Rewarding Learning

ADVANCED
General Certificate of Education
2024

Centre Number

Candidate Number

Mathematics

Assessment Unit A2 1
assessing
Pure Mathematics

MV18

[AMT11]

THURSDAY 30 MAY, MORNING

Time

2 hours 30 minutes, plus your additional time allowance.

Instructions to Candidates

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all twelve** questions in the spaces provided.
Do not write on blank pages or tracing paper.
Complete in black ink only.
Questions which require drawing or sketching should be completed using an HB pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit.**
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

Information for Candidates

The total mark for this paper is 150

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

1 (a) Write as a single fraction [4 marks]

$$\frac{x+3}{x^2-2x-15}-\frac{1}{2x-7}$$

(b) Simplify the following algebraic expression. [5 marks]

$$\frac{3x^2 - 27}{3x^2 - 8x - 3} \div \frac{3x^2 + 10x + 3}{x - 7}$$

2 (a) Write each of the following series using sigma notation.

(i) $-7 - 3 + 1 + 5 + 9 + \ldots + 25$ [2 marks]

(ii) $6 + \frac{7}{4} + \frac{8}{9} + \frac{9}{16} + \dots$ [3 marks]

(b) A sequence is defined using the recurrence relation

$$u_{n+1} = au_n + b$$

If the first three terms are

$u_1 = 150$, $u_2 = 210$, $u_3 = 288$

find the values of a and b. [6 marks]

3 (a) Differentiate the following expressions with respect to x, simplifying where possible.

(i) $x^2 \ln x$ [4 marks]

(ii) $\frac{3e^{x^2}}{2x^4}$ [5 marks]

(b) Integrate the following expressions with respect to x.

(i) $4x^5 + \frac{1}{2x} - 3e^{-x}$ [3 marks]

(ii) $\sin x \cos^3 x$ [2 marks]

4 (a) Prove that the sum of the first n terms of a geometric series is given by

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$ [5 marks]

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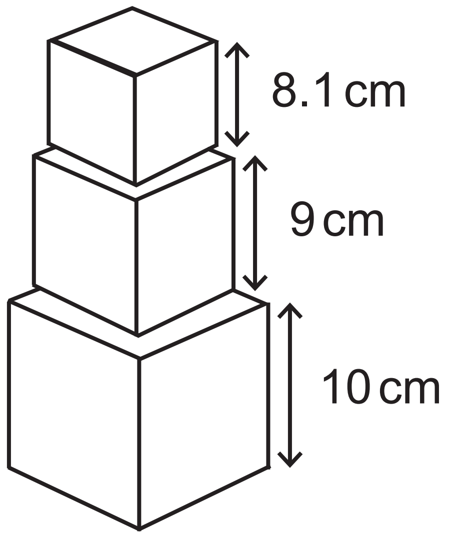
(b) A child builds a tower from cube-shaped blocks which decrease in size.

The largest block has a height of 10 cm and each subsequent block has a height 90**%** the size of the previous block. The first three such blocks are shown in **Fig. 1** below.

Fig. 1



(i) What is the height of the 8th block in the tower? [2 marks]

(ii) Find the minimum number of blocks the child will need to build a tower with a height of at least 90 cm. [4 marks]

In theory, the maximum possible height of the tower is 1 metre.

(iii) Why is this not feasible in practice? [1 mark]

5 (a) (i) Prove that $\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta$
[4 marks]

(ii) Hence, or otherwise, solve the following equation for $0 \leqslant \theta \leqslant 2\pi$ [6 marks]

$\operatorname{cosec}^4\theta - \cot^4\theta = -3\cot\theta$

(b) (i) Write $3\cos\theta - 5\sin\theta$ in the form $R\cos(\theta + \alpha)$ where $0° \leqslant \alpha \leqslant 90°$ [6 marks]

(ii) Hence find the maximum value of the expression

$8 - 3\cos\theta + 5\sin\theta$

giving the smallest positive value of θ for which this occurs. [4 marks]

6 (i) Find the binomial expansion of

$$(8 - 2x)^{\frac{1}{3}}$$

in ascending powers of x, up to and including the term in x^2 [5 marks]

(ii) State the range of values of x for which this expansion is valid. [1 mark]

(iii) Use the expansion obtained in part **(i)** to show that $\sqrt[3]{7.1} \approx 1.9222$ (5 sf). [2 marks]

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(Questions continue overleaf)

When a calculator is used to find $\sqrt[3]{7.1}$, the answer is 1.9220 (5 sf).

