use Hooke's law as a model, relating the force in an elastic string or spring to the extension or compression, and understand the term 'modulus of elasticity'; • demonstrate understanding of and use the modelling assumptions in problems involving the application of Hooke's law, including familiarity with the idea of elastic limits;
Work and energy	• calculate work done by a force when its point of application undergoes a displacement (including use of the scalar product); • calculate the work done by a variable force, where the force is given as a simple function of displacement: $\text{Work} = \int_a^b F \, \mathrm{d}x$ • demonstrate understanding of the concepts of kinetic energy, gravitational potential energy and elastic potential energy, and use the formulae to calculate these.

Content	Learning Outcomes
Work and energy (cont.)	Students should be able to: • demonstrate understanding of and use the relationship between the change in energy of a system and the work done by the external forces, and use the Principle of Conservation of Mechanical Energy in appropriate cases;
Power	• use the definition of power as the rate at which a force does work (leading to $P = Fv$) and the rate of increase of energy; • solve problems involving power, including vehicles in motion and pumps raising and ejecting water;
Circular motion	• demonstrate understanding of the concept of angular speed for a particle moving in a circle, and use the relation $v = r\omega$ • demonstrate understanding that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae: $a = r\omega^2$ and $a = \frac{v^2}{r}$ • solve problems that can be modelled by the motion of a particle moving in a horizontal circle with constant speed, including the conical pendulum and banked corners (but excluding sliding or overturning problems).

Section B: Mechanics 2

Content	Learning Outcomes
Further particle equilibrium	Students should be able to: • solve more complex problems involving particle equilibrium, including those involving a particle attached to elastic strings or springs on a rough plane; and
Resultant and relative velocity	• solve problems involving resultant velocity (using graphical or vector component method).

Content	Learning Outcomes
Resultant and relative velocity (cont.)	Students should be able to: • solve problems involving relative velocity, including problems involving minimum distance or interception, but not involving course for closest approach;
Further circular motion	• solve problems involving motion in a vertical circle, including proofs of standard results;
Gravitation	• demonstrate understanding of and use the universal law of gravitation; • solve problems involving satellite motion;
Dimensions	• check expressions and equations for dimensional consistency; and • derive equations connecting physical quantities where a product relationship is assumed.

Section C: Statistics

Content	Learning Outcomes
Sampling	Students should be able to: • demonstrate understanding of the techniques of: – simple random sampling; – stratified sampling; – opportunity sampling; – quota sampling; and – cluster sampling; • select and critique sampling techniques in the context of solving statistical problems, including describing the advantages and disadvantages associated with the different techniques; and
Probability	• demonstrate understanding of and enumerate the arrangements of r objects from n objects, including the use of the formula: $$nPr = \frac{n!}{(n-r)!}$$

Content	**Learning Outcomes**
Probability (cont.)	Students should be able to: • demonstrate understanding of and enumerate the combinations of r objects from n objects, including the use of the binomial coefficients $n\mathrm{C}r$ • demonstrate understanding of and enumerate arrangements and combinations with repetitions and/or restrictions; • evaluate probabilities in simple cases using permutations and combinations;
Statistical distributions	• demonstrate understanding of and use the geometric distribution as a model, including the calculation of probabilities using the geometric distribution; • demonstrate understanding of and use discrete probability distributions, including: – probability functions; – mean; – variance; and – standard deviation; • calculate probabilities such as $\mathrm{P}(a \le X \le b)$, $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ for simple cases of a discrete random variable X • understand and use continuous probability distributions, including: – probability density functions; – mean; – variance; and – standard deviation; • calculate probabilities such as $\mathrm{P}(a < X < b)$, $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ for a continuous random variable X, where the probability density function of X is given as a simple function of x; and • understand and use the expressions for $\mathrm{E}(aX + b)$ and $\mathrm{Var}(aX + b)$, where X is a discrete or continuous random variable.

Content	Learning Outcomes
Statistical distributions (cont.)	Students should be able to: • demonstrate understanding of and use the Poisson distribution as a model, including the calculation of probabilities using the Poisson distribution; • use the expressions for the mean and variance of the binomial, geometric and Poisson distributions;
Bivariate distributions	• calculate the product-moment correlation coefficient and understand its use, interpretation and limitations; • demonstrate understanding of explanatory (independent) and response (dependent) variables; • calculate the equation of a regression line using the method of least squares; • use the equation of the regression line to make predictions within the range of the explanatory variable; and • demonstrate understanding of the dangers of extrapolation.

