Rewarding Learning

ADVANCED SUBSIDIARY (AS)

General Certificate of Education

Further Mathematics

Assessment Unit AS 2

assessing

Applied Mathematics

[SFM21]

SFM21

EXEMPLAR PAPER

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the **Answer Booklet** provided.
You must answer the questions in the dedicated spaces provided in the Answer Booklet.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an HB pencil.
You must answer **all** questions from sections A and B **or** A and C **or** A and D **or** C and D.
You should spend equal time on each of the two sections.
Show clearly the full development of your answers. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
The total mark for each section of this paper is 50.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{ms}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

SECTION A Mechanics 1

Answer all five questions in this section.

1 A bead B of mass 0.5 kg is attached to one end of a light inextensible string of length L metres.

The other end of the string is attached to a fixed point O.

The bead is made to describe horizontal circles with constant angular velocity 3 rad s^{-1} with centre vertically below O as shown in **Fig. 1** below.

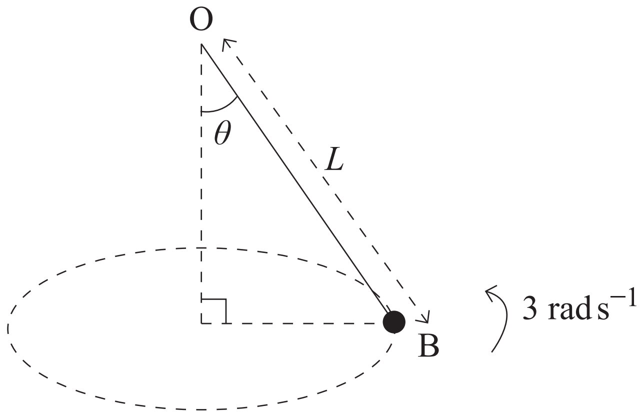


Fig. 1

The string makes an angle θ with the downward vertical.

The tension in the string is 12 N.

(i) Draw a diagram showing all the external forces acting on the bead. [2]

(ii) Find the values of θ and L. [7]

2 A car of mass 1200 kg is travelling along a straight horizontal road.

The car experiences a constant resistance to motion of 300 N.

At a certain instant, the car is accelerating at $0.4\,\text{m}\,\text{s}^{-2}$ and the engine of the car is working at a rate of 19.5 kW.

(i) Find the speed of the car at this instant. [4]

The car then moves up a hill inclined at 30° to the horizontal at a constant speed of $15\,\text{m}\,\text{s}^{-1}$

The resistance to motion remains 300 N.

(ii) Find the power of the car's engine as it moves up the line of greatest slope of the hill. [4]

3 A particle of mass 6 kg moves x metres from a fixed point O, in a positive direction along the x-axis.

It is acted upon by a variable force, F newtons, given by

$$F = px + 1$$

where p is a positive constant.

The particle is at rest at O.

When $x = 3$ the work done by F is $3p^2$ joules.

Find the speed of the particle at this point. [7]

4 A climber of mass 80 kg is securely attached by a light elastic rope to a fixed point O on a vertical rock face.

The rope has natural length 20 metres and modulus of elasticity 18 500 newtons.

He climbs to a point which is d metres vertically above O where $d < 20$

He slips and falls from rest, but is brought to instantaneous rest by the force in the rope when the rope has extended by 8 metres.

(i) By taking the Gravitational Potential Energy to equal zero at O, use the Principle of Conservation of Mechanical Energy to find the value of d. [8]

(ii) Find the speed of the climber when he is 25 metres below O. [6]

(iii) Explain briefly why, in reality, his speed at this point is less than the value found. [1]

5 A light elastic string is attached to a fixed point A on a plane inclined at an angle θ to the horizontal where $\sin\theta = \frac{3}{5}$

A particle of mass 10 kg is attached to the other end of the string.

The string has natural length 5 metres and modulus of elasticity 147 newtons.

The particle is first held at a point B on the plane, where AB = 5 metres and B is below A as shown in **Fig. 2** below.

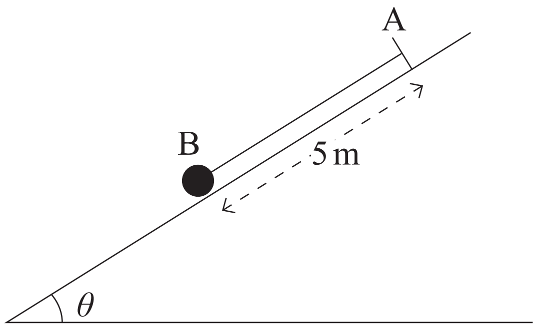


Fig. 2

The string is parallel to a line of greatest slope of the plane.

