Rewarding Learning

ADVANCED

General Certificate of Education

Mathematics

AMF31

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

[AMF31]

Assessment

Assessment Level of Control:

Tick the relevant box (✓)

Controlled Conditions	
Other	

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$, where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Straight lines l_1 and l_2 have equations

$$l_1 \qquad \frac{x-3}{2} = \frac{y-p}{3} = \frac{z-1}{-1}$$

$$l_2 \qquad \frac{x-3}{1} = \frac{y+1}{-2} = \frac{z-4}{1}$$

where p is a scalar constant.
The lines intersect at the point A.

Find the value of p and the coordinates of the point A. [8]

2 For each non-negative integer n, let

$$I_n = \int x^n \cos x \, \mathrm{d}x$$

(i) Show that if $n \geqslant 2$

$$I_n = x^n \sin x + nx^{n-1} \cos x - n(n-1)I_{n-2}$$ [7]

(ii) Hence find

$$\int x^4 \cos x \, \mathrm{d}x$$ [4]

3 **(i)** Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left\{\frac{1}{2}\left(\sin^{-1}x + x\sqrt{1-x^2}\right)\right\} = \sqrt{1-x^2} \qquad [4]$$

(ii) Write $4x - x^2 - 3$ in the form $a - (x - b)^2$ [1]

(iii) Hence find the exact value of

$$\int_2^3 \sqrt{4x - x^2 - 3}\,\mathrm{d}x \qquad [5]$$

4 **(a)** Find

$$\int \frac{\mathrm{d}x}{5 - 4x + 4x^2} \qquad [4]$$

(b) Differentiate

$$\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x-3}\right) \qquad [6]$$

5 **(i)** Sketch the curve $y = \sinh^{-1}x$ [1]

(ii) Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}} \qquad [4]$$

(iii) Show that

$$\sinh^{-1}x \equiv \ln\left(x + \sqrt{1+x^2}\right) \qquad [4]$$

(iv) Find, in fraction form, the exact solution to the equation

$$\sinh^{-1}\frac{3}{4} + \sinh^{-1}x = \sinh^{-1}\frac{4}{3} \qquad [4]$$

6 **(i)** Using the exponential definitions of $\sinh x$ and $\cosh x$, show that

$$\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$ [4]

(ii) If $$\sinh x - 2\coth y = 1$$

and $$2\sinh x - 5\coth y = -1$$

find x and y, giving your answers in logarithmic form. [6]

7 With reference to a fixed origin O, the points A(4, 1, 3), B(−2, 7, 6) and C(5, −3, 2) determine the plane ABC.

(i) Find $\overrightarrow{AB} \times \overrightarrow{AC}$ [4]

(ii) Hence or otherwise find, in the form **r.n** = d, an equation of the plane ABC. [3]

The point D with position vector $\overrightarrow{OD} = 11\mathbf{i} - 9\mathbf{j} + \lambda\mathbf{k}$ is in the plane ABC.

(iii) Find the value of λ. [2]

(iv) What kind of quadrilateral is ABCD? Justify your answer. [2]

(v) Find, in surd form, the area of the quadrilateral ABCD. [2]

THIS IS THE END OF THE QUESTION PAPER