



Rewarding Learning

**ADVANCED
General Certificate of Education
2024**

Mathematics

Assessment Unit A2 1

assessing

Pure Mathematics

[AMT11]

THURSDAY 30 MAY, MORNING

**MARK
SCHEME**

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

- (a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

		AVAILABLE MARKS
1	(a) $\frac{x+3}{(x-5)(x+3)} - \frac{1}{2x-7}$	MW1
	$= \frac{1}{x-5} - \frac{1}{2x-7}$	
	$= \frac{(2x-7) - (x-5)}{(x-5)(2x-7)}$	M1 W1
	$= \frac{2x-7-x+5}{(x-5)(2x-7)}$	
	$= \frac{x-2}{(x-5)(2x-7)}$	W1
(b)	$\frac{3(x+3)(x-3)}{(3x+1)(x-3)} \times \frac{x-7}{(3x+1)(x+3)}$	M1 W2
	$= \frac{3(x-7)}{(3x+1)^2}$	M1
		W1
2	(a) (i) $\sum_{r=1}^9 (4r-11)$	MW1
		MW1
	(ii) $\sum_{r=1}^{\infty} \frac{r+5}{r^2}$	MW1
		MW1
		MW1
	(b) $u_{n+1} = au_n + b$	
	$u_2 = au_1 + b$	M1
	$210 = 150a + b$	W1
	$u_3 = au_2 + b$	
	$288 = 210a + b$	MW1
$288 = 210a + b$		
$-210 = 150a + b$	M1	
$\frac{78 = 60a}{a = 1.3}$	W1	
$b = 15$	MW1	
		9
		11

		M1 MW2	AVAILABLE MARKS	
3	(a) (i)	$\frac{dy}{dx} = 2x \ln x + x^2 \left(\frac{1}{x}\right)$		
		$= 2x \ln x + x$	W1	
	(ii)	$\frac{dy}{dx} = \frac{2x^4 (6xe^{x^2}) - 3e^{x^2} (8x^3)}{(2x^4)^2}$	M1 MW2	
		$\frac{dy}{dx} = \frac{12x^5 e^{x^2} - 24x^3 e^{x^2}}{4x^8}$	MW1	
		$\frac{dy}{dx} = \frac{12x^3 e^{x^2} (x^2 - 2)}{4x^8}$		
		$\frac{dy}{dx} = \frac{3e^{x^2} (x^2 - 2)}{x^5}$	W1	
	(b) (i)	$\int \left(4x^5 + \frac{1}{2x} - 3e^{-x}\right) dx$		
		$= \frac{2}{3} x^6 + \frac{1}{2} \ln x + 3e^{-x} + c$	MW3	
	(ii)	$\int \sin x \cos^3 x dx$		
		$= -\frac{1}{4} \cos^4 x + c$	MW2	14
4	(a)	$S_n = a + ar + ar^2 + ar^3 + \dots + ar^{n-1}$	MW1	
		$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-1} + ar^n$	M1W1	
	Subtract			
	$S_n - rS_n = a - ar^n$		MW1	
	$S_n (1 - r) = a(1 - r^n)$			
	$S_n = \frac{a(1 - r^n)}{(1 - r)}$		W1	
	(b) (i)	$a = 10, r = 0.9$		
		$u_n = a \times r^{n-1}$ $u_8 = 10(0.9)^7$ $u_8 = 4.78 \text{ cm (3sf)}$	M1 W1	
	(ii)	$S_n \geq 90$		
		$\frac{10(1 - 0.9^n)}{(1 - 0.9)} \geq 90$	M1 W1	
$1 - 0.9^n \geq 0.9$				
$0.1 \geq 0.9^n$				
$\log 0.1 \geq n \log 0.9$		M1		
$n \geq \frac{\log 0.1}{\log 0.9}$ $n \geq 21.85$ Therefore, the child needs 22 blocks.		W1		

- (iii) You need an infinite number of blocks to reach this limit which is not practically possible in this context.

or

Blocks would become so small that it would be impossible to manufacture them/hold them/build with them.