Section D: Discrete and Decision Mathematics

Content	Learning Outcomes
Group theory	Students should be able to: • recall that a group consists of a set of elements together with a binary operation which is closed and associative, for which an identity exists in the set and for which every element has an inverse in the set; • use the basic group properties to show that a given structure is, or is not, a group (questions may be set on, for example, symmetry groups, permutation groups, groups of 2 × 2 matrices or the group of residue classes mod m); • recall the meaning of the term 'order of a group'; • determine the period of elements in a given group; • demonstrate understanding of the idea of a subgroup of a group, find subgroups in simple cases and show that given subsets are, or are not, (proper) subgroups; • recall and apply Lagrange's theorem concerning the order of a subgroup of a finite group (the proof of the theorem is not required); • demonstrate understanding of the meaning of the term 'cyclic' as applied to groups; • use the term 'generator' in relation to cyclic groups; • demonstrate understanding of the idea of isomorphism between groups and determine whether given groups are, or are not, isomorphic;
Graph theory	• demonstrate understanding of and use the basic concepts of graph theory, including vertex, edge, degree, planarity and subgraph; and • demonstrate understanding of certain basic graphs, including the complete graph on n vertices (K_n), the complete bipartite graph ($K_{m,n}$) and the star on n vertices (S_n).

Content	Learning Outcomes
Graph theory (cont.)	Students should be able to: • demonstrate understanding of and use the traversability of graphs including the terms ‘circuits’, ‘Eulerian circuits’ and ‘Hamiltonian paths’, and basic conditions necessary for their existence; • demonstrate understanding of and deal with weighted edges and digraphs; • demonstrate understanding of and use the basic concepts associated with trees: root, connectedness, binary tree and spanning tree;
Algorithms on graphs	• demonstrate understanding of the definition of an algorithm, including the term ‘greedy algorithm’; • solve problems involving critical path analysis, including: – a precedence table for an activity network; – event times and float times; and – an algorithm for finding the critical path; • recall and use Prim’s algorithm to find a minimal spanning tree for a connected weighted graph; • recall binary trees and traversing them using breadth first search and depth first search; • recall and use Dijkstra’s algorithm to find a shortest path;
Recurrence relationships	• demonstrate understanding of and apply the basic structure of recurrence models, namely a recurrence relation together with initial conditions; • solve homogenous, constant coefficient and linear recurrence relations, including Fibonacci-type relations; and
Boolean algebra	• use truth tables to prove the equivalence of propositional statements (involving no more than three variables).

3.4 Unit A2 1: Pure Mathematics

This unit, which assumes knowledge of GCE Mathematics Units AS 1 and A2 7 and Unit AS 1 of GCE Further Mathematics, covers the pure content of A2 Further Mathematics and is compulsory for A level Further Mathematics. The unit is assessed by a 2 hour 15 minute external examination, with 7–12 questions worth 150 raw marks.

Content	**Learning Outcomes**
Proof	Students should be able to: • construct proofs using mathematical induction (contexts may, for example, include sums of series, divisibility and powers of matrices);
Further algebra and functions	• decompose rational functions into partial fractions (including denominators with quadratic factors); • demonstrate understanding of and use formulae for the sums of integers, squares and cubes and use these to sum other series; • demonstrate understanding of and use the method of differences for summation of series including use of partial fractions; • find the Maclaurin series of a function, including the general term; • recognise and use the Maclaurin series for e^x, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$ and be aware of the range of values of x for which they are valid; • derive the series expansions of simple compound functions; • demonstrate understanding of and use the standard small angle approximations of sine, cosine and tangent $\sin x \approx x$ $\cos x \approx 1 - \frac{x^2}{2}$ $\tan x \approx x$ where x is in radians; and
Complex numbers	• demonstrate understanding of De Moivre's theorem and use it to find multiple angle formulae and sums of series.

