Rewarding Learning

ADVANCED
General Certificate of Education
2024

Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AMT21]

WEDNESDAY 12 JUNE, AFTERNOON

MARK SCHEME

General Marking Instructions

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A: Mechanics

AVAILABLE MARKS

1 **(i)** Before: ⟶ +ve

⟶ 8 m s^{-1} 5 m s^{-1} ⟵

Ⓐ Ⓑ

After:

4 m s^{-1} ⟵ ⟶ 4 m s^{-1}

Ⓐ Ⓑ

A: $I = Mv - Mu$

$I = 3(-4) - 3(8)$ M1W1

$I = -36$ Ns so $|I| = 36$ Ns W1

(ii) B: $I = mv - mu$

$36 = m(4 - (-5))$ M1W1

$36 = 9m$

$m = 4$ W1

Alternative solution:

Momentum before = Momentum after

$3(8) + m(-5) = 3(-4) + m(4)$ M1W1

$36 = 9m$

$m = 4$ W1 6

2 **(i)** $v = \int a \, dt$

$= \int (6 - 2t) \, dt$ M1

$= 6t - t^2 + c$

$t = 0$, $v = 0$ so $c = 0$

$v = 6t - t^2$ MW1

When $t = 3$, $v = 6(3) - (3)^2$

$= 9$

Since $a = 0$ for $t > 3$, $v = 9$ for $t > 3$ W1

(ii) $s = \int v \, dt$

$= \int (6t - t^2) \, dt$ M1

$= 3t^2 - \frac{1}{3}t^3 + c$

$t = 0$ when $s = 0$ so $c = 0$

$s = 3t^2 - \frac{1}{3}t^3$ W1

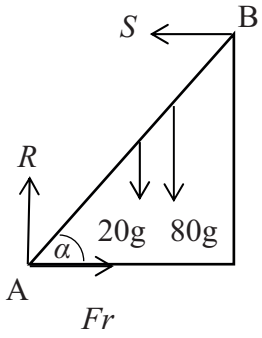
When $t = 3$, $s = 3(3)^2 - \frac{1}{3}(3)^3$

$s = 18$ m W1

(iii) $18 + 9(t_1) = 24 + 8(t_1)$ where t_1 is the time after Jerry's acceleration becomes zero. M1

$t_1 = 6$ seconds W1

Total time = 9 seconds MW1 9

			AVAILABLE MARKS
3	**(i)** 	MW2	
	(ii) Resolving forces vertically $R = 100g$	MW1	
	Resolving forces horizontally $S = Fr$	MW1	
	At the point of slipping $Fr = \mu R$	M1	
	Taking moments about A $20g \cos \alpha\ (2) + 80g \cos \alpha\ (2.5) = \mu\ 100g \sin \alpha\ (4)$	M2W2	
	$20g\left(\frac{3}{5}\right)(2) + 80g\left(\frac{3}{5}\right)(2.5) \quad = \mu\ 100g\left(\frac{4}{5}\right)(4)$	MW1	
	$\mu = 0.45$	W1	11
4	**(i)** ↑ at greatest height $v = 0$ $u = u \sin \theta$, $v = 0$, $a = -g$, $s = h$ $v^2 = u^2 + 2as$ $0 = u^2 \sin^2 \theta - 2gh$	M2W1	
	$h = \dfrac{u^2 \sin^2 \theta}{2g}$	W1	
	(ii) $\dfrac{25^2 \sin^2 \theta}{2(9.8)} < 8$	M1W1	
	$\sin^2 \theta < 0.25088$ $\theta < 30.06$		
	The greatest angle is 30°	W1	
	(iii) $s = ut + \frac{1}{2} at^2$ $-2 = 40 \sin (-5°)\ t - \frac{1}{2}\ (9.8)\ t^2$	M1W1	
	$4.9t^2 + (3.4862\ldots)\ t - 2 = 0$	W1	
	$t = \dfrac{-3.4862... \pm \sqrt{3.4862...^2 - 4(4.9)(-2)}}{9.8}$ $t = 0.3755\ldots$		
	$t = 0.376$ s	W1	11