(iv) Suggest one way to improve on the approximation found in part **(iii)**, showing that this agrees with the calculator answer to 5 sf. [3 marks]

7 Use the substitution $u = 2 - x$ to find

$\int x(2 - x)^{12}\,dx$ [7 marks]

8 (i) Using partial fractions, show that

$$\frac{1}{P(5-P)} \equiv \frac{1}{5P} + \frac{1}{5(5-P)}$$ [4 marks]

A planner models the population growth of a city by

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0.006P\,(5 - P)$$

where P is the population (in hundreds of thousands) at time t years.

When $t = 0$, $P = 4$

(ii) By solving the differential equation, find the value of t when the population is 450 000 [8 marks]

9 A curve is defined by the equation

$x^2 + 4xy + y^2 + 12 = 0$

(i) Show that $\frac{dy}{dx}$ is

$\frac{-x-2y}{2x+y}$ [5 marks]

Point A lies on the curve.

The tangent to the curve at the point A is parallel to the y axis.

The x coordinate of A is negative.

(ii) Hence find the coordinates of A. [5 marks]

10 A curve is defined parametrically by

$$x = 3 + \tan\theta, \qquad \frac{1}{y+3} = \cos\theta$$

(i) Find the Cartesian equation of the curve. [6 marks]

The curve has two turning points.

(ii) Use parametric differentiation to find the coordinates of the curve's turning points.

Show working to identify the nature of each turning point. [13 marks]

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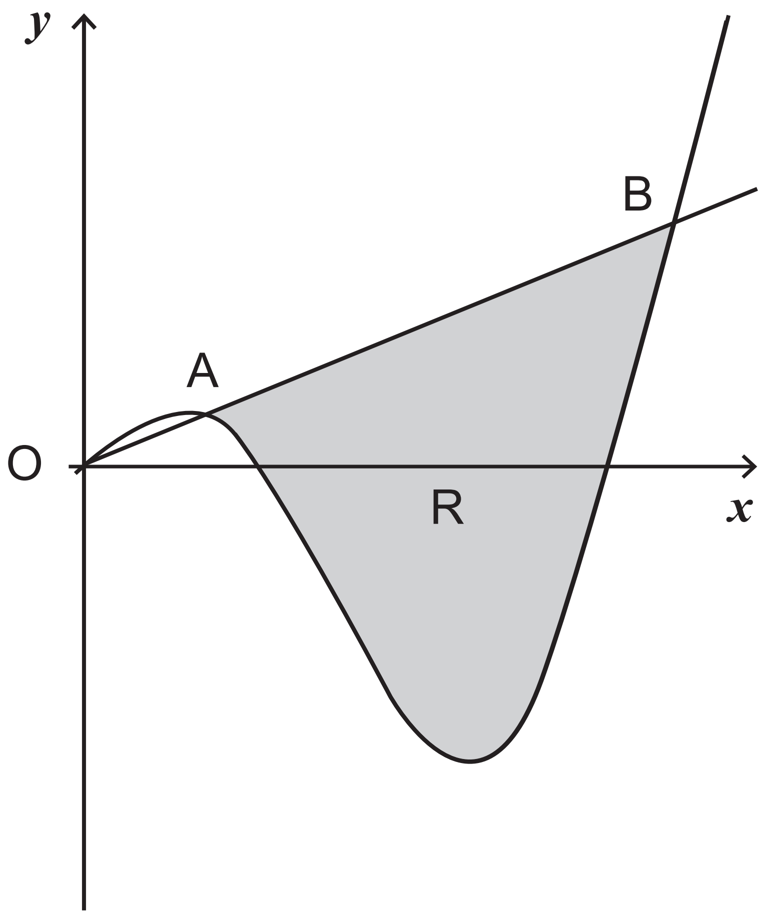
(Questions continue overleaf)

11 A landscape gardener has designed a patio.

The patio can be modelled as the shaded region R enclosed between the line $y = 2x$ and the curve $y = 4x \cos 2x$ as shown in **Fig. 2** below.

Fig. 2



The line and curve intersect at the points O, A and B.

(i) Show that the x coordinates of A and B are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ respectively. [3 marks]

(ii) Find the exact area of the shaded region R.
[11 marks]

12 Two functions are defined as

$$f(x) = \frac{3}{2x + 5} \qquad x \in \mathbb{R}, \quad x \neq -\frac{5}{2}$$

$$g(x) = |x| \qquad x \in \mathbb{R}$$

(i) Find the composite function $gf(x)$ stating its domain and range. [4 marks]

A third function is defined as

$h(x) = 2x^2 + 5x + 2 \qquad x \in \mathbb{R}, \quad x > 0$

(ii) State the range of $h(x)$. [1 mark]

(iii) Show that the inverse function $h^{-1}(x)$ is given by

$$h^{-1}(x) = \frac{-5 + \sqrt{8x + 9}}{4}$$

State the domain and range of this inverse function.
[6 marks]

This is the end of the question paper

SOURCES

For Examiner's use only	
Question Number	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total Marks	

Examiner Number