Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2024

Further Mathematics

Unit 1 **(With calculator)**

Pure Mathematics

[GFM11]

GFM11

TUESDAY 21 MAY, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page.
Complete in black ink only. **Do not write with a gel pen.**
All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all thirteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on page 2.

Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation: If $y = ax^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \,\mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

1 Given that $\frac{dy}{dx} = 6x^2 - \frac{2}{5x^2} + 4$

(i) find an expression for $\frac{d^2y}{dx^2}$,

Answer _____ [2]

(ii) find an expression for y, given that $y = 4$ when $x = -2$

Answer _____ [5]

[Turn over

2 Solve the equation $x^2 + 14x + 10 = 7$ by **completing the square**.

Give your answer in surd form.

Answer _____ [4]

3 Solve the inequality

$$(x + 1)^2 \geqslant 2x + 5$$

You **must** show clearly each stage of your solution.

Answer _____ [4]

[Turn over

4 **(i)** Solve the equation

$$\cos x = -0.6$$

for $-180° \leqslant x \leqslant 180°$

Answer _____ [2]

(ii) Hence solve the equation

$$\cos\left(\frac{\theta}{2} - 20°\right) = -0.6$$

for $-360° \leqslant \theta \leqslant 360°$

Answer _____ [3]

[Turn over

5 Matrices **P** and **Q** are defined by

$$\mathbf{P} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

(i) Find the matrix $\mathbf{P}^{-1}$, the inverse of the matrix **P**.

Answer _____ [2]

(ii) Hence find the matrix **X** such that $\mathbf{PX} = \mathbf{Q}$

Show clearly each stage of your working.

Answer _____ [2]

[Turn over

6 **(a)** Express $3\log 2x - 2\log x$ as a single log.

Answer _____ [3]

(b) Solve the equation

$$4^{2x+1} = 6^{3x-2}$$

Answer _____ [5]

[Turn over

7 **(a)** Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} 4 & 5 \\ -1 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} 1 & 5 \\ 0 & 3 \end{bmatrix}$$

Calculate

(i) $\mathbf{B} - 2\mathbf{C}$

Answer _____ [1]

(ii) $\mathbf{A}^2$

Answer _____ [2]

(b) Find the values of a, b and c in the following matrix equation.

$$\begin{bmatrix} 2 & -1 \\ a & 1 \end{bmatrix} \begin{bmatrix} 4 & b \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 4 \\ c & 6 \end{bmatrix}$$

Answer $a =$ _____

$b =$ _____

$c =$ _____ [3]

[Turn over

8 Simplify **fully** the following algebraic expressions.

(a) $\dfrac{2x^2 + 6x}{x^2 - 1} \div \dfrac{4x + 12}{2x^2 + 3x - 5}$

Answer _____ [4]

(b) $\left(\frac{x}{x+3} + \frac{2}{x^2-9}\right) \times \frac{x+3}{x-1}$

Answer _____ [5]

[Turn over

9 At the point P, where $x = -1$, the tangent to the curve $y = 4 + \frac{a}{x^2}$ is parallel to the straight line $y = 3x + 1$

(i) Find the value of a.

Answer _____ [4]

(ii) Find the equation of the tangent to the curve at the point P.

Answer _____ [3]

(iii) Find the equation of the normal to the curve at the point P.

Answer _____ [2]

[Turn over

10 A curve is defined by the equation

$$y = 2x^3 - 5x^2 - 3x$$

(i) Find the **coordinates** of the points where the curve meets the x-axis.

Answer _____ [3]

(ii) Find the coordinates of the turning points of the curve and, using calculus, identify each turning point as either a maximum or minimum point.

You **must** show working to justify your answer.