			AVAILABLE MARKS
5	**(i)** $\mathbf{a} = \frac{d\mathbf{v}}{dt}$		
	$= -2t\,\mathbf{i} - 7\mathbf{j}$	M1W1	
	(ii) $\mathbf{s} = \int \mathbf{v}\, dt$		
	$\mathbf{s} = \left(8t - \frac{t^3}{3}\right)\mathbf{i} + \left(18t - \frac{7t^2}{2}\right)\mathbf{j} + \mathbf{c}$	M1W1	
	$t = 0$ when $\mathbf{s} = 2\mathbf{i} + \mathbf{j}$ so $\mathbf{c} = 2\mathbf{i} + \mathbf{j}$	M1	
	$\mathbf{s} = \left(2 + 8t - \frac{t^3}{3}\right)\mathbf{i} + \left(1 + 18t - \frac{7t^2}{2}\right)\mathbf{j}$	W1	
	(iii) **i** and **j** components of **v** must be equal for a bearing of 045°		
	$8 - t^2 = 18 - 7t$	MW1	
	$t^2 - 7t + 10 = 0$		
	$(t - 2)(t - 5) = 0$	MW1	
	$t = 2$ or $t = 5$	W1	
	When $t = 2$, $\mathbf{v} = 4\mathbf{i} + 4\mathbf{j}$		
	When $t = 5$, $\mathbf{v} = -17\mathbf{i} - 17\mathbf{j}$	M1	
	For a bearing of 045° the components of **i** and **j** need to be positive so $t = 2$	W1	
	(iv) speed $= \sqrt{4^2 + 4^2}$	M1	
	$= 5.66\ \text{m s}^{-1}$	W1	13
		Section A	**50**

Section B (Statistics)

AVAILABLE MARKS

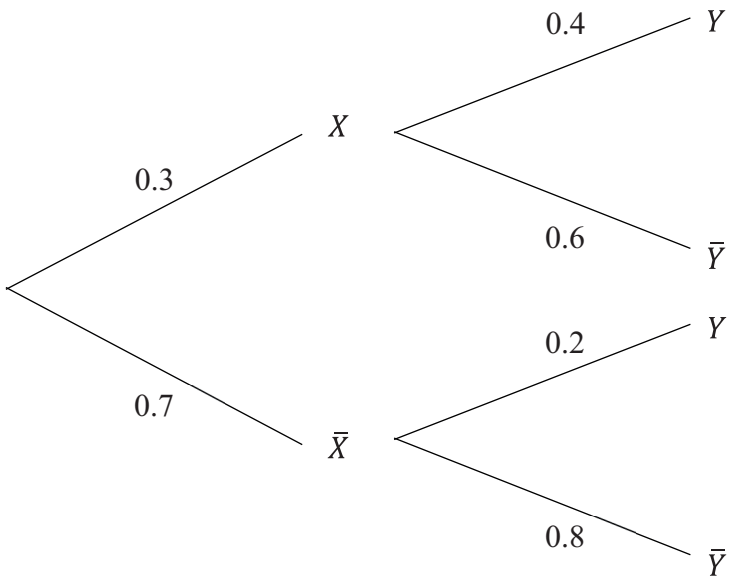
6 **(i)** 0.4973 MW1

(ii) Since 0.516 > 0.4973 there is sufficient evidence of a positive correlation at the 5% level of significance. MW2 — 3

7 **(a)** **(i)** $P(\overline{A}|B) = P(\overline{A}) = 0.7$ MW1

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= P(A) + P(B) - P(A)P(B)$
$= 0.3 + 0.65 - 0.3 \times 0.65$ M2
$= 0.755$ W1

(b) **(i)**



MW2

$P(Y) = P(X)P(Y|X) + P(\overline{X})P(Y|\overline{X})$ M1
$= 0.3 \times 0.4 + 0.7 \times 0.2$ MW1
$= 0.26$ W1

(ii) $P(\overline{X}|\overline{Y}) = \dfrac{P(\overline{X} \cap \overline{Y})}{P(\overline{Y})}$ M1