Content	Learning Outcomes
Complex numbers (cont.)	Students should be able to: • demonstrate understanding of and use the definition $e^{i\theta} = \cos\theta + i\sin\theta$ and the form $z = re^{i\theta}$ • find the n^{th} roots of $re^{i\theta}$ for $r \neq 0$ and demonstrate knowledge that they form the vertices of a regular *n*-gon in the Argand diagram; • use complex roots of unity to solve geometric problems;
Further calculus	• evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity; • integrate using partial fractions (extend to include quadratic factors in the denominator); • differentiate inverse trigonometric functions; • integrate functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 + x^2)^{-1}$ and choose trigonometric substitutions to integrate associated functions; • use repeated integration by parts; • demonstrate understanding of and use simple reduction formulae in integration;
Polar co-ordinates	• demonstrate understanding of and use polar co-ordinates and convert between polar and Cartesian co-ordinates; • sketch curves with r given as a function of θ (including use of trigonometric functions); • find the area enclosed by a polar curve;
Hyperbolic functions	• demonstrate understanding of the definitions of hyperbolic functions $\sinh x$, $\cosh x$ and $\tanh x$, including their domains and ranges, and sketch their graphs; and • differentiate and integrate hyperbolic functions.

Content	Learning Outcomes
Hyperbolic functions (cont.)	Students should be able to: • demonstrate understanding of and use the definitions of the inverse hyperbolic functions and their domains and ranges; • derive and use the logarithmic forms of the inverse hyperbolic functions; • integrate functions of the form $(x^2+a^2)^{-\frac{1}{2}}$ and $(x^2-a^2)^{-\frac{1}{2}}$ and choose substitutions to integrate associated functions;
Differential equations	• find and use an integrating factor to solve differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{P}(x)y=\mathrm{Q}(x)$ and recognise when it is appropriate to do so; • find both general and particular solutions to differential equations; • use differential equations in modelling; • solve differential equations of the form $y''+ay'+by=0$ where a and b are constants, by using the auxiliary equation; • solve differential equations of the form $y''+ay'+by=\mathrm{f}(x)$ where a and b are constants, by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $\mathrm{f}(x)$ is a polynomial, exponential or trigonometric function, but not a solution of the corresponding homogeneous equation); and • demonstrate understanding of and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of the solution of the differential equation.

3.5 Unit A2 2: Applied Mathematics

This unit, which assumes knowledge of all GCE Mathematics units and Units AS 1 and A2 1 of GCE Further Mathematics, covers the applied content of A2 Further Mathematics and is compulsory for A level Further Mathematics. The unit is assessed by a 2 hour 15 minute external examination, with 7–12 questions worth 150 raw marks. The unit has four sections and students must answer questions from two of the sections (A and B **or** A and C **or** A and D **or** C and D):

- Section A: Mechanics 1 (also assumes knowledge of Section A of Unit AS 2);
- Section B: Mechanics 2 (also assumes knowledge of Section A of Unit AS 2 and Section A of this unit);
- Section C: Statistics (also assumes knowledge of Section C of Unit AS 2); and
- Section D: Discrete and Decision Mathematics (also assumes knowledge of Section D of Unit AS 2).

Each section is worth 50 percent to the assessment of the unit.

Section A: Mechanics 1

Content	Learning Outcomes
Simple harmonic motion	Students should be able to: • use the definition of and standard results for simple harmonic motion; • solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and relate the solution to the motion; • solve problems involving the simple pendulum, including a seconds pendulum; • solve problems involving oscillations of a particle attached to the end of an elastic spring or string (oscillations will be in the direction of the spring or string);
Damped oscillations	• model damped oscillations using 2^{nd} order differential equations and interpret their solutions;
Centre of mass	• demonstrate understanding of the concept of centre of mass; and • find the centre of mass of systems of particles at fixed points and rods (including use of symmetry but excluding use of calculus or variable density problems).

Content	Learning Outcomes
Centre of mass (cont.)	Students should be able to: • find the centre of mass of rectangular, triangular and circular laminae; • find the centre of mass of a composite lamina; • solve problems involving suspended laminae;
Frameworks	• solve problems involving light pin-jointed frameworks (use of Bow's notation is optional; questions may involve identifying forces as tension or thrust as well as calculating their magnitude); and
Further circular motion	• solve more complex problems involving motion on banked corners, including questions on sliding or overturning.

