



Rewarding Learning

**General Certificate of Secondary Education
January 2019**

Further Mathematics

Unit 1

Pure Mathematics

[GMF11]

WEDNESDAY 16 JANUARY, AFTERNOON

**MARK
SCHEME**

General Marking Instructions

Introduction

Mark schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of students in schools and colleges.

The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes, therefore, are regarded as part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

			AVAILABLE MARKS
1 (a) (i)	$\mathbf{P} + 3\mathbf{Q} = \begin{bmatrix} 2 & -5 \\ -1 & 4 \end{bmatrix} + 3 \begin{bmatrix} 4 & -2 \\ 5 & -3 \end{bmatrix}$ $= \begin{bmatrix} 2 & -5 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 12 & -6 \\ 15 & -9 \end{bmatrix} = \begin{bmatrix} 14 & -11 \\ 14 & -5 \end{bmatrix}$	MW1	
	$\text{(ii) } \mathbf{P}^2 = \begin{bmatrix} 2 & -5 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 9 & -30 \\ -6 & 21 \end{bmatrix}$	M1 W1	
(b) AB		MW1	
(c) E + F		MW1	5
2	$x^2 + 8x - 7 = 0$ $(x + 4)^2 - 16 - 7 = 0$ $(x + 4)^2 - 23 = 0$ $(x + 4)^2 = 23$ $(x + 4) = \pm \sqrt{23}$ $x = -4 \pm \sqrt{23}$	M1 W1 W1 W1	4
3 (a)	$y = 7x^2 - \frac{3}{x^3} + \frac{5x^4}{4}$ $= 7x^2 - 3x^{-3} + \frac{5}{4}x^4$ $\frac{dy}{dx} = 14x + 9x^{-4} + 5x^3$ <p style="margin-left: 2em;">or $14x + \frac{9}{x^4} + 5x^3$</p>	MW1 × 3	
(b)	$\int \left(9x^3 + \frac{3}{7x^4} - 5 \right) dx$ $= \int \left(9x^3 + \frac{3}{7}x^{-4} - 5 \right) dx$ $= \frac{9}{4}x^4 - \frac{1}{7}x^{-3} - 5x + c$ <p style="margin-left: 2em;">or $\frac{9}{4}x^4 - \frac{1}{7x^3} - 5x + c$</p>	MW1 × 4	7

4 (a) $x + 2b = c + a$

$x = c + a - 2b$

$x = \begin{bmatrix} -2 \\ 7 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2 \begin{bmatrix} 1 \\ -4 \end{bmatrix}$

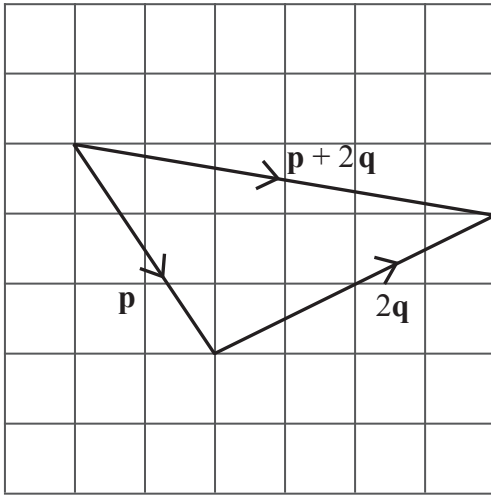
$x = \begin{bmatrix} 1 \\ 5 \end{bmatrix} - \begin{bmatrix} 2 \\ -8 \end{bmatrix}$

$x = \begin{bmatrix} -1 \\ 13 \end{bmatrix}$

M1

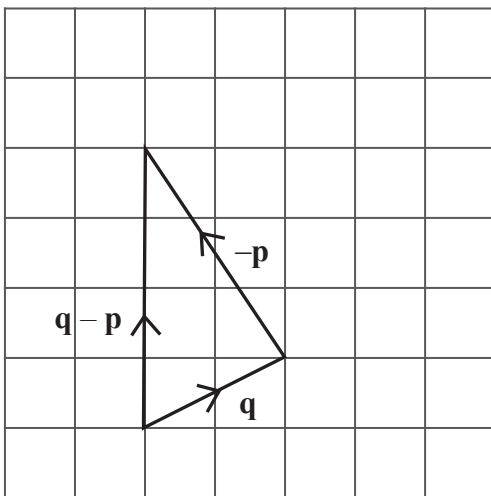
W1

(b) (i)



W1

(ii)



W1

4

5 (i) $\cos \theta = 0.3$

$$\theta = 72.542^\circ \text{ or } -72.542^\circ$$

$$\rightarrow 72.54^\circ \text{ or } -72.54^\circ$$

W1 W1

(ii) $\cos\left(\frac{x}{2} + 20^\circ\right) = 0.3$

From (i)

$$\frac{x}{2} + 20^\circ = 72.542^\circ \text{ or } -72.542^\circ$$

M1

$$\frac{x}{2} = 52.542^\circ \text{ or } -92.542^\circ$$

$$x = 105.084^\circ \text{ or } -185.084^\circ$$

$$\rightarrow 105.08^\circ \text{ or } -185.08^\circ$$

W1 W1

5

6 $\mathbf{YX} = \mathbf{Z}$

$$\mathbf{X} = \mathbf{Y}^{-1}\mathbf{Z}$$

$$\mathbf{Y}^{-1} = -\frac{1}{13} \begin{bmatrix} 1 & -2 \\ -5 & -3 \end{bmatrix}$$