MW1

AVAILABLE
MARKS

12

5 (a) (i) $\operatorname{cosec}^4 \theta - \cot^4 \theta$
 $\equiv (\operatorname{cosec}^2 \theta - \cot^2 \theta)(\operatorname{cosec}^2 \theta + \cot^2 \theta)$ M1 W1
 $\equiv (1)(\operatorname{cosec}^2 \theta + \cot^2 \theta)$ M1
 $\equiv \operatorname{cosec}^2 \theta + \cot^2 \theta$ MW1

(ii) $\operatorname{cosec}^2 \theta + \cot^2 \theta = -3 \cot \theta$ M1
 $2 \cot^2 \theta + 3 \cot \theta + 1 = 0$ W1
 $(2 \cot \theta + 1)(\cot \theta + 1) = 0$
 $\cot \theta = -\frac{1}{2}$ MW1
 $\tan \theta = -2$ MW1
 $\cot \theta = -1$
 $\tan \theta = -1$

$\theta = 2.03 \text{ rad}, 5.18 \text{ rad}, \frac{3\pi}{4}, \frac{7\pi}{4}$ W2

(b) (i) $3 \cos \theta - 5 \sin \theta = R \cos \theta \cos \alpha - R \sin \theta \sin \alpha$ MW1

$R \cos \alpha = 3$ MW1
 $R \sin \alpha = 5$
 $\tan \alpha = \frac{5}{3}$ M1
 $\alpha = 59.0362^\circ$ W1

Also
 $R = \sqrt{3^2 + 5^2}$ MW1
 $R = \sqrt{34}$

$3 \cos \theta - 5 \sin \theta \equiv \sqrt{34} \cos(\theta + 59.0^\circ)$ W1

(ii) Function can be re-written as
 $8 - (3 \cos \theta - 5 \sin \theta)$
 $= 8 - \sqrt{34} \cos(\theta + 59.0362^\circ)$ M1

Maximum value is $8 + \sqrt{34}$ W1

This occurs when $\cos(\theta + 59.0362^\circ) = -1$ M1
 $\theta + 59.0362^\circ = 180^\circ$

$\theta = 121^\circ$ (3sf) MW1

20

<p>6 (i) $(8 - 2x)^{\frac{1}{3}}$</p> $= 8^{\frac{1}{3}} \left(1 - \frac{x}{4}\right)^{\frac{1}{3}}$ $= 2 \left[1 + \frac{1}{3} \left(-\frac{x}{4}\right) + \frac{\frac{1}{3} \left(-\frac{2}{3}\right)}{2!} \left(-\frac{x}{4}\right)^2 + \dots \right]$ $= 2 \left(1 - \frac{x}{12} - \frac{x^2}{144} + \dots\right)$ $= 2 - \frac{x}{6} - \frac{x^2}{72}$	<p>M1W1</p> <p>MW2</p> <p>W1</p>	
<p>(ii) Valid for $\left \frac{x}{4}\right < 1$</p> $-4 < x < 4$	<p>MW1</p>	
<p>(iii) $8 - 2x = 7.1$</p> $x = 0.45$ $(7.1)^{\frac{1}{3}} \approx 2 - \frac{0.45}{6} - \frac{(0.45)^2}{72} = 1.9222 \text{ (5sf)}$	<p>MW1</p> <p>MW1</p>	
<p>(iv) Suggestions: Increase the number of terms in your expansion</p> <p>e.g. adding one further term would give</p> $2 \left[1 + \frac{1}{3} \left(-\frac{x}{4}\right) + \frac{\frac{1}{3} \left(-\frac{2}{3}\right)}{2!} \left(-\frac{x}{4}\right)^2 + \frac{\frac{1}{3} \left(-\frac{2}{3}\right) \left(-\frac{5}{3}\right)}{3!} \left(-\frac{x}{4}\right)^3 + \dots \right]$ <p>If $x = 0.45$</p> $(7.1)^{\frac{1}{3}} \approx 2 - \frac{0.45}{6} - \frac{(0.45)^2}{72} - \frac{5}{2592} (0.45)^3 = 1.9220 \text{ (5sf)}$	<p>M1</p> <p>MW1</p>	