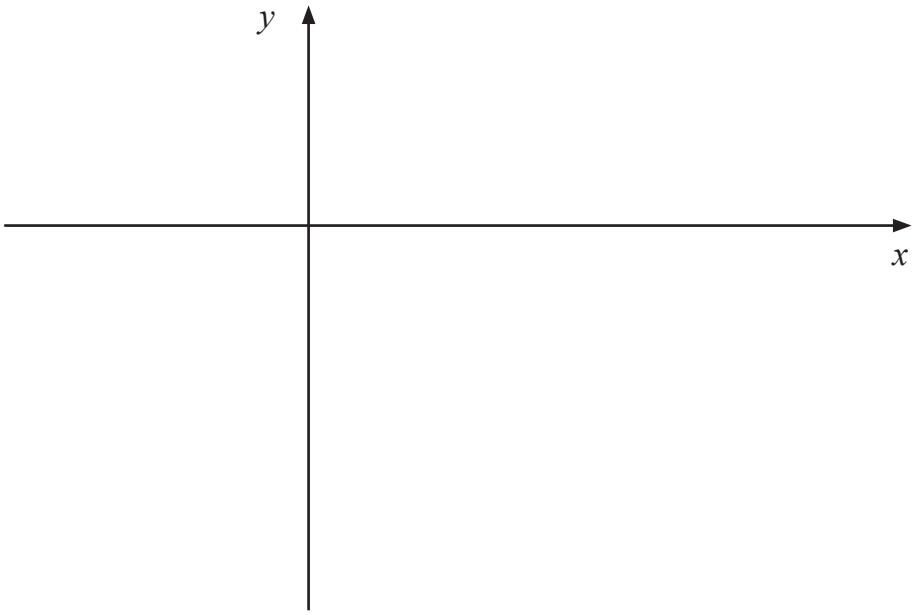
Answer _____ [7]

Q10 continues on page 21

BLANK PAGE

DO NOT WRITE ON THIS PAGE

(iii) Sketch the curve on the axes below.



[2]

[Turn over

11 A car showroom sells the new car models Hurricane, Iguana and Jeronimo.

The profit for each Hurricane sold is x,

the profit for each Iguana sold is y,

the profit for each Jeronimo sold is z,

where x, y and z are in thousands of pounds.

In February, 10 Hurricanes, 12 Iguanas and 16 Jeronimos were sold.

The total profit was £94,000

(i) Show that x, y and z satisfy the equation

$$5x + 6y + 8z = 47$$

[1]

In March, 16 Hurricanes, 6 Iguanas and 4 Jeronimos were sold.

The total profit was £82,000

(ii) Show that x, y and z also satisfy the equation

$$8x + 3y + 2z = 41$$

[1]

In April, 18 Hurricanes, 3 Iguanas and 12 Jeronimos were sold.

The total profit was £96,000

(iii) Show that x, y and z also satisfy the equation

$$6x + y + 4z = 32$$

[1]

[Turn over

(iv) Solve the equations

$$\begin{aligned} 5x + 6y + 8z &= 47 \\ 8x + 3y + 2z &= 41 \\ 6x + y + 4z &= 32 \end{aligned}$$

to find the profit the showroom makes for the sale of each model of car.

Show clearly each stage of your solution.

Answer Hurricane £ _____

Iguana £ _____

Jeronimo £ _____ [8]