Section B: Mechanics 2

Content	Learning Outcomes
Further kinematics	Students should be able to: • solve problems involving kinematics in three dimensions, including use of calculus and **i**, **j** and **k** unit vectors; • solve problems involving variable acceleration along a straight line, where acceleration is given as a function of time, velocity or displacement (including examples involving constant power);
Further centre of mass	• find the centre of mass of laminae and solids, including the use of calculus (proof of standard results for solid cone and solid hemisphere only may be required; table of standard results may be used); • find the centre of mass of composite bodies; • solve problems involving suspended bodies; and • solve sliding/toppling problems.

Content	Learning Outcomes
Force systems in two dimensions	Students should be able to: • find the general resultant of a system of coplanar forces; • solve problems involving the replacement of a force system by a single force, by a couple or by a single force acting at a specific point together with a couple;
Restitution	• demonstrate understanding of and use Newton's law of restitution; and • solve problems involving direct elastic collisions between smooth spheres or between a smooth sphere and a fixed plane.

Section C: Statistics

Content	Learning Outcomes
Linear combinations of independent variables	Students should be able to: • demonstrate understanding of and use the expressions for $\mathrm{E}(aX + bY)$ and $\mathrm{Var}(aX + bY)$, where X and Y are independent random variables; • solve problems involving linear combinations of independent normally distributed variables, including the expressions for the mean and variance of the sum of a number of independent observations from a given population; • demonstrate understanding of and use the distribution of a multiple of a single observation from a given population;
Sampling and estimation	• demonstrate understanding of and use the central limit theorem for samples of 30 or more observations; and • calculate point estimates of the population mean and variance, including use of $S^2 = \frac{\Sigma(X_i - \bar{X})^2}{n-1}$ as an unbiased estimator of σ^2

Content	**Learning Outcomes**
Sampling and estimation (cont.)	Students should be able to: • demonstrate understanding of and use the standard error of the mean; • calculate confidence intervals for the population mean;
The *t*-distribution	• demonstrate understanding of when it is appropriate to use the *t*-distribution; • carry out a hypothesis test for the population mean using a small sample drawn from a normally distributed variable; • formulate a hypothesis and carry out either a two-sample or paired-sample *t*-test (as appropriate) for the difference of the sample means and demonstrate understanding of the conditions for these tests to be valid;
χ^2 tests	• fit a theoretical distribution, as prescribed by a given hypothesis, to given data (questions set will not involve lengthy calculations); • use a χ^2 test with the appropriate number of degrees of freedom to carry out the corresponding goodness of fit test (classes should be combined so that each expected frequency is at least 5); and • use a χ^2 test with the appropriate number of degrees of freedom to test for independence in a contingency table (rows or columns, as appropriate, should be combined so that each expected frequency is at least 5, and Yates' correction should be used in the special case of a 2 × 2 table).

Section D: Discrete and Decision Mathematics

Content	Learning Outcomes
Counting	Students should be able to: • demonstrate understanding of and use the Principle of Inclusion and Exclusion (PIE), including the connection to Venn diagrams and the problem of derangements; • enumerate restricted positions, including the use of Rook polynomials;
Graph theory	• demonstrate understanding of the concept of vertex colouring and edge colouring of graphs; • demonstrate understanding of cutsets and use the max-flow min-cut theorem; • demonstrate understanding of and work with bipartite graphs, including matchings and complete matchings; • demonstrate understanding of and use Hall's marriage theorem;
Algorithms on graphs	• recall and use the nearest neighbour algorithm to construct a Hamiltonian cycle; • solve problems involving program evaluation and review technique (PERT), including: – precedence table for an activity network; – algorithm for finding the critical path; and – calculating the overall probability of completing a project by a certain time; • use the simplex algorithm and tableau to solve two variable linear programming problems;
Generating functions	• demonstrate understanding of the meaning of a generating function; • formulate a generating function to solve simple summation problems; and • use combinatorial arguments and elementary generating functions to prove simple formulae involving, for example, binomial coefficients.

Content	**Learning Outcomes**
Group theory	Students should be able to: • demonstrate understanding of and work with symmetry groups, including the cyclic group (C_n), the dihedral group (D_{2n}) and the symmetry groups of the cube, octahedron and tetrahedron; • understand the concept of a cycle index $P_G(\times 1, \times 2, ..., \times n)$ • use a table of cycle indices for simple symmetry groups; • use Polya's Enumeration Theorem to enumerate colourings in simple symmetrical structures; and • demonstrate understanding of the use of the pattern inventory $P_G((b+w), (b^2+w^2), ..., (b^n+w^n))$ for 2 colours and the similar result for 3 colours.

