Rewarding Learning

ADVANCED

General Certificate of Education

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

AMM41

[AMM41]

Assessment

Assessment Level of Control:

Tick the relevant box (✓)

Controlled Conditions	
Other	

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows a framework ABCDE consisting of seven equal light rigid pin-jointed rods, AB, BC, CD, DE, EA, AC and CE, in the shape of a trapezium.
BCD is horizontal.
The framework rests on two smooth fixed supports at B and D and carries vertical loads of 300 N at A and 700 N at E.
The supports supply vertical thrusts of P newtons at B and Q newtons at D.

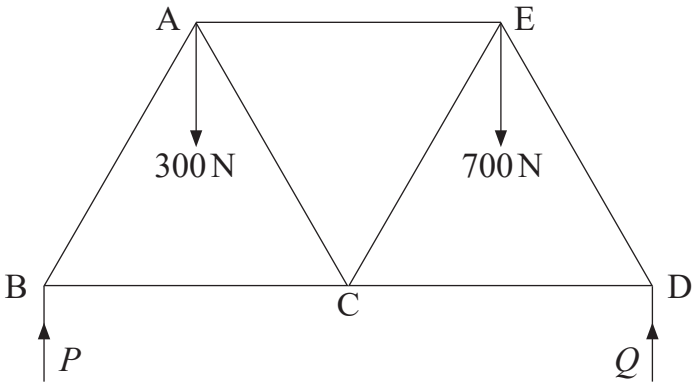

Fig. 1

(i) Show that $P = 400$ [4]

(ii) By resolving forces at A and one other suitable point, find the force in the rod AC, stating whether it is a tension or a thrust. [6]

2 **Fig. 2** below shows the system of forces, in newtons, acting along the sides of the quadrilateral ABCD.

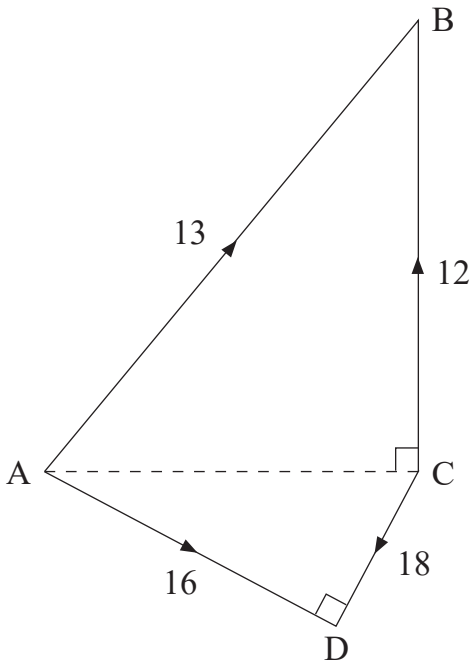

Fig. 2

AB = 2.6 m, BC = 2.4 m, AC = 1 m, AD = 0.8 m and CD = 0.6 m
The system of forces reduces to a single force R newtons.

(i) Find the magnitude of R and show that it acts parallel to AC. [7]

The line of action of R cuts CB at P, where CP = x metres.

(ii) Find x. [3]

3 **Fig. 3** below shows a uniform lamina which is bounded by the curve $y = ax(4 - x)$ and the x-axis.

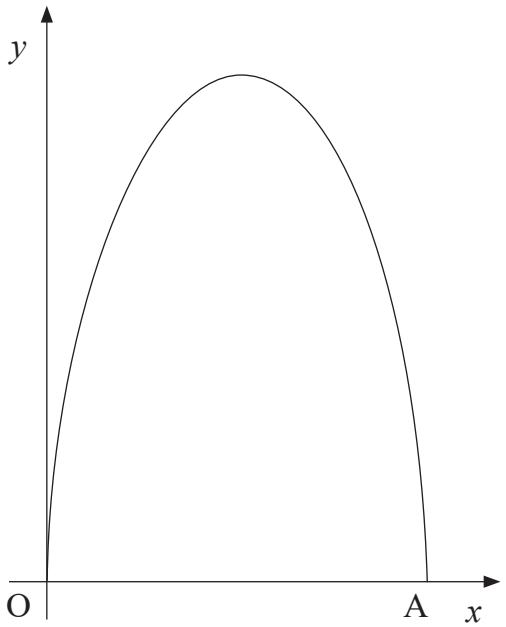

Fig. 3

The density of the lamina is $1\,\text{kg}\,\text{m}^{-2}$
The mass of the lamina is $\frac{32a}{3}$ kg.