[Turn over

12 The table below shows the power, P (watts), and the voltage, V (volts), in a particular circuit.

Power P (watts)	**Voltage** V (volts)		
11	1.5		
45	3		
125	5		
245	7		
405	9		

It is believed that a relationship of the form

$$P = kV^n$$

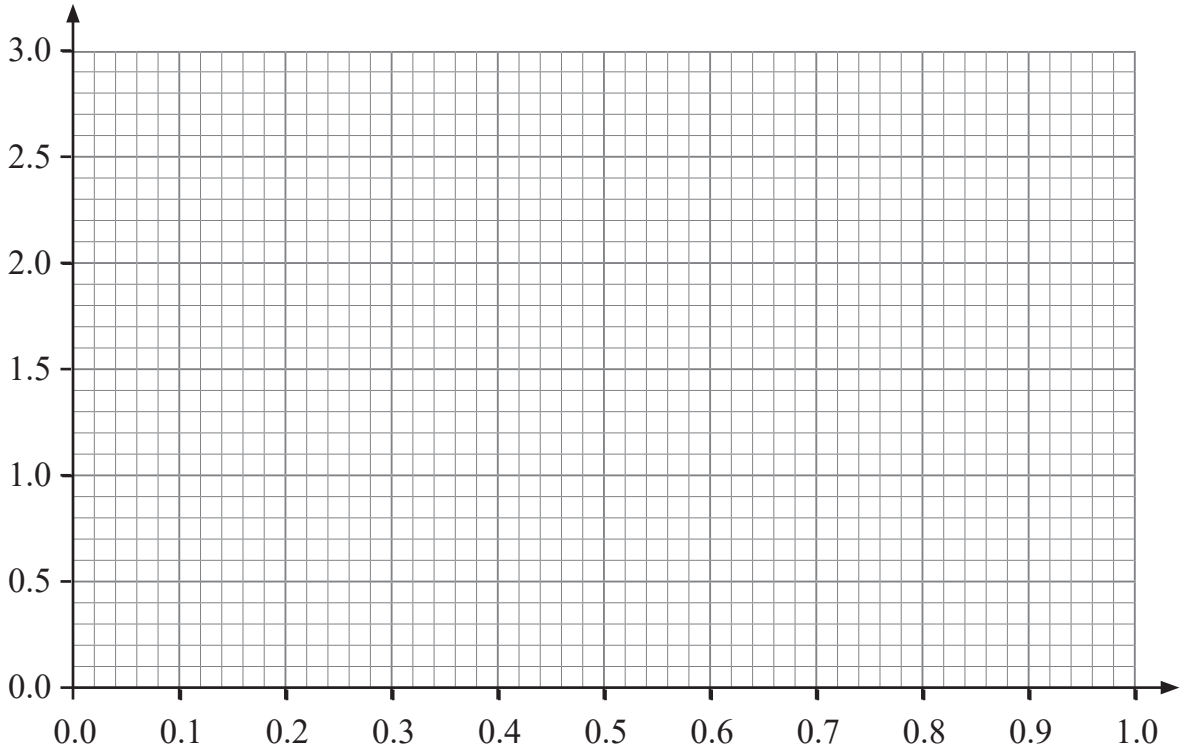
exists, where k and n are constants.

(i) Verify that a relationship of the form

$$P = kV^n$$

exists by drawing a suitable straight line graph on the grid opposite.

Show clearly the values used, correct to 3 decimal places, in the table above.



Label the axes clearly. [6]

[Turn over

(ii) Hence find the values of k and n, correct to 2 decimal places.

Answer k = _____ , n = _____ [4]

(iii) Use the formula $P = kV^n$, with your values for k and n, to calculate the minimum voltage required if the power must exceed 1000 watts.

Give your answer correct to the nearest volt and state any assumption that you make.

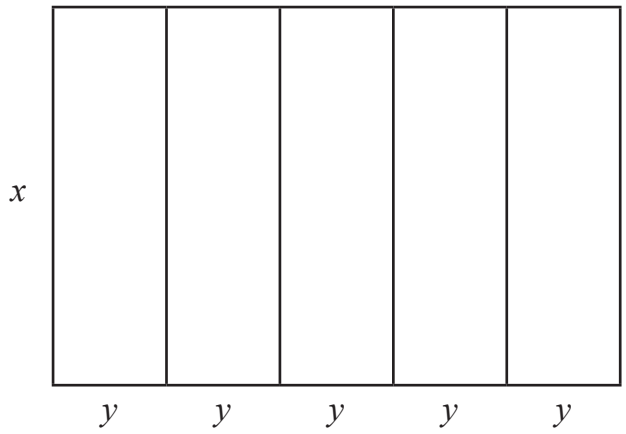
You **must** show your working.

Answer _____ volts [2]

Assumption _____ [1]

[Turn over

13 A gardener fences 5 adjacent plots, each x m by y m, with shared fencing, as shown in the diagram below.



A total of 300 m of fencing is available.

(i) Show that $y = 30 - \frac{3}{5}x$

[1]

(ii) Hence show that the total area of the 5 plots is

$$A = 150x - 3x^2$$

[2]

(iii) Find the maximum total area of the 5 plots, proving by calculus that it is a maximum.

Answer_____ m^2 [5]

THIS IS THE END OF THE QUESTION PAPER

DO NOT WRITE ON THIS PAGE

For Examiner's use only	
Question Number	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
Total Marks	

Examiner Number