Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
January 2019

Further Mathematics

Unit 1 **(With calculator)**
Pure Mathematics

[GMF11]

GMF11

WEDNESDAY 16 JANUARY, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen.**
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all thirteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

12015

Formula Sheet

PURE MATHEMATICS

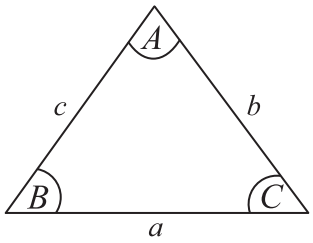
Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \dfrac{1}{2} ab \sin C$



Differentiation: If $y = ax^n$ then $\dfrac{dy}{dx} = nax^{n-1}$

Integration: $\int ax^n \, dx = \dfrac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\frac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $\quad s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $\quad s = ut + \frac{1}{2}at^2$

where u is initial velocity, v is final velocity, a is acceleration, t is time, s is change in displacement

Newton's Second Law: $F = ma$

where F is resultant force, a is acceleration, m is mass

STATISTICS

Statistical measures: Mean $= \dfrac{\Sigma fx}{\Sigma f}$ $\quad$ Median $= L_1 + \dfrac{\left\{\frac{N}{2} - (\Sigma f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
N is total frequency
$(\Sigma f)_1$ is the sum of the frequencies up to but not including the median class
f_{median} is the frequency of the median class
c is the width of the median class

Standard deviation $= \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$ where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

[Turn over

1 **(a)** Matrices **P** and **Q** are defined by

$$\mathbf{P} = \begin{bmatrix} 2 & -5 \\ -1 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}$$

Express as a single matrix:

(i) $\mathbf{P} + 3\mathbf{Q}$

Answer _____ [1]

(ii) $\mathbf{P}^2$

Answer _____ [2]

(b) Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 6 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

Which one of the products

$$\mathbf{AB}, \quad \mathbf{AC}, \quad \mathbf{BC} \quad \text{and} \quad \mathbf{CB}$$

is impossible to form?

Answer _____ [1]

(c) Matrices **D**, **E**, **F** and **G** are defined by

$$\mathbf{D} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} 6 \\ 4 \end{bmatrix} \quad \text{and} \quad \mathbf{G} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Which one of the following combinations

$$\mathbf{D} + \mathbf{E}, \quad \mathbf{E} - \mathbf{D}, \quad \mathbf{E} + \mathbf{F} \quad \text{or} \quad \mathbf{G} - \mathbf{F}$$

is impossible to form?

Answer _____ [1]

[Turn over

2 Solve the equation

$$x^2 + 8x - 7 = 0$$

by completing the square.

Give your answer in the form $a \pm \sqrt{b}$, where a and b are whole numbers.

Answer _____ [4]

3 **(a)** Find $\frac{dy}{dx}$ if $y = 7x^2 - \frac{3}{x^3} + \frac{5x^4}{4}$

Answer _____ [3]

[Turn over

(b) Find $\int \left(9x^3 + \frac{3}{7x^4} - 5\right) \mathrm{d}x$

Answer _____ [4]

4 **(a)** The vectors **a**, **b** and **c** are defined by

$$\mathbf{a} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -4 \end{bmatrix} \text{ and } \quad \mathbf{c} = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$$

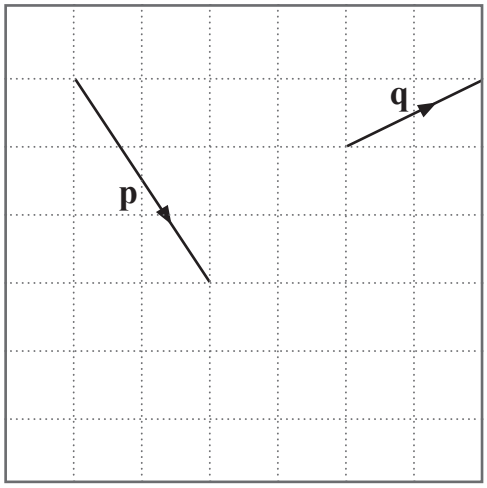
Find the vector **x** if

$$\mathbf{x} + 2\mathbf{b} = \mathbf{c} + \mathbf{a}$$

Answer _____ [2]

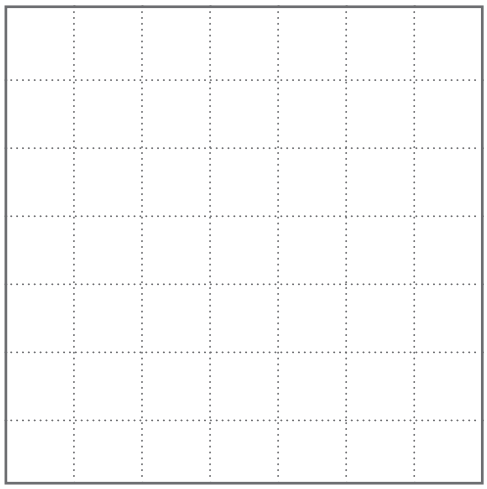
[Turn over

(b) The vectors **p** and **q** are shown below.