The particle is released from rest.

(i) Given that the plane is smooth, find the distance down the slope from B that the particle moves before next coming to instantaneous rest. [5]

If instead the plane is rough, the particle next comes to instantaneous rest 2 metres down the slope from B.

(ii) Find the coefficient of friction between the particle and the plane. [6]

SECTION B Mechanics 2

Answer all five questions in this section.

1 A violin has four strings, each of which has a pitch determined by its fundamental frequency.

The frequency f depends on the tension S in the string, the mass per unit length ρ of the string and the length l of the string.

It is known that the frequency is related to the other string properties as follows:

$$f = kS^x \rho^y l^z$$

where k is a dimensionless constant.

The dimensions of frequency are T^{-1}

(i) Use the method of dimensions to find x, y and z. [6]

The violinist holds one of the strings which effectively makes its length 15% shorter.

(ii) Find the percentage change in its frequency, given that the tension and mass per unit length are unchanged. [2]

2 A satellite of mass m kg is moving in a circular orbit of radius r metres around a planet of mass M kg.

(i) Show that the angular speed of the satellite around the planet can be expressed as

$$\sqrt{\frac{GM}{r^3}} \text{ rad s}^{-1}$$

where G is the universal gravitational constant. [6]

A satellite orbits Saturn in a circular path of radius 133 600 km.

The mass of Saturn is 5.683×10^{26} kg and the value of G is $6.674 \times 10^{-11}\,\text{m}^3\,\text{s}^{-2}\,\text{kg}^{-1}$

(ii) Find the period of orbit of the satellite in hours. [4]

3 A pendulum consists of a bob P of mass 4 kg, attached to one end of a light rod of length 4 m.

The other end of the rod is freely hinged at the fixed point O.

The bob is at rest at its lowest point and is then given a horizontal impulse so that it starts to move with speed $u\,\mathrm{m\,s^{-1}}$

Find the possible values of u if the pendulum is to perform complete circles. [8]

4 A teacher on lunchtime duty notices Pauline standing 5 m North East of Roisin.

At the same instant, each girl starts to run.

Pauline has a velocity of $4\,\mathrm{m\,s^{-1}}$ due South.

Roisin has a velocity of $3\sqrt{2}\,\mathrm{m\,s^{-1}}$ South East.

If they maintain these velocities, find the shortest distance between Pauline and Roisin in the subsequent motion. [9]

5 Two small identical rings, A and B, each of mass 0.6 kg are threaded on to a fixed rough horizontal wire.

The rings are attached to opposite ends of a light inextensible string of length 2*L* metres, as shown in **Fig. 1** below.

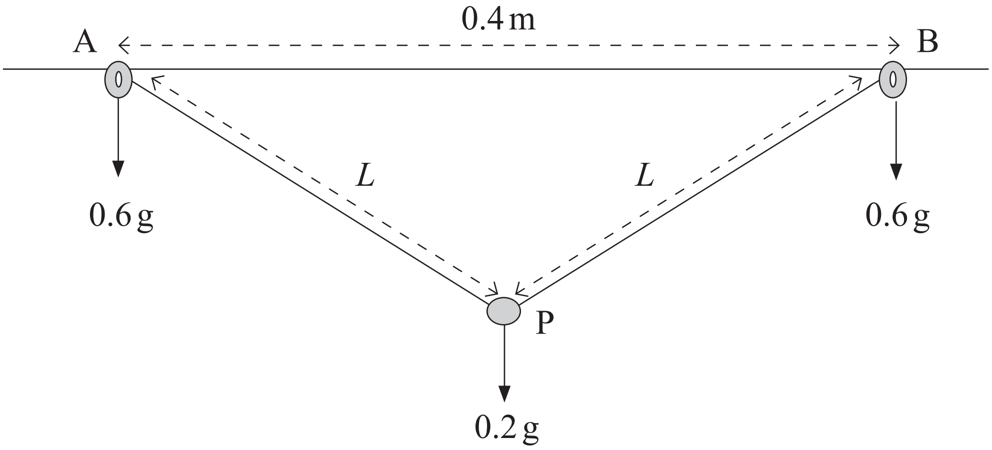


Fig. 1

A particle P of mass 0.2 kg is attached to the midpoint of the string.

