Rewarding Learning

ADVANCED

General Certificate of Education

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

AMM21

[AMM21]

Assessment

Assessment Level of Control:

Tick the relevant box (✓)

Controlled Conditions	
Other	

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A particle P moves in the x–y plane so that its displacement, $\mathbf{r}$, from a fixed point O at any time t can be modelled by

$$\mathbf{r} = (2t - 3t^2)\,\mathbf{i} + 5t\,\mathbf{j}$$

(i) Find the velocity of P at any time t. [2]

(ii) Find the exact speed of P when $t = \frac{1}{2}$ [3]

(iii) Find the acceleration of P and explain why it is constant. [3]

2 At time $t = 0$ a particle P is at the point with position vector $(-\mathbf{i} - 3\mathbf{j})$ and is moving with constant velocity $(2\mathbf{i} + 4\mathbf{j})$.

At $t = 0$ a particle Q is at the point with position vector $(4\mathbf{i} + \mathbf{j})$ and is moving with constant velocity $(a\mathbf{i} + b\mathbf{j})$.

The particles collide when $t = 2$

Find a and b. [7]

3 A train of mass 30 000 kg is travelling along a straight horizontal track against a constant resistance of 10 000 N. The train's engine works at a constant rate of 700 kW.

(i) Find the acceleration of the train when it is travelling at $20\,\text{m s}^{-1}$ [5]

The train now ascends a slope inclined at θ to the horizontal, where $\sin\theta = \frac{1}{50}$

The resistance remains at 10 000 N.

(ii) Draw a diagram showing the forces acting on the train. [2]

(iii) Find the maximum speed of the train up the slope. [6]

4 A smooth bead of mass m kilograms is threaded onto a smooth circular wire of radius r metres and centre O.
The wire is fixed in a vertical plane as shown in **Fig. 1** below.

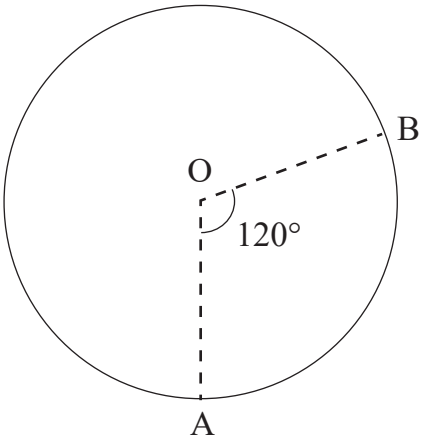

Fig. 1

The bead is projected from its lowest point A with a speed of $\sqrt{5gr}\ \text{m s}^{-1}$
Take the potential energy of the bead to be zero at A.

Using the conservation of mechanical energy, find, in terms of g and r, the speed of the bead as it passes through the point B, where $A\hat{O}B = 120°$ [7]

5 A cyclist and her cycle have a total mass of 60 kg and at $t = 0$ seconds are at rest on a straight horizontal road. She experiences a forward force of $\frac{120}{v}$ N and a resistive force of $\frac{4v}{3}$ N, where v is her speed at any time t.

(i) Show that the acceleration of the cyclist, $a\,\text{m s}^{-2}$, is

$$a = \frac{90 - v^2}{45v}$$ [3]

(ii) Find the time taken for her to reach a speed of $4\,\text{m s}^{-1}$ [9]

6 **Fig. 2** below shows a light inextensible string attached to two fixed points A and B, with A vertically above B. A smooth ring, C, is fixed to the midpoint of the string and C moves in a horizontal circle about AB.

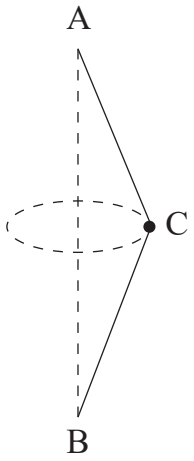

Fig. 2

AB is 3 m long and the string is 4 m long. C has a mass of M kilograms and moves with a speed of $\sqrt{6g}\,\text{m s}^{-1}$

(i) Draw a diagram showing the external forces acting on C. [2]

(ii) Find, in terms of M and g, the tensions in the string. [11]

7 **(i)** A particle is projected under gravity from a point O with speed u at an angle θ above the horizontal. At time t the particle has horizontal displacement x and vertical displacement y from O as shown in **Fig. 3** below.

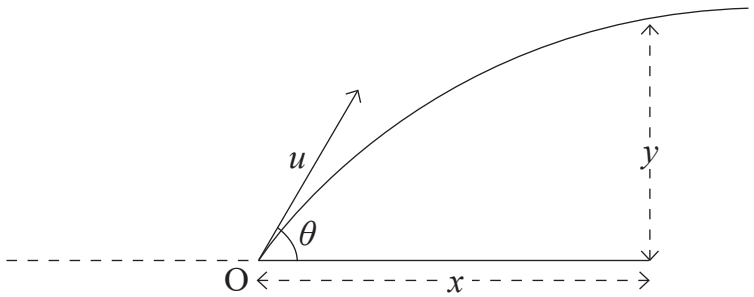

Fig. 3

Show that an equation for the path of the particle is

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta) \qquad [6]$$

Take $g = 10\,\text{m}\,\text{s}^{-2}$ in the remaining parts of this question.

Bill is trying to throw a tennis ball over a vertical school fence. The fence is 2 m high and is 2 m horizontally from him. Bill throws the ball with a speed of $6\,\text{m}\,\text{s}^{-1}$ from a point 1 m vertically above the ground. The ball just clears the fence.

(ii) Find the two possible angles of projection. [5]

(iii) Find the time taken for the ball to reach its greatest possible height above the ground. [4]

THIS IS THE END OF THE QUESTION PAPER