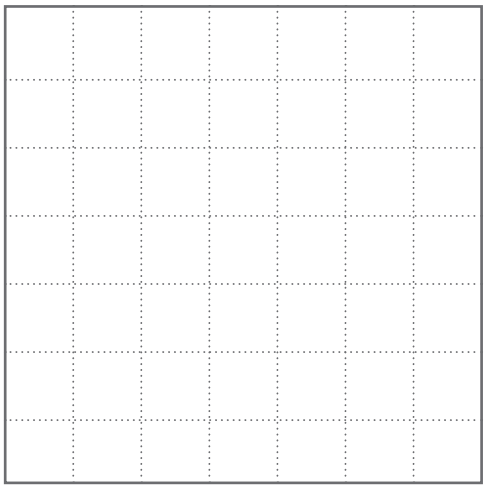
On the grids below, draw diagrams to show the vectors

(i) **p** + 2**q**



[1]

(ii) q − p



[1]

[Turn over

5 **(i)** Solve the equation

$$\cos \theta = 0.3$$

for $-180° \leq \theta \leq 180°$

Answer _____ [2]

(ii) Hence solve the equation

$$\cos\left(\frac{x}{2} + 20°\right) = 0.3$$

for $-360° \leq x \leq 360°$

Answer _____ [3]

[Turn over

6 Matrices **Y** and **Z** are defined by

$$\mathbf{Y} = \begin{bmatrix} -3 & 2 \\ 5 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{Z} = \begin{bmatrix} 16 \\ -5 \end{bmatrix}$$

Find the matrix **X** such that $\mathbf{YX} = \mathbf{Z}$

Answer _____ [5]

7 **(a)** Solve the equation

$$4^{2x-3} = 6^{x+4}$$

Answer _____ [5]

[Turn over

(b) Given that

$$\log_3 5 = x \quad \text{and} \quad \log_3 2 = y$$

express $\log_3 90$ in terms of x and y.

Answer _____ [3]

(c) Given that

$$\log z = 3 \log x - \log y$$

express z in terms of x and y.

Answer _____ [2]

[Turn over

8 Simplify **fully** the algebraic expressions

(a) $$\frac{x - 6}{3x^2 - 11x - 4} \div \frac{2x^2 - 12x}{2x - 8}$$

Answer _____ [4]

(b) $\dfrac{2}{x + 2} - \dfrac{x - 7}{2x^2 + 3x - 2}$

Answer _____ [4]

[Turn over

9 A curve is defined by the equation $y = 3x^2 - x + 4$

The tangent to this curve at a point P is parallel to the line $y = 11x + 5$

(i) Find the coordinates of P.

Answer _____ [5]

(ii) Find the equation of the **normal** to this curve at the point (−1, 8).

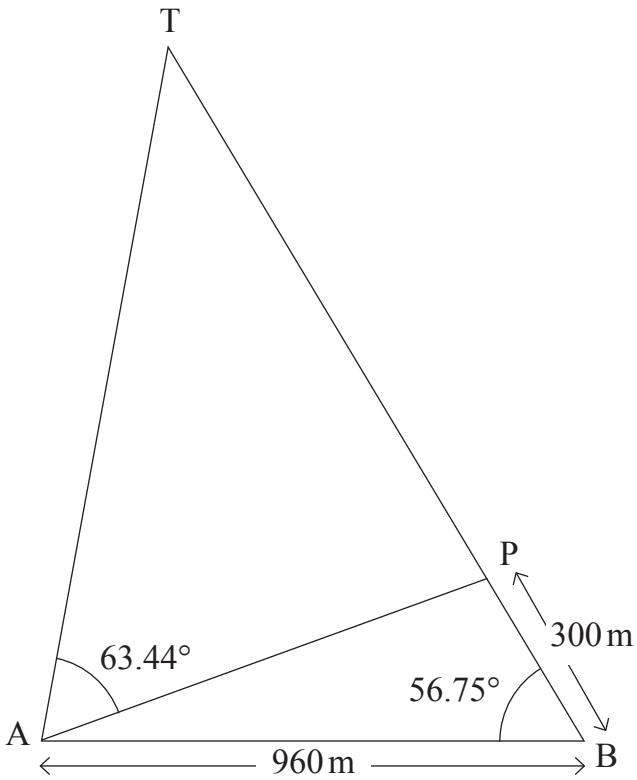
Answer _____ [4]

[Turn over

10 Andrea and Bill were at positions A and B on horizontal ground. The distance between A and B was 960 m.

Bill noticed that a pole at position P, 300 m from B, was directly in line with a tower T.

Andrea measured the angle $T\hat{A}P$ as 63.44° and Bill measured the angle $A\hat{B}P$ as 56.75°, as shown in the diagram below.



(i) Calculate the distance AP.

Answer _____ m [2]

(ii) Calculate the size of the angle $P\hat{A}B$.

Answer _____ ° [2]

[Turn over

(iii) Write down the size of the angle $T\hat{A}B$.

Answer _____° [1]

(iv) Write down the size of the angle $A\hat{T}B$.

Answer _____° [1]

(v) Calculate the distance AT.