$= \dfrac{0.7 \times 0.8}{1 - 0.26}$ MW2

$= \dfrac{28}{37}$ W1 — 13

8 SOLUTION 1: using calculator functions

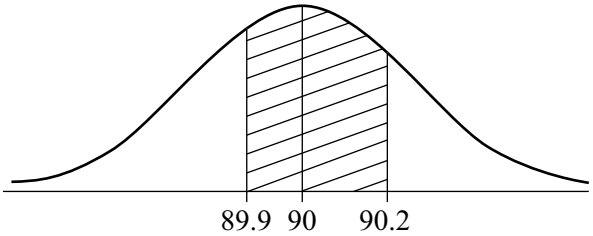
AVAILABLE MARKS

(i) $\Phi^{-1}(0.791) = 0.81$ MW1

$\frac{90.324 - \mu}{0.4} = 0.81$ M1 MW1

$\mu = 90$ W1

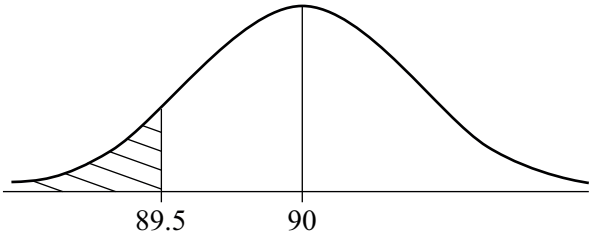
(ii) $\mu = 90$, $\sigma = 0.4$ MW1



MW1

$P(89.9 < L < 90.2) = 0.290$ MW1

(iii)



MW1

$P(L < 89.5) = 0.1056$ MW1

P(neither is usable) $= 0.1056^2$ M1

$= 0.0112$ W1

(iv) The lengths of wood are independent of each other. MW1

8 SOLUTION 2: Using tables

AVAILABLE MARKS

(i) $\Phi^{-1}(0.791) = 0.81$ MW1

$\dfrac{90.324 - \mu}{0.4} = 0.81$ M1 MW1

$\mu = 90$ W1

(ii) $P\left(\dfrac{89.9 - 90}{0.4} < Z < \dfrac{90.2 - 90}{0.4}\right)$ MW1

$= P(-0.25 < Z < 0.5)$

$= \Phi(0.25) + \Phi(0.5) - 1$ M1

$= 0.5987 + 0.6915 - 1$

$= 0.290$ W1

(iii) $P\left(Z < \dfrac{89.5 - 90}{0.4}\right)$

$= P(Z < -1.25)$

$= 1 - \Phi(1.25)$ MW1

$= 1 - 0.8944$

$= 0.1056$ W1

P(neither is usable) $= 0.1056^2$ M1

$= 0.0112$ W1

(iv) The lengths of wood are independent of each other. MW1 12

9 (i) $H_0 : p = 0.2$ MW1

$H_1 : p > 0.2$ (one-tailed test) MW1

(ii) The critical region of a test is the set of values of the test statistic which would result in the null hypothesis being rejected. MW1

(iii) Let D be the number of DVD players returned and c the critical value.

$P(D \geqslant c) < 0.05$ M1

$1 - P(D \leqslant c - 1) < 0.05$

$P(D \leqslant c - 1) > 0.95$ M1

Under H_0 $D \sim \text{Bin}(10, 0.2)$ M1

Tables: $P(D \leqslant 3) = 0.8791$ MW1

$P(D \leqslant 4) = 0.9672$ MW1

$c - 1 = 4$

$c = 5$ W1

Therefore the critical region for the test is $D \geqslant 5$ MW1

(iv) Three DVD players were returned but three is outside the critical region. MW1

Do not reject H_0. MW1

We conclude that there is insufficient evidence at the 5% level of significance to suggest an increase in the probability that a DVD player is returned. MW1 13

			AVAILABLE MARKS
10	$T \sim N(35, 3.1^2)$		
	$H_0: \mu = 35$	MW1	
	$H_1: \mu > 35$ (one-tailed test)	MW1	
	Critical value is 1.645	MW1	
	$\bar{z} = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$		
	$= \dfrac{36 - 35}{\dfrac{3.1}{\sqrt{40}}}$	M1 MW1	
	$= 2.04$	W1	
	$\bar{z} > 1.645$	MW1	
	Reject H_0.	MW1	
	We conclude that there is sufficient evidence at the 5% level of significance to support Tom's claim that his journey time increased during the roadworks.	MW1	9
		Section B	**50**
		Total	**100**