		AVAILABLE MARKS		
7	$u = 2 - x, \quad x = 2 - u$ $\frac{du}{dx} = -1$ $dx = -du$ $\int -(2 - u)u^{12} du$ $\int (-2u^{12} + u^{13}) du$ $= -\frac{2}{13} u^{13} + \frac{1}{14} u^{14} + c$ Therefore $\int x(2 - x)^{12} dx = -\frac{2}{13} (2 - x)^{13} + \frac{1}{14} (2 - x)^{14} + c$	MW1 M1 MW1 W2 MW1 W1	7	
8	(i) $\frac{1}{P(5 - P)} \equiv \frac{A}{P} + \frac{B}{5 - P}$ $1 \equiv A(5 - P) + BP$ $P = 0$ $1 = 5A$ $A = \frac{1}{5}$ $P = 5$ $1 = 5B$ $B = \frac{1}{5}$ $\frac{1}{P(5 - P)} \equiv \frac{1}{5P} + \frac{1}{5(5 - P)}$	M1 W1 MW1 MW1		12
	(ii) $\frac{dP}{dt} = 0.006P(5 - P)$ $\int \frac{dP}{P(5 - P)} = \int 0.006 dt$ $\int \frac{1}{5P} + \frac{1}{5(5 - P)} dP = \int 0.006 dt$ $\frac{1}{5} \ln P - \frac{1}{5} \ln 5 - P = 0.006t + c$ When $t = 0, P = 4$ $\frac{1}{5} \ln 4 - \frac{1}{5} \ln 1 = 0.006(0) + c$ $c = \frac{1}{5} \ln 4$ When $P = 4.5, t = ?$ $\frac{1}{5} \ln 4.5 - \frac{1}{5} \ln 0.5 = 0.006t + \frac{1}{5} \ln 4$ $t = 27.0$	M1 W1 MW1 MW2 M1 W1 MW1		

9 (i) $x^2 + 4xy + y^2 + 12 = 0$

$$2x + 4y + 4x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

M2 W2

$$\frac{dy}{dx} (4x + 2y) = -2x - 4y$$

$$\frac{dy}{dx} = \frac{2(-x - 2y)}{2(2x + y)}$$

$$\frac{dy}{dx} = \frac{-x - 2y}{2x + y}$$

MW1

(ii) If tangent is parallel to y-axis, normal is parallel to x-axis therefore

$$2x + y = 0$$

M1

$$y = -2x$$

W1

Sub into first equation

$$x^2 + 4x(-2x) + (-2x)^2 + 12 = 0$$

M1

$$x^2 - 8x^2 + 4x^2 + 12 = 0$$

$$-3x^2 + 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

W1

x is negative $\Rightarrow x = -2$

If $x = -2, y = 4$

Therefore A is the point $(-2, 4)$.

W1

AVAILABLE
MARKS

10

			AVAILABLE MARKS
10 (i)	$x = 3 + \tan \theta$ $\tan^2 \theta = (x - 3)^2$	$y + 3 = \sec \theta$ $\sec^2 \theta = (y + 3)^2$	M1 W1 MW2
	$\tan^2 \theta + 1 = \sec^2 \theta$ $(x - 3)^2 + 1 = (y + 3)^2$		M1 W1
(ii)	$\frac{dx}{d\theta} = \sec^2 \theta$		MW1
	$\frac{dy}{d\theta} = \sec \theta \tan \theta$		MW1
	$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$		M1
	$= \sec \theta \tan \theta \times \frac{1}{\sec^2 \theta}$		W1
	$= \sin \theta$		
	Turning points when $\frac{dy}{dx} = 0$		
	$\sin \theta = 0$		M1
	$\theta = 0, \pi$		W2
	When $\theta = 0$		
	$x = 3, y = -2$		M1 W1
	When $\theta = \pi$		
	$x = 3, y = -4$		MW1
	Nature		
	$\frac{d^2y}{dx^2} = \frac{d}{d\theta} (\sin \theta) \times \frac{d\theta}{dx}$		M1
	$= \cos \theta \times \frac{1}{\sec^2 \theta}$		
	$= \cos^3 \theta$		W1
	When $\theta = 0, \frac{d^2y}{dx^2} > 0$ When $\theta = \pi, \frac{d^2y}{dx^2} < 0$		
	Maximum at $(3, -4)$		
	Minimum at $(3, -2)$		MW1