M1 W1 W1

$$\mathbf{X} = -\frac{1}{13} \begin{bmatrix} 1 & -2 \\ -5 & -3 \end{bmatrix} \begin{bmatrix} 16 \\ -5 \end{bmatrix} = -\frac{1}{13} \begin{bmatrix} 26 \\ -65 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

M1 W1

5

AVAILABLE
MARKS

		AVAILABLE MARKS
7 (a)	$4^{2x-3} = 6^{x+4}$	
	$\log 4^{2x-3} = \log 6^{x+4}$	M1
	$(2x - 3) \log 4 = (x + 4) \log 6$	M1
	$2x \log 4 - 3 \log 4 = x \log 6 + 4 \log 6$	
	$x(2 \log 4 - \log 6) = 4 \log 6 + 3 \log 4$	MW1
	$x = \frac{4 \log 6 + 3 \log 4}{2 \log 4 - \log 6} = 11.55$	M1 W1
(b)	$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$	M1
	$\log_3 90 = \log_3 2 + \log_3 3^2 + \log_3 5$	
	$= \log_3 2 + 2 \log_3 3 + \log_3 5$	MW1
	$= y + 2 + x$	W1
(c)	$\log z = 3 \log x - \log y$	
	$= \log x^3 - \log y$	
	$= \log \left(\frac{x^3}{y} \right)$	MW1
	$z = \frac{x^3}{y}$	W1
		10

		AVAILABLE MARKS
8 (a)	$\frac{x-6}{3x^2-11x-4} \div \frac{2x^2-12x}{2x-8}$	
	$= \frac{x-6}{3x^2-11x-4} \times \frac{2x-8}{2x^2-12x}$	M1
	$= \frac{x-6}{(3x+1)(x-4)} \times \frac{2(x-4)}{2x(x-6)}$	MW1 MW1
	$= \frac{1}{x(3x+1)}$	W1
(b)	$\frac{2}{x+2} - \frac{x-7}{2x^2+3x-2}$	
	$= \frac{2}{x+2} - \frac{x-7}{(2x-1)(x+2)}$	MW1
	$= \frac{2(2x-1) - (x-7)}{(2x-1)(x+2)}$	M1 W1
	$= \frac{3x+5}{(2x-1)(x+2)}$	W1
9 (i)	$y = 3x^2 - x + 4$	
	$\frac{dy}{dx} = 6x - 1$	M1 W1
	$= 11$	MW1
	$x = 2$	W1
	$y = 3(2^2) - 2 + 4 = 14$	
	$\therefore P = (2, 14)$	W1
(ii) At (-1, 8)		
$\frac{dy}{dx} = 6x - 1 = -7$	W1	
gradient of normal $= \frac{1}{7}$	M1	
Equation of normal is $y = \frac{1}{7}x + c$		
$8 = \frac{1}{7}(-1) + c$	MW1	
$c = \frac{57}{7}$		
$y = \frac{1}{7}x + \frac{57}{7}$	W1	
		8
		9

10 (i) $AP^2 = AB^2 + BP^2 - 2 \cdot AB \cdot BP \cos \widehat{ABP}$

$$AP^2 = 960^2 + 300^2 - 2(960)(300) \cos 56.75^\circ$$

MW1

$$AP = 834.136 \rightarrow 834.14 \text{ m}$$

W1

(ii) $\frac{\sin \widehat{PAB}}{300} = \frac{\sin 56.75^\circ}{834.136}$

$$\sin \widehat{PAB} = \frac{300 \sin 56.75^\circ}{834.136}$$

MW1

$$\widehat{PAB} = 17.504^\circ \rightarrow 17.50^\circ$$

W1

(iii) $\widehat{TAB} = 63.44^\circ + 17.504^\circ = 80.944^\circ \rightarrow 80.94^\circ$

W1

(iv) $\widehat{ATB} = 180^\circ - (80.944^\circ + 56.75^\circ) = 42.306^\circ \rightarrow 42.31^\circ$

