Rewarding Learning

ADVANCED

General Certificate of Education

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

AMF21

[AMF21]

Assessment

TIME

1 hour 30 minutes.

Assessment Level of Control:

Tick the relevant box (✓)

Controlled Conditions	
Other	

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find, in radians, the general solution of the equation

$$2\sec^2\theta - 3\tan\theta - 1 = 0$$ [6]

2 **(i)** Show that

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{n}{3}(4n^2-1)$$ [4]

(ii) Using this result, evaluate

$$\sum_{r=5}^{40}(2r-1)^2$$ [3]

3 Use mathematical induction to prove that

$$1^3 + 2^3 + \ldots + n^3 = \frac{1}{4}n^2(n+1)^2$$

for all positive integers n. [7]

4 **(i)** By using de Moivre's theorem prove that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$$ [5]

(ii) Hence, or otherwise, find the general solution of the equation

$$\cos 3\theta + \cos\theta = 0$$ [6]

5 A parabola with equation $y^2 = 4ax$ has general point $P(at^2, 2at)$.

(i) Show that the equation of the normal at P is

$$y + tx = at^3 + 2at$$ [4]

The point $Q(ak^2, 2ak)$ lies on the parabola. The chord PQ passes through the focus of the parabola.

(ii) Show that $k = -\frac{1}{t}$ [6]

(iii) Using **(i)** or otherwise, write down the equation of the normal at Q. [2]

The normals at P and Q intersect the directrix at M and N.

(iv) Prove that

$$\text{MN} = a\left(t + \frac{1}{t}\right)^3$$ [5]

6 **(i)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 13 \sin 2x$$ [10]

(ii) If $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$, find the particular solution. [4]

7 **(i)** Use Maclaurin's theorem to show that the series expansion for $\frac{1}{1+x}$ up to the term in x^3 is given by

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 \qquad \text{for } |x| < 1$$ [5]

(ii) Hence use partial fractions to find the series expansion of

$$\frac{3 - 3x + 4x^2}{(1 + 2x^2)(1 - 3x)}$$

up to and including the term in x^3 [8]

THIS IS THE END OF THE QUESTION PAPER