4 Scheme of Assessment

4.1 Assessment opportunities

Each unit is available for assessment in summer each year. It is possible to resit individual AS and A2 assessment units once and count the better result for each unit towards an AS or A level qualification. Candidates' results for individual assessment units can count towards a qualification until we withdraw the specification.

4.2 Assessment objectives

There are three assessment objectives for this specification. Candidates must:

AO1 use and apply standard techniques, by:

- selecting and correctly carrying out routine procedures; and
- accurately recalling facts, terminology and definitions;

AO2 reason, interpret and communicate mathematically, by:

- constructing rigorous mathematical arguments (including proofs);
- making deductions and inferences;
- assessing the validity of mathematical arguments;
- explaining their reasoning; and
- using mathematical language and notation correctly;

AO3 solve problems within mathematics and in other contexts, by:

- translating problems in mathematical and non-mathematical contexts into mathematical processes;
- interpreting solutions to problems in their original context, and, where appropriate, evaluating their accuracy and limitations;
- translating situations in context into mathematical models;
- using mathematical models; and
- evaluating the outcomes of modelling in context, recognising the limitations of models and, where appropriate, explaining how to refine them.

4.3 Assessment objective weightings

The table below sets out the assessment objective weightings for each assessment unit and the overall A level qualifications:

Assessment Objective	Assessment Unit Weighting			
	AS 1	AS 2	A2 1	A2 2
AO1	35–50%	25–40%	30–45%	25–40%
AO2	30–50%	25–40%	30–45%	25–40%
AO3	10–25%	20–40%	15–30%	30–50%
Total	**100%**	**100%**	**100%**	**100%**

4.4 Synoptic assessment at A2

The A2 assessment units include some synoptic assessment, which encourages candidates to develop their understanding of the subject as a whole. In our GCE Further Mathematics, synoptic assessment involves:

- building on material from the AS units; and
- bringing together and making connections between areas of knowledge, understanding and skills that they have explored throughout the course.

4.5 Higher order thinking skills

The A2 assessment units provide opportunities to demonstrate higher order thinking skills by incorporating:

- more demanding unstructured questions; and
- questions that require candidates to make more connections between sections of the specification.

4.6 Reporting and grading

We report the results of individual assessment units on a uniform mark scale that reflects the assessment weighting of each unit.

We award AS qualifications on a five grade scale from A to E, with A being the highest. We award A level qualifications on a six grade scale from A* to E, with A* being the highest. To determine candidates' grades, we add the uniform marks obtained in individual assessment units.

To be awarded an A*, candidates need to achieve a grade A on their full A level qualification and at least 90 percent of the maximum uniform marks available for the A2 units. If candidates fail to attain a grade E, we report their results as unclassified (U).

The grades we award match the grade descriptions in Section 5 of this specification.

5 Grade Descriptions

Grade descriptions are provided to give a general indication of the standards of achievement likely to have been shown by candidates awarded particular grades. The descriptions must be interpreted in relation to the content in the specification; they are not designed to define that content. The grade awarded depends in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of candidates' performance in the assessment may be balanced by better performances in others.

AS Grade Descriptions

Grade	Description
AS **A Grade**	For AO1, candidates characteristically: • select and accurately carry out almost all routine procedures correctly; and • accurately recall almost all facts, terminology and definitions. For AO2, candidates characteristically: • independently construct rigorous mathematical arguments in almost all relevant contexts; • make valid deductions and inferences in almost all relevant contexts; • assess, critique and improve the validity of a mathematical argument in almost all relevant contexts; • construct extended chains of reasoning to achieve a given result, find and correct errors and explain their reasoning, evaluating evidence in almost all relevant contexts; and • use mathematical language and notation correctly in almost all relevant contexts. For AO3, candidates characteristically: • translate problems in mathematical or non-mathematical contexts into mathematical processes in almost all relevant contexts; • interpret solutions to problems in their original context and, where appropriate, evaluate their accuracy and limitations in almost all relevant contexts; • translate situations in context into mathematical models in almost all relevant contexts; • use mathematical models in almost all relevant contexts; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them in almost all relevant contexts.

