Rewarding Learning

ADVANCED
General Certificate of Education
2024

Centre Number

Candidate Number

Mathematics

Assessment Unit A2 2
assessing
Applied Mathematics

MV18

[AMT21]

WEDNESDAY 12 JUNE, AFTERNOON

Time

1 hour 30 minutes, plus your additional time allowance.

Instructions to Candidates

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided. Do not write on blank pages.
Complete in black ink only.
Questions which require drawing or sketching should be completed using an HB pencil.
Candidates must answer **all** questions from sections A and B. Equal time should be spent on each section. Show clearly the full development of your answers.
Answers without working may not gain full credit.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

Information for Candidates

The total mark for this paper is 100. The total available mark for each section of this paper is 50.

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_{\mathbf{e}} z$

Answer all questions.

Section A

Mechanics

1 Two particles, A and B, of masses 3 kg and m kg respectively, are moving in opposite directions along the same straight line so that the particles collide directly.

Before the collision, the speeds of A and B are $8\,\mathrm{m\,s^{-1}}$ and $5\,\mathrm{m\,s^{-1}}$ respectively.

After the collision, the direction of motion of each particle is reversed and the speed of each particle is $4\,\mathrm{m\,s^{-1}}$

(i) Find the magnitude of the impulse exerted by B on A in the collision. [3 marks]

(ii) Find the value of m. [3 marks]

2 Tom and Jerry are competitive sprinters.

Tom is running along a straight horizontal road at a constant speed of $8\,\text{m}\,\text{s}^{-1}$

Jerry is stationary at a point on the road until Tom passes him.
Jerry immediately runs after Tom until he catches up with him.

The time, t seconds, is measured from the moment when Tom passes Jerry on the road.

Jerry's acceleration, $a\,\text{m}\,\text{s}^{-2}$, is given by

$$a = \begin{cases} 6 - 2t & \text{for} \quad 0 \leqslant t \leqslant 3 \\ 0 & \text{for} \quad t > 3 \end{cases}$$

(i) Show that Jerry's speed, $v\,\text{m}\,\text{s}^{-1}$, is given by

$$v = \begin{cases} 6t - t^2 & \text{for} \quad 0 \leqslant t \leqslant 3 \\ 9 & \text{for} \quad t > 3 \end{cases}$$ [3 marks]

(ii) Find how far Jerry has run when $t = 3$ [3 marks]

(iii) Find the time taken for Jerry to catch up with Tom.
[3 marks]

3 A uniform ladder, of length 4 m and mass 20 kg, has one end A resting on rough horizontal ground and its other end B resting against a smooth vertical wall.

The ladder is inclined at α degrees to the horizontal, where $\sin \alpha = \frac{4}{5}$

The coefficient of friction between the ladder and the ground is μ.

A man, of mass 80 kg, stands on the ladder at the point C, where AC = 2.5 m.

The ladder is on the point of slipping.

(i) Draw a diagram below showing the external forces acting on the ladder. [2 marks]

(ii) Find the value of μ. [9 marks]

4 A ball is projected at an angle θ above the horizontal with an initial velocity of u m s^{-1}

(i) Prove that the greatest height, h metres, of the ball above the horizontal is given by

$$h = \frac{u^2 \sin^2\theta}{2g}$$ [4 marks]

A tennis player is playing tennis on an indoor court.

The height of the ceiling above the floor of the court is 9 m.

The player hits the ball with a speed of $25\,\mathrm{m\,s^{-1}}$ into the air from a height of 1 m above the floor of the court.

(ii) Find the greatest angle of projection at which he can hit the ball if he wants to ensure it never hits the ceiling.

Give your answer to the nearest degree. [3 marks]

Another player hits the ball from a height of 2 m above the court at an angle of 5° below the horizontal.

The initial speed of the ball is 40 m s^{-1}

(iii) Find the time taken until it hits the horizontal floor.
[4 marks]

5 [**i** and **j** are unit vectors in the directions of East and North respectively.]

The velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ of a particle P at time t seconds is given by

$$\mathbf{v} = (8 - t^2)\,\mathbf{i} + (18 - 7t)\,\mathbf{j}$$

(i) Find an expression for the acceleration of P at time t. [2 marks]

When $t = 0$, P is passing through the point which has a displacement of $(2\mathbf{i} + \mathbf{j})$ m from a fixed point O.

(ii) Find an expression for the displacement of P from O at time t. [4 marks]

(iii) Find the time at which the particle is travelling on a bearing of 045° [5 marks]

(iv) Hence find the speed of P at this time. [2 marks]

Section B

Statistics

6 A researcher is testing whether the product–moment correlation coefficient of a bivariate population is positive. For a random sample of size 12, he calculates that $r = 0.516$ and decides to use a 5% level of significance.

(i) Write down the critical value for a one-tailed hypothesis test. [1 mark]

(ii) Based on this value, what conclusion can the researcher draw? [2 marks]

7 **(a)** Independent events A and B are such that $P(A) = 0.3$ and $P(B) = 0.65$

(i) Write down the value of $P(\overline{A} \mid B)$. [1 mark]

(ii) Find $P(A \cup B)$. [3 marks]

(b) Events X and Y are such that
$P(X) = 0.3$, $P(Y \mid X) = 0.4$ and $P(\overline{Y} \mid \overline{X}) = 0.8$

Using a tree diagram, or otherwise, calculate:

(i) $P(Y)$; [5 marks]

(ii) $P(\overline{X} \mid \overline{Y})$. [4 marks]

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(Questions continue overleaf)

8 A machine cuts lengths of wood for use as shelves on a bookcase. The lengths of wood are normally distributed with mean μ and standard deviation 0.4 cm.

(i) Given that 20.9% of the lengths of wood are longer than 90.324 cm, find the value of μ. [4 marks]

A length of wood is selected at random.

(ii) Find the probability that the length of this piece of wood is between 89.9 cm and 90.2 cm. [3 marks]

To be used as a shelf, a length of wood must be longer than 89.5 cm.

Two lengths of wood are selected at random.

(iii) Calculate the probability that neither length of wood can be used as a shelf. [4 marks]

(iv) What assumption must be made in calculating the probability in part **(iii)**? [1 mark]

9 Over time, the manager of an electrical goods store notices that 20**%** of customers who buy a DVD player return it.

Following a sales promotion, the manager suspects that the probability of a DVD player being returned has increased.

He decides to test his suspicion.

He takes a random sample of ten DVD players which were sold following the promotion, and finds that three of these were returned.

(i) State the null and alternative hypotheses for this test. [2 marks]

(ii) Explain the meaning of the term **critical region**.
[1 mark]

(iii) Find the critical region for a 5% level of significance.
[7 marks]

(iv) Complete the hypothesis test, interpreting your conclusion in context. [3 marks]

10 The length of time Tom takes to travel to work is normally distributed with mean 35 minutes and standard deviation 3.1 minutes.

Over a period of time, there were roadworks on Tom's journey.

Tom claims that the roadworks significantly increased his travel time to work.

For a random sample of 40 of these days, he calculates the mean time for him to travel to work to be 36 minutes.

Test Tom's claim at the 5% level of significance.
[9 marks]

This is the end of the question paper

For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
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10	
Total Marks	

Examiner Number

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