W1

(v) $\frac{AT}{\sin 56.75^\circ} = \frac{960}{\sin 42.306^\circ}$

MW1

$$AT = 1192.760 \rightarrow 1192.76 \text{ m}$$

W1

Alternative solution

$$\widehat{APT} = 180^\circ - 63.44^\circ - 42.306^\circ = 74.254^\circ$$

$$\frac{AT}{\sin 74.254^\circ} = \frac{834.136}{\sin 42.306^\circ}$$

MW1

$$AT = 1192.760 \rightarrow 1192.76 \text{ m}$$

W1

(vi) $\text{Speed} = \frac{AT}{3.6} = \frac{1192.760}{3.6}$

M1

$$= 331.32 \text{ m/s}$$

W1

10

11 (i) $y = x^3 - 2x^2 - 8x = 0$

M1

$$x(x^2 - 2x - 8) = 0$$

$$x(x - 4)(x + 2) = 0$$

$$x = 0, 4 \text{ or } -2$$

meets x -axis at $(0,0)$, $(4,0)$, $(-2,0)$

MW1 MW1

(ii) $\frac{dy}{dx} = 3x^2 - 4x - 8 = 0$

M1 W1 M1

$$x = \frac{4 \pm \sqrt{4^2 - 4(3)(-8)}}{6}$$

$$x = 2.431 \text{ or } -1.097$$

$$y = -16.901 \text{ or } 5.049$$

MW1

Turning points at $(2.43, -16.90)$, $(-1.10, 5.05)$

MW1

(iii) $\frac{d^2y}{dx^2} = 6x - 4$

MW1

When $x = 2.431$, $\frac{d^2y}{dx^2} = 10.586 > 0$

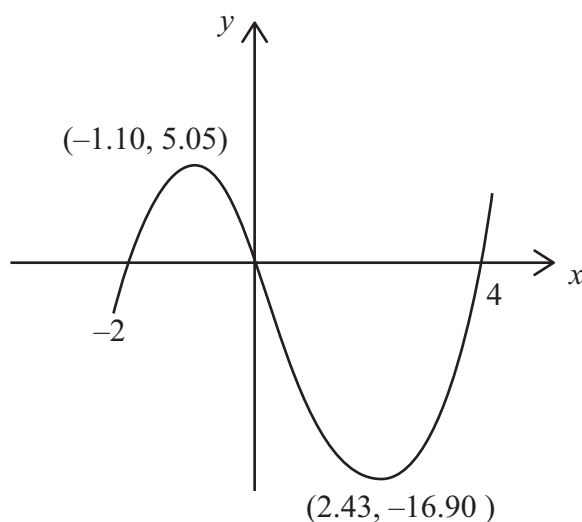
\therefore minimum at $(2.43, -16.90)$

When $x = -1.097$, $\frac{d^2y}{dx^2} = -10.582 < 0$

maximum at $(-1.10, 5.05)$

MW1

(iv)



M1 (shape)

W1 (points)

(v) $\int_0^4 (x^3 - 2x^2 - 8x) dx$

M1 M1

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - 4x^2 \right]_0^4$$

MW1

$$= \left[64 - \frac{128}{3} - 64 \right] - [0]$$

$$= -42\frac{2}{3}$$

area = $42\frac{2}{3}$

W1

AVAILABLE
MARKS

12 (i) $12(2x + 2y) = 1800$

MW1

$$x + y = 75$$

(ii) $5xy = 6750$

$$xy = 1350$$

MW1

(iii) $y = 75 - x$ (or $y = \frac{1350}{x}$)

MW1

$$x(75 - x) = 1350 \quad \left(\text{or } x + \frac{1350}{x} = 75\right)$$

M1

$$75x - x^2 = 1350 \quad (\text{or } x^2 + 1350 = 75x)$$

$$x^2 - 75x + 1350 = 0$$

W1

$$x = \frac{75 \pm \sqrt{75^2 - 4(1)(1350)}}{2} = \frac{75 \pm 15}{2} \quad [\text{or } (x - 45)(x - 30) = 0]$$

M1

$$x = 45 \text{ or } 30$$

$$y = 30 \text{ or } 45$$

Dimensions are 45 cm \times 30 cm

W1

7

AVAILABLE
MARKS

13 (i) $A = xy + 2x$

MW1

(ii) $P = 4x + 2y + 4$

MW1

(iii) $A = xy + 2x = 30$

$$xy = 30 - 2x$$

$$y = \frac{30}{x} - 2$$

MW1

$$P = 4x + 2\left(\frac{30}{x} - 2\right) + 4$$

M1

$$P = 4x + \frac{60}{x} - 4 + 4$$

W1

$$P = 4x + \frac{60}{x}$$

(iv) $\frac{dP}{dx} = 4 - \frac{60}{x^2} = 0$

MW1

$$x^2 = 15$$

$$x = \sqrt{15} = 3.873 \rightarrow 3.87 \text{ m}$$

W1

$$y = \frac{30}{x} - 2 = \frac{30}{\sqrt{15}} - 2 = 2\sqrt{15} - 2$$

$$= 5.746 \rightarrow 5.75 \text{ m}$$

W1

$$\frac{d^2P}{dx^2} = \frac{120}{x^3} > 0 \therefore \text{minimum}$$

MW1

(v) $P = 4x + 2y + 4$

$$= 4\sqrt{15} + 2(2\sqrt{15} - 2) + 4$$

$$= 8\sqrt{15} = 30.98 \text{ m}$$

MW1

Alternative solution

$$P = 4x + \frac{60}{x}$$

$$= 4\sqrt{15} + \frac{60}{\sqrt{15}} = 4\sqrt{15} + 4\sqrt{15}$$

$$= 8\sqrt{15} = 30.98 \text{ m}$$

MW1

Total

AVAILABLE
MARKS

10

100