Grade	Description
AS **E Grade**	For AO1, candidates characteristically: • select and accurately carry out some routine procedures correctly; and • accurately recall some facts, terminology and definitions. For AO2, candidates characteristically: • independently construct rigorous mathematical arguments in some relevant contexts; • make valid deductions and inferences in some relevant contexts; • assess, critique and improve the validity of a mathematical argument in some relevant contexts; • construct extended chains of reasoning to achieve a given result, find and correct errors and explain their reasoning, evaluating evidence in some relevant contexts; and • use mathematical language and notation correctly in some relevant contexts. For AO3, candidates characteristically: • translate problems in mathematical or non-mathematical contexts into mathematical processes in some relevant contexts; • interpret solutions to problems in their original context and, where appropriate, evaluate their accuracy and limitations in some relevant contexts; • translate situations in context into mathematical models in some relevant contexts; • use mathematical models in some relevant contexts; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them in some relevant contexts.

A2 Grade Descriptions

Grade	Description
A2 **A Grade**	For AO1, candidates characteristically: • select and accurately carry out almost all routine procedures correctly; and • accurately recall almost all facts, terminology and definitions. For AO2, candidates characteristically: • independently construct rigorous mathematical arguments in almost all relevant contexts; • make valid deductions and inferences in almost all relevant contexts; • assess, critique and improve the validity of a mathematical argument in almost all relevant contexts; • construct extended chains of reasoning to achieve a given result, find and correct errors and explain their reasoning, evaluating evidence in almost all relevant contexts; and • use mathematical language and notation correctly in almost all relevant contexts. For AO3, candidates characteristically: • translate problems in mathematical or non-mathematical contexts into mathematical processes in almost all relevant contexts; • interpret solutions to problems in their original context and, where appropriate, evaluate their accuracy and limitations in almost all relevant contexts; • translate situations in context into mathematical models in almost all relevant contexts; • use mathematical models in almost all relevant contexts; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them in almost all relevant contexts.

Grade	Description
A2 **E Grade**	For AO1, candidates characteristically: • select and accurately carry out some routine procedures correctly; and • accurately recall some facts, terminology and definitions. For AO2, candidates characteristically: • independently construct rigorous mathematical arguments in some relevant contexts; • make valid deductions and inferences in some relevant contexts; • assess, critique and improve the validity of a mathematical argument in some relevant contexts; • construct extended chains of reasoning to achieve a given result, find and correct errors and explain their reasoning, evaluating evidence in some relevant contexts; and • use mathematical language and notation correctly in some relevant contexts. For AO3, candidates characteristically: • translate problems in mathematical or non-mathematical contexts into mathematical processes in some relevant contexts; • interpret solutions to problems in their original context and, where appropriate, evaluate their accuracy and limitations in some relevant contexts; • translate situations in context into mathematical models in some relevant contexts; • use mathematical models in some relevant contexts; and • evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them in some relevant contexts.

6 Guidance on Assessment

There are four external assessment units in this specification, two at AS level and two at A2:

- Unit AS 1: Pure Mathematics;
- Unit AS 2: Applied Mathematics;
- Unit A2 1: Pure Mathematics; and
- Unit A2 2: Applied Mathematics.

6.1 Unit AS 1: Pure Mathematics

This unit assumes knowledge of Unit AS 1 of GCE Mathematics. The unit is assessed by a 1 hour 30 minute external examination, with 6–10 questions worth 100 raw marks.

6.2 Unit AS 2: Applied Mathematics

This unit assumes knowledge of Units AS 1 and AS 2 of GCE Mathematics and Unit AS 1 of GCE Further Mathematics. The unit is assessed by a 1 hour 30 minute external examination, with 6–10 questions worth 100 raw marks.
The unit has four sections and candidates must answer questions from two of these sections (each question is worth 50 percent of the overall marks for this assessment):

- Section A: Mechanics 1;
- Section B: Mechanics 2 (also assumes knowledge of Section A of this unit);
- Section C: Statistics; and
- Section D: Discrete and Decision Mathematics.

6.3 Unit A2 1: Pure Mathematics

This unit assumes knowledge of all GCE Mathematics units and Unit AS 1 of GCE Further Mathematics. The unit is assessed by a 2 hour 15 minute external examination, with 7–12 questions worth 150 raw marks.

