Rewarding Learning

ADVANCED

General Certificate of Education

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

AMM31

[AMM31]

Assessment

Assessment Level of Control:

Tick the relevant box (✓)

Controlled Conditions	
Other	

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take g = $9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows a uniform lamina consisting of the trapezium ABCD with the square PQRS removed.

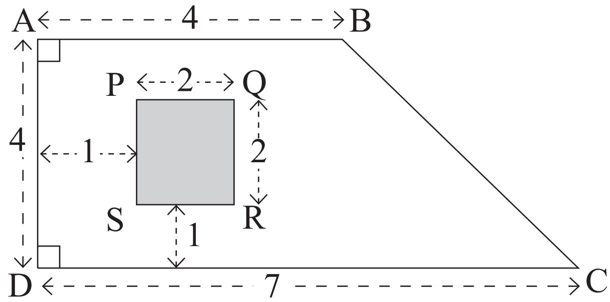

Fig. 1

The density of the material of the lamina is $1\,\text{kg}\,\text{m}^{-2}$
All dimensions are in metres.

(i) Find the distance of the centre of mass of the lamina from the edges AD and DC. [8]

The lamina is freely suspended at A and hangs in equilibrium.

(ii) Find the angle the edge AD makes with the vertical. [4]

2 A particle P is acted on by the forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ newtons.
P moves along the line whose vector equation is

$$\mathbf{r} = \lambda \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \text{ metres}$$

Given that $$\mathbf{F_1} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

(i) find the work done on P by $\mathbf{F_1}$ as P moves along the line. [2]

(ii) State what this result tells you about $\mathbf{F_1}$ and $\mathbf{r}$. [1]

A and B are the points on the line where λ has the values 1 and 4 respectively.
The total work done on P by all three forces as P moves from A to B is 25 joules.

Given that $\mathbf{F_2} = \begin{pmatrix} 4 \\ a \\ 5 \end{pmatrix}$ and $\mathbf{F_3} = \begin{pmatrix} -a \\ 2a \\ 4 \end{pmatrix}$

where a is a scalar constant,

(iii) show that $a = \frac{31}{6}$ [5]

3 **Fig. 2** below shows a ring Q of mass m kg threaded on to a rough horizontal wire.

The coefficient of friction between Q and the wire is $\frac{1}{3}$

A light inextensible string is fastened to Q at one end and to a fixed point A on the wire at the other end.
A smooth ring P of mass $4m$ kg is threaded on the string.
P hangs in equilibrium with angle APQ = 2θ.

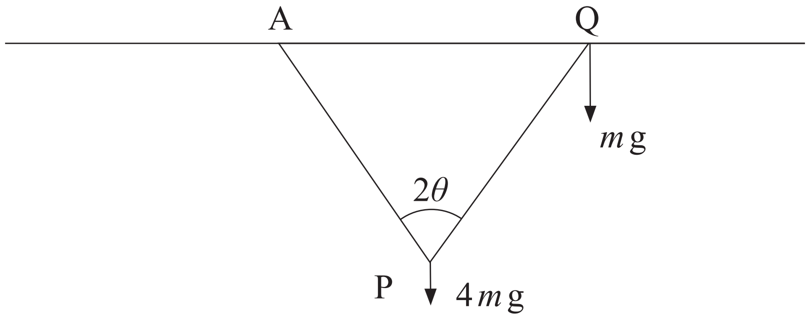

Fig. 2

Q is about to slip along the wire towards A.

(i) Draw a diagram showing the external forces acting on P and Q. [2]

(ii) Find θ. [10]

4 **Fig. 3** below shows a crate Q of weight W suspended from a rigid fixed horizontal beam by two vertical light elastic strings, S_1 and S_2

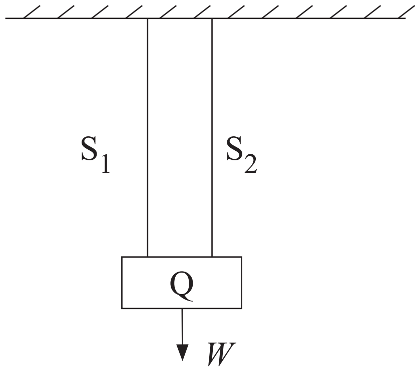

Fig. 3

S_1 has natural length $3l$ and modulus of elasticity 2λ.
S_2 has natural length $4l$ and modulus of elasticity λ.
Model Q as a particle.
Q hangs in equilibrium a distance $6l$ vertically below the beam.

(i) Find λ in terms of W. [6]

The string S_2 is removed.
Q now hangs in equilibrium a distance x vertically below the beam.

(ii) Find x in terms of l. [3]

5 In this question take **i** and **j** as unit vectors acting due East and due North respectively.

A boat is sailing due North at $20\,\text{km}\,\text{h}^{-1}$
The wind has constant velocity $\mathbf{v} = x\mathbf{i} + y\mathbf{j}\,\text{km}\,\text{h}^{-1}$ where x and y are scalar constants.
To the crew the wind appears to be blowing from due West at $u\,\text{km}\,\text{h}^{-1}$

(i) Find **v** in terms of u, **i** and **j**. [4]

The boat changes course to sail at $26\,\text{km}\,\text{h}^{-1}$ on a bearing of $\tan^{-1}\left(\frac{5}{12}\right)$
The wind now appears to blow from due North at $w\,\text{km}\,\text{h}^{-1}$

(ii) Find **v** in terms of w, **i** and **j**. [4]

(iii) Find the values of x and y. [2]

(iv) Briefly explain why the crew might think that the wind is stronger when the boat is sailing due North. [3]

6 A particle S moves along a straight line with simple harmonic motion.
O is a fixed point on the line.
At time t seconds S is x metres from O, where

$$x = 4\sin\left(\frac{\pi}{3}t + \varepsilon\right)$$

where ε is a constant.

At time $t = 0$, S is at the point P on the line moving towards O.

Given that $\text{OP} = 2\sqrt{2}\,\text{m}$,

(i) show that $\varepsilon = \frac{3\pi}{4}$ [4]

(ii) Find t when S passes through P again for the third time. [6]

7 Take g = 10 m s^{-2} in this question.

Fig. 4 below shows a particle R of mass 2 kg, which is fastened to one end of a light elastic string. The other end of the string is fastened to the fixed point A on a rough horizontal plane.

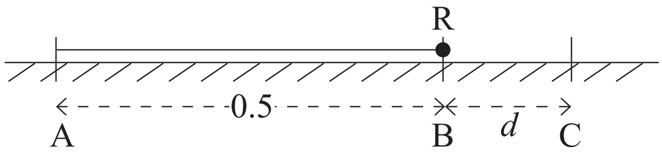

Fig. 4

The string has natural length 0.5 m and modulus of elasticity 16 N.
The coefficient of friction between R and the plane is 0.5
R is placed at the point B on the plane, where AB = 0.5 m.

R is given a speed of 6 m s^{-1} in the direction **AB**.
R comes to rest again at the point C, where BC = d metres.

Find d. [11]

THIS IS THE END OF THE QUESTION PAPER