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11 (i)	$2x = 4x \cos 2x$ $2x - 4x \cos 2x = 0$ $2x(1 - 2 \cos 2x) = 0$	M1
	$x = 0 \quad \text{or} \quad \cos 2x = \frac{1}{2}$	
	$\cos 2x = \frac{1}{2}$	W1
	$x = \frac{\pi}{6}, \frac{5\pi}{6}$	MW1
(ii)	$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (2x - 4x \cos 2x) dx$	M2 MW1
	$\int 4x \cos 2x dx$	
	$u = 4x$	
	$\frac{du}{dx} = 4$	M2
	$\frac{dv}{dx} = \cos 2x$	
	$v = \frac{1}{2} \sin 2x$	MW1
	$\int 4x \cos 2x dx = \frac{1}{2} (4x)(\sin 2x) - \int 4\left(\frac{1}{2}\right) \sin 2x dx$	MW2
	$= 2x \sin 2x - \int 2 \sin 2x dx$	
	$= 2x \sin 2x + \cos 2x$	W1
	$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (2x - 4x \cos 2x) dx = [x^2 - 2x \sin 2x - \cos 2x]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$	MW1
	$= \left[\left(\frac{5\pi}{6}\right)^2 - 2\left(\frac{5\pi}{6}\right) \sin\left(\frac{5\pi}{3}\right) - \cos\left(\frac{5\pi}{3}\right) \right] - \left[\left(\frac{\pi}{6}\right)^2 - 2\left(\frac{\pi}{6}\right) \sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{3}\right) \right]$	
	$= \frac{2\pi^2}{3} + \pi\sqrt{3}$	MW1
	$\text{Area} = \left(\frac{2\pi^2}{3} + \pi\sqrt{3}\right) \text{ square units}$	

AVAILABLE MARKS

14

<p>12 (i) $gf(x) = g\left(\frac{3}{2x+5}\right)$ $= \left \frac{3}{2x+5}\right ,$ $x \in \mathbb{R}, x \neq -\frac{5}{2}$ $gf(x) \in \mathbb{R}, gf(x) > 0$</p> <p>(ii) $h(x) \in \mathbb{R}, h(x) > 2$</p> <p>(iii) $y = 2x^2 + 5x + 2$ $y = 2\left[x^2 + \frac{5}{2}x + 1\right]$ $y = 2\left[\left(x + \frac{5}{4}\right)^2 - \frac{25}{16} + 1\right]$ $y = 2\left(x + \frac{5}{4}\right)^2 - \frac{9}{8}$ $\frac{y}{2} + \frac{9}{16} = \left(x + \frac{5}{4}\right)^2$ $x = \frac{-5 \pm \sqrt{8y+9}}{4}$</p>	<p>M1W1</p> <p>MW1</p> <p>MW1</p> <p>MW1</p> <p>M1</p> <p>M1 W1</p>
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As function must be one to one need to restrict inverse to only positive side for the required domain.

$h^{-1}(x) = \frac{-5 + \sqrt{8x+9}}{4}, x \in \mathbb{R}, x > 2$ $h^{-1}(x) \in \mathbb{R}, h^{-1}(x) > 0$	<p>W1 MW1</p> <p>MW1</p>
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Total

11

150

AVAILABLE
MARKS