When the rings are in limiting equilibrium P hangs vertically below the wire, with AB = 0.4 m.

The coefficient of friction between each ring and the wire is 0.5

(i) Draw a diagram showing all the forces acting on each of the three masses. [3]

(ii) Show that the normal reaction between each ring and the wire is 0.7 g N. [4]

(iii) Find the value of *L*. [8]

SECTION C Statistics

Answer all five questions in this section.

1 The owners of a football stadium would like to find out the views of the supporters who attend matches with regard to changing some of the stadium seating areas to standing sections.

By looking at previous attendance statistics, the owners have identified that the stadium crowd typically comprises 80% home supporters and 20% away supporters.

They intend to place a number of interviewers around the stadium at the next game.

Each interviewer has been instructed to interview 10 supporters, of which 8 must be home supporters and 2 must be away supporters.

(i) State the non-random sampling technique that is being used. [1]

(ii) Describe one advantage and one disadvantage of using this particular sampling technique. [4]

2 A researcher is investigating how the mass of a chimpanzee is related to its age.

She took a sample of 10 chimpanzees from a nature reserve with ages ranging from birth to 12 weeks old.

For each chimpanzee she recorded the age x, number of weeks since birth, and the mass y kg.

The largest mass recorded was 4 kg.

Some of the summary statistics are shown below.

$$\Sigma x = 60 \qquad \Sigma y = 24 \qquad \Sigma x^2 = 480 \qquad \Sigma y^2 = 62.4$$

From the data obtained the researcher calculated the product–moment correlation coefficient to be 0.9

(i) Find the equation of the least squares regression line of chimpanzee mass y on age x, in the form $y = a + bx$ where a and b are constants. [9]

(ii) Suggest a reason why it would not be appropriate to use the regression line calculated in part **(i)** to:

(a) estimate the mass of a chimpanzee at 14 weeks old; [1]

(b) estimate the age of a chimpanzee with a mass of 3 kg. [1]

3 A fairground game involves the contestant rolling two fair tetrahedral dice at the same time.

Each tetrahedral die has four faces numbered 1, 2, 3 and 4

The positive difference between the scores shown on the two dice determines the prize won by the contestant.

The possible prizes that the contestant can win are given below.

- £5 if the difference is 3
- £1 if the difference is 1
- £0 if the difference is 0 or 2

Let the amount, in pounds, won by a contestant on a single attempt be modelled by the random variable X.

(i) Using a sample space, or otherwise, copy and complete **Table 1** below showing the probability distribution of X. [5]

Table 1

x	0	1	5
$\mathrm{P}(X = x)$			

The game organiser would like to make an average profit of 50p per contestant attempt.

(ii) Find the price that the organiser should charge a contestant for a single attempt in order to achieve this average profit. [3]

(iii) Calculate $\mathrm{Var}(X)$. [3]

The random variable Y is related to X by the formula $Y = 3X - 4$

(iv) Find the mean and variance of Y. [3]

4 An experiment involves a biased coin being tossed a number of times.

Let p be the probability that the coin lands on tails on any given throw.

The probability that the coin first lands on tails on the second throw is 0.24

(i) Find the two possible values of p. [4]

Using the smaller value of p, find the probability that:

(ii) the coin lands on tails for the first time on the eighth throw; [3]

(iii) the coin does not land on tails before the fifth throw. [2]

5 Each letter of the word DESTINY is written on a separate card.

The word DESTINY contains two vowels (E and I) and five consonants.

Four cards out of the seven are to be chosen at random.

(i) Find the number of different combinations possible. [1]

(ii) Find the probability that exactly three consonants are on the four chosen cards. [3]

The seven cards are to be arranged in a random order in a line.

(iii) Find the number of arrangements possible. [1]

(iv) Find the probability that the two cards containing vowels are next to each other in either order. [3]

Another word has n different letters, where $n \geqslant 3$

Two of the letters are vowels and the remainder are consonants.

Each letter is written on a separate card and the cards are arranged in a random order in a line.