6.4 Unit A2 2: Applied Mathematics

This unit assumes knowledge of all GCE Mathematics units and Units AS 1 and A2 1 of GCE Further Mathematics. The unit is assessed by a 2 hour 15 minute external examination, with 7–12 questions worth 150 raw marks. The unit has four sections and candidates must answer questions from two of these sections (each question is worth 50 percent of the overall marks for this assessment):

- Section A: Mechanics 1 (also assumes knowledge of Section A of Unit AS 2);
- Section B: Mechanics 2 (also assumes knowledge of Section A of Unit AS 2 and Section A of this unit);
- Section C: Statistics (also assumes knowledge of Section C of Unit AS 2); and
- Section D: Discrete and Decision Mathematics (also assumes knowledge of Section D of Unit AS 2).

7 Links and Support

7.1 Support

The following resources are available to support this specification:

- our Mathematics microsite at www.ccea.org.uk
- specimen assessment materials; and
- guidance notes for teachers.

We also intend to provide:

- past papers and mark schemes;
- Chief Examiner's reports;
- planning frameworks;
- support days for teachers;
- a resource list; and
- exemplification of standards.

7.2 Curriculum objectives

This specification supports centres to build on the broader Northern Ireland Curriculum objectives to develop the young person:

- as an individual;
- as a contributor to society; and
- as a contributor to the economy and environment.

It can contribute to meeting the requirements of the Northern Ireland Entitlement Framework at post-16 and the provision of a broad and balanced curriculum.

Curriculum Progression from Key Stage 4

This specification builds on learning from Key Stage 4 and gives students opportunities to develop their subject knowledge and understanding further.

Students will also have opportunities to continue to develop the **Cross-Curricular Skills** and the **Thinking Skills and Personal Capabilities** shown below. The extent of this development depends on the teaching and learning methodology the teacher uses.

Cross-Curricular Skills

- Communication:
  - Talking and Listening
  - Reading
  - Writing
- Using Mathematics
- Using ICT

Thinking Skills and Personal Capabilities
- Problem Solving
- Working with Others
- Self-Management

For further guidance on the skills and capabilities in this subject, please refer to the support materials on the subject microsite.

7.3 Examination entries

Entry codes for this subject and details on how to make entries are available on our Qualifications Administration Handbook microsite, which you can access at www.ccea.org.uk

Alternatively, you can telephone our Examination Entries, Results and Certification team using the contact details provided.

7.4 Equality and inclusion

We have considered the requirements of equality legislation in developing this specification and designed it to be as free as possible from ethnic, gender, religious, political and other forms of bias.

GCE qualifications often require the assessment of a broad range of competences. This is because they are general qualifications that prepare students for a wide range of occupations and higher level courses.

During the development process, an external equality panel reviewed the specification to identify any potential barriers to equality and inclusion. Where appropriate, we have considered measures to support access and mitigate barriers.

We can make reasonable adjustments for students with disabilities to reduce barriers to accessing assessments. For this reason, very few students will have a complete barrier to any part of the assessment.

It is important to note that where access arrangements are permitted, they must not be used in any way that undermines the integrity of the assessment. You can find information on reasonable adjustments in the Joint Council for Qualifications document *Access Arrangements and Reasonable Adjustments*, available at www.jcq.org.uk

7.5 Contact details

If you have any queries about this specification, please contact the relevant CCEA staff member or department:

- Specification Support Officer: Nuala Tierney
  (telephone: (028) 9026 1200, extension 2292, email: ntierney@ccea.org.uk)

- Subject Officer: Joe McGurk
  (telephone: (028) 9026 1200, extension 2106, email: jmcgurk@ccea.org.uk)

- Examination Entries, Results and Certification
  (telephone: (028) 9026 1262, email: entriesandresults@ccea.org.uk)

- Examiner Recruitment
  (telephone: (028) 9026 1243, email: appointments@ccea.org.uk)

- Distribution
  (telephone: (028) 9026 1242, email: cceadistribution@ccea.org.uk)

- Support Events Administration
  (telephone: (028) 9026 1401, email: events@ccea.org.uk)

- Moderation
  (telephone: (028) 9026 1200, extension 2236, email: moderationteam@ccea.org.uk)

- Business Assurance (Complaints and Appeals)
  (telephone: (028) 9026 1244, email: complaints@ccea.org.uk or appealsmanager@ccea.org.uk).

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