(i) Show that the y coordinate of G, the centre of mass of the lamina, is $1.6a$ [9]

G has coordinates $(b, 8)$.

(ii) Find a and b. [2]

4 Aine is conducting experiments on a newly discovered material. She believes that the relationship between F, the force exerted, and the properties Birl B, Curl C, Gurl G and Knurl K is of the form

$$F = aB^wC^xG^yK^z$$

where a is a dimensionless constant.

The dimensions of the properties are given in **Table 1** below:

Table 1

Property	Dimensions
Birl	$[ML^{-1}T^{-1}]$
Curl	$[ML^2]$
Gurl	$[M^{-1}T]$
Knurl	$[M\,LT^{-1}]$

(i) Use the Method of Dimensions to show that $x = -1$ and find y and z in terms of w. [8]

After treating the material with a catalyst Aine finds that the Birl is doubled, the Gurl is halved and the Curl and Knurl remain unchanged.

(ii) Given that F remains unchanged, find w. [4]

5 A satellite of mass m kg is placed in a circular orbit about the Earth in a plane through the equator.
The mass of the Earth is M kg.
The distance of the satellite from the centre of the Earth is r metres.
The periodic time of the satellite's orbit is T seconds.

(i) Show that

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

where G is the universal gravitational constant. [5]

(ii) Given that $T = 86\,400\,\text{s}$
$G = 6.67 \times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$
$M = 5.97 \times 10^{24}\ \text{kg}$

find r. [2]

6 A boy is playing on a rope swing fastened to a fixed point C on a branch of a tree in his garden.
The mass of the boy is m kg and the length of the rope swing is l metres.
When the rope is vertical the boy is at A and has speed u m s^{-1}
When the rope is at an angle θ to the downward vertical the boy is at B and has speed v m s^{-1} as shown in **Fig. 4** below:

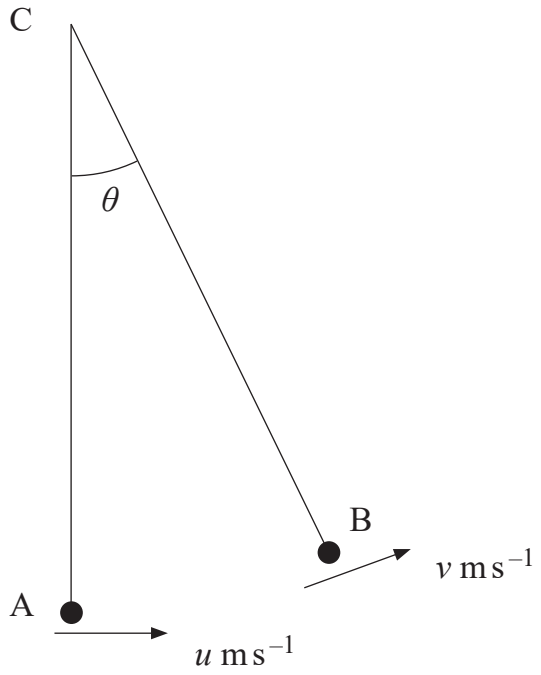

Fig. 4

The boy may be modelled as a particle.
Take gravitational potential energy to be zero at A.

(i) Find v in terms of θ, l, u and g, the acceleration due to gravity. [6]

(ii) Given $m = 30$, $l = 4$, $u = 10$ and $v = 6$, find the tension in the rope when the boy is at B. [6]

(iii) State one assumption you have made about the rope when answering this question. [1]

7 Two smooth spheres, P and Q, of equal radii are at rest in a straight smooth horizontal groove.
P has mass 2 kg and Q has mass m kg.
P is propelled towards Q at $12\,\text{m s}^{-1}$
After P and Q collide, Q moves forward at $2v\,\text{m s}^{-1}$ while P rebounds at $v\,\text{m s}^{-1}$

(i) Show that $m = \dfrac{12 + v}{v}$ [3]

As a result of the collision $\dfrac{5}{16}$ of the energy is lost.

(ii) Find v. [6]

The coefficient of restitution between P and Q is e.

(iii) Find e. [3]

THIS IS THE END OF THE QUESTION PAPER