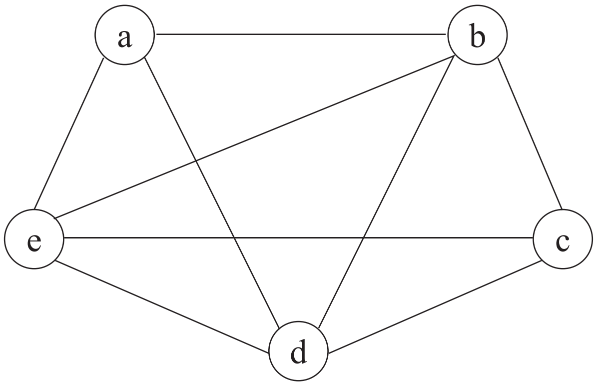
(v) Find, in terms of n, the probability that the two cards containing the vowels are next to each other. [3]

SECTION D Discrete and Decision Mathematics

Answer all five questions in this section.

1 **(a)** The graph G_1 on 5 vertices is shown in **Fig. 1** below.

Draw a new version of G_1 to demonstrate that it is planar.



[2]

Fig. 1

(b) The graph G_2 on 6 vertices is shown in **Fig. 2** below.

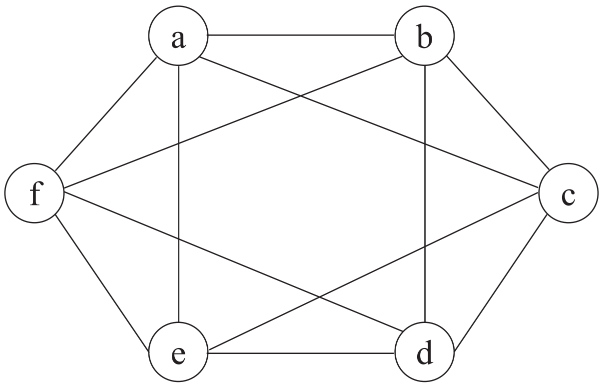


Fig. 2

(i) How many edges must be removed from G_2 to leave a spanning tree? [1]

(ii) Draw a spanning tree that is a subgraph of G_2 [2]

(iii) How many edges must be added to G_2 to form K_6, the complete graph on 6 vertices? [2]

(c) Write down an Eulerian trail of the graph $K_{3,2}$ shown in **Fig. 3** below. [2]

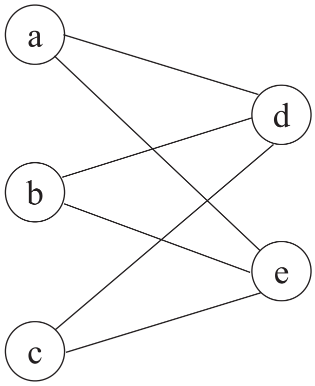


Fig. 3

2 The sequence u_n is defined by the relationship

$$u_{n+2} = \frac{5}{3}u_{n+1} + \frac{2}{3}u_n$$

where $u_0 = 2$ and $u_1 = \frac{5}{3}$ and n is an integer.

(i) Show that

$$u_n = \alpha^n + \beta^n$$

where α and β are real numbers to be found. [7]

(ii) Given that $u_t = 4096.000002$ to 10 significant figures, find the value of t. [3]

3 The travel times in minutes between seven towns are given in the road network in **Fig. 4** below.

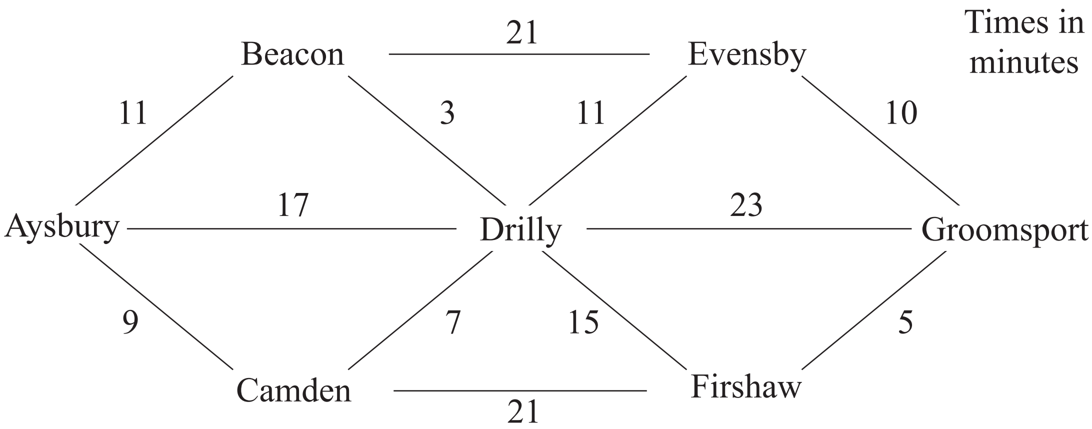


Fig. 4

A race is planned from Aysbury to Groomsport.