Answer _____ m [2]

Andrea observed lightning strike the tower and exactly 3.6 seconds later she heard thunder.

(vi) Calculate the speed of sound.

Answer _____ m/s [2]

11 A curve is defined by the equation $y = x^3 - 2x^2 - 8x$

(i) Find the **coordinates** of the points where the curve meets the x-axis.

Answer _____ [3]

(ii) Find the coordinates of the turning points of the curve.

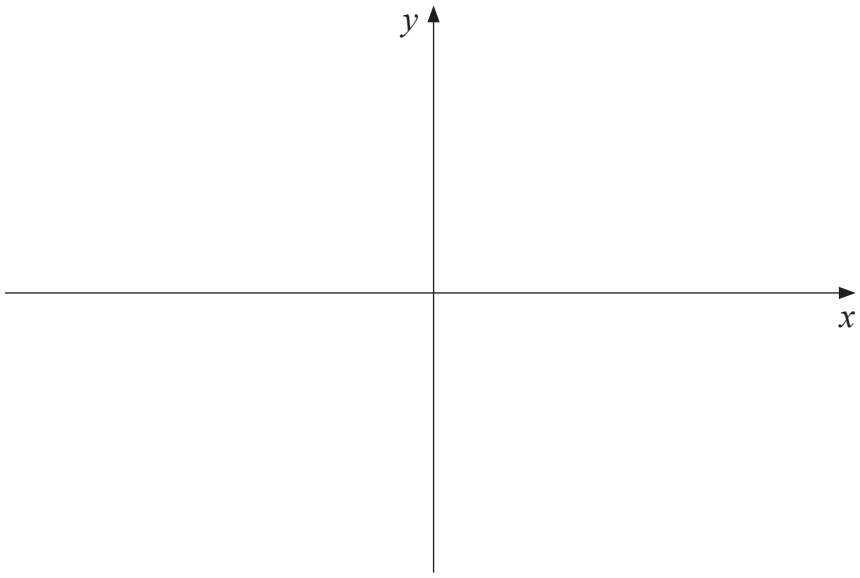
Answer _____ [5]

[Turn over

(iii) Using calculus, identify each turning point as either a maximum or a minimum point. You **must** show working to justify your answer.

Answer _____ [2]

(iv) Sketch the curve on the axes below.



[2]

(v) Find the area enclosed by the curve and the **positive** x-axis.

Answer _____ [4]

12 Brian wants to frame a rectangular family photograph.

The photograph has a width of x cm and a length of y cm.

The price for framing the photograph is made up of

- the cost of the wooden frame at 12p per cm and
- the cost of the glass at 5p per cm^2.

The cost of the wooden frame is £18.00

(i) Show that $x + y = 75$

[1]

The cost of the glass is £67.50

(ii) Show that $xy = 1350$

[1]

(iii) Hence, by solving these equations simultaneously, calculate the dimensions of Brian's photograph.

Answer _____ cm × _____ cm [5]

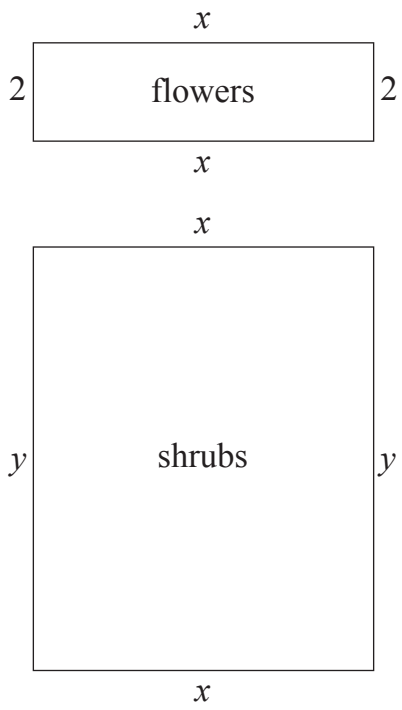
[Turn over

13 Fred plans to dig two plots in his garden.

He wishes to have

- one small rectangular plot, 2 m wide and x m long for flowers, and
- a larger rectangular plot, x m wide and y m long for shrubs,

as shown in the diagram below.



Write down, in terms of x and y,

(i) an expression for the total area A of both plots,

Answer _____ [1]

(ii) an expression for the total perimeter P around both plots.

Answer _____ [1]

Fred has enough topsoil to cover an area of $30\,m^2$, which is the total area of both plots.

(iii) Express y in terms of x and hence show that the total perimeter is given by

$$P = 4x + \frac{60}{x}$$

[3]

[Turn over

To cut down his work of trimming round the edges of the plots, Fred wishes to keep the total perimeter to a minimum.

(iv) Find the values of x and y which will minimise the total perimeter, showing that these values give a minimum.

Answer $x =$ _____ m, $y =$ _____ m [4]

(v) Hence find the minimum total length round the edges of both plots that Fred will have to trim.

Answer _____ m [1]

THIS IS THE END OF THE QUESTION PAPER

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For Examiner's use only	
Question Number	**Marks**
1	
2	
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Examiner Number