(i) Apply Dijkstra's algorithm to this network to calculate the shortest time from Aysbury to Groomsport.

Use the version of the road network **Diagram 1** given in the Answer Booklet.

Be careful to fill the algorithm details into the box at each vertex.

Do not delete any of the temporary times. [5]

(ii) Write down the fastest route from Aysbury to Groomsport and the corresponding minimum time. [2]

(iii) Write down the minimum time from Aysbury to Evensby. [1]

Due to road works, the road from Drilly to Beacon is temporarily one way, as shown in **Fig. 5** below.

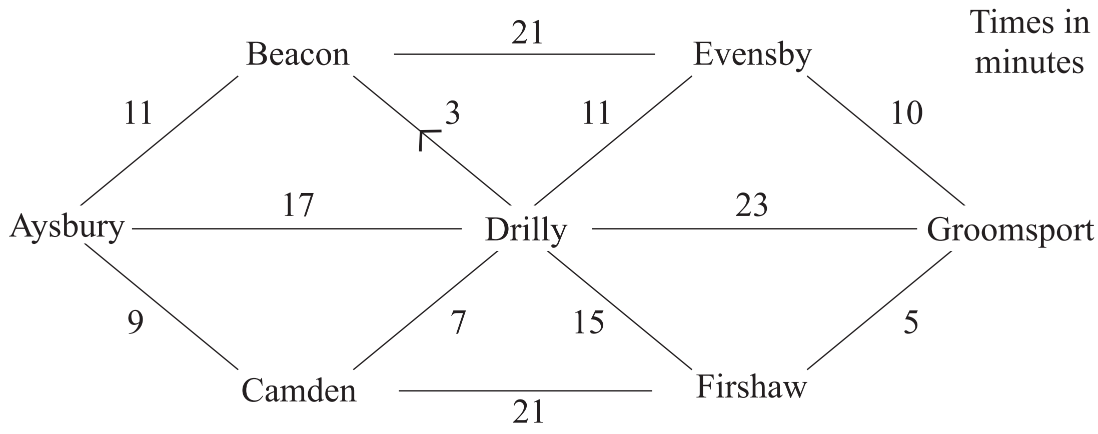


Fig. 5

(iv) Apply Dijkstra's algorithm to **Fig. 5** to recalculate the new shortest time from Aysbury to Groomsport, taking into account the one-way road.

Use the version of the road network **Diagram 2** given in the Answer Booklet.

Be careful to fill the algorithm details into the box at each vertex.

Do not delete any of the temporary times. [3]

4 Let p, q and r be propositional statements.

The Boolean implication operator

$$p \Rightarrow q$$

is defined by the truth table in **Fig. 6** below.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Fig. 6

(i) Use truth tables to show that $p \Rightarrow q$ and $\sim q \Rightarrow \sim p$ are equivalent. [3]

(ii) Use truth tables to prove that

$$[(p \Rightarrow q) \text{ and } (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$$

is a tautology, i.e. is always true. [7]

5 The set of six permutations p, q, r, s, t and u forms a group G under the operation $\circ$, the composition of permutations.

The permutations are defined by

$$p = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \qquad q = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \qquad r = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

$$s = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \qquad t = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \qquad u = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

An example composition is

$$q \circ s = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = t$$

so $q \circ s = t$.

(i) Write down the identity permutation. [1]

A partially complete Cayley table for this group is given in **Fig. 7** below.

$\circ$	p	q	r	s	t	u
p		u	t	r	s	
q	u	p	s	t		
r	s		u			
s	t	r	q			
t						
u						

Fig. 7

(ii) Copy and complete the Cayley table in **Fig. 7** [4]

(iii) Copy and complete the table of elements and their periods in **Fig. 8** below.

Element	p	q	r	s	t	u
Period						

[2]

Fig. 8

(iv) List all the subgroups of G. [3]

THIS IS THE END OF THE QUESTION PAPER

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