Rewarding Learning

ADVANCED
General Certificate of Education

Centre Number

Candidate Number

Mathematics

Assessment Unit C4
assessing
Module C4:
Core Mathematics 4

AMC41

[AMC41]
Assessment

TIME

1 hour 30 minutes.

Assessment Level of Control:
Tick the relevant box (✓)

Controlled Conditions	
Other	

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all eight** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen**.
Questions which require drawing or sketching should be completed using an H.B. pencil.
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions. **Answers without working may not gain full credit**.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

1 **(i)** Write

$$15\cos x - 8\sin x$$

in the form $R\cos(x + \alpha)$, where R is a positive integer and $0° < \alpha < 90°$ [3]

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(ii) Hence find the maximum value of the expression

$$30\cos x - 16\sin x$$ [2]

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[Turn over

2 **(a)** Find the acute angle between the two lines given by the vector equations:

$$\mathbf{r}_1 = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - \mathbf{k})$$

$$\mathbf{r}_2 = 2\mathbf{i} - \mathbf{k} + \lambda(3\mathbf{i} - 5\mathbf{k}) \qquad [5]$$

(b) The position vectors of the three points A, B and C relative to a fixed origin O are

$$\begin{pmatrix}0\\4\\4\end{pmatrix} \quad \begin{pmatrix}-1\\6\\5\end{pmatrix} \text{ and } \begin{pmatrix}-4\\12\\8\end{pmatrix}$$

respectively.

Show that the points A, B and C all lie on the same straight line. [5]

3 **(a)** The function $f(x) = \frac{3x}{x-1}$ has domain $x \in \mathbb{R}, x \neq a$

(i) Write down the value of a. [1]

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(ii) Find the inverse function $f^{-1}(x)$ and state its domain. [5]

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(iii) Hence, using function notation, write down the range of f(x). [1]

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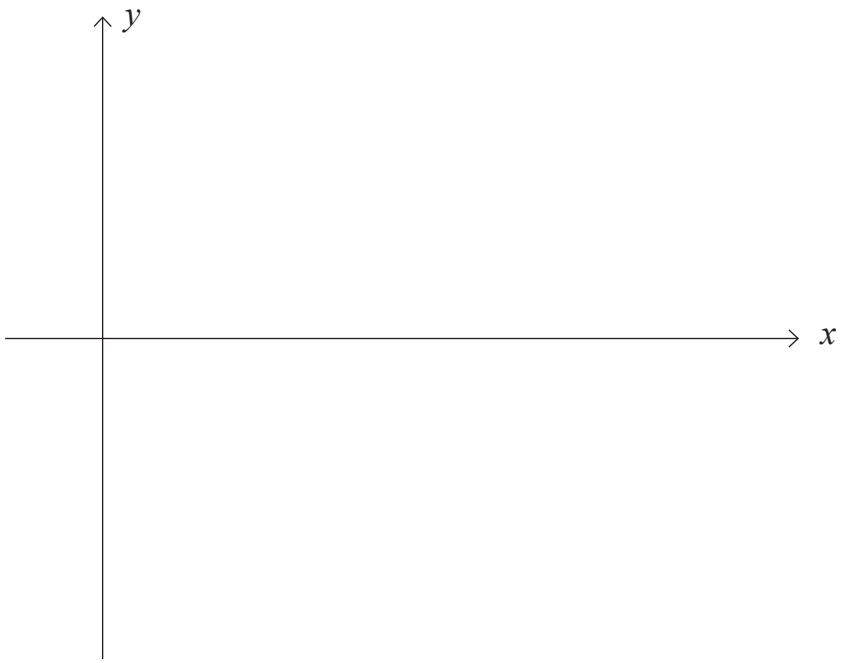
(b) The function g is defined as g(x) = cosec x for $0 < x < \pi$

(i) On the axes below, sketch the graph of y = g(x). [2]



(ii) Briefly explain why we cannot find an inverse function $g^{-1}(x)$. [1]

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[Turn over

4 **(a)** Use the substitution $u = \sin x + 3$ to find the exact value of

$$\int_0^{\frac{\pi}{2}} e^{(\sin x+3)} \cos x \, dx \qquad [5]$$

(b) On part of his ascent, the position of a mountaineer as he climbs a slope can be modelled by the parametric equations

$$x = 3\text{cosec } 2t \quad \text{and} \quad y = 7 - \cot 2t$$

where $\frac{\pi}{16} \leqslant t \leqslant \frac{\pi}{4}$

Find $\frac{dy}{dx}$ when $t = \frac{\pi}{6}$ [6]

5 **(i)** Starting with the formula for $\sin(A + B)$ prove that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x \qquad [6]$$

(ii) Hence find the exact solutions of

$\sin 3x - 2\sin x = 0$ where $0 \leqslant x \leqslant \pi$ [4]

[Turn over

6 **(i)** Find $\int \frac{3x+4}{x}\,dx \qquad x > 0$ [3]

(ii) Given that $y = 9$ when $x = 1$, solve the differential equation

$$x\frac{dy}{dx} = (3x + 4)\sqrt{y}$$

Give your answer in the form $y = f(x)$. [7]

7 The metal drip tray for under a barbecue can be modelled by rotating the curve

$$y = x^2 \sqrt{\ln x}$$

through 2π radians about the x-axis, between $x = 1$ and $x = 2$, as shown in **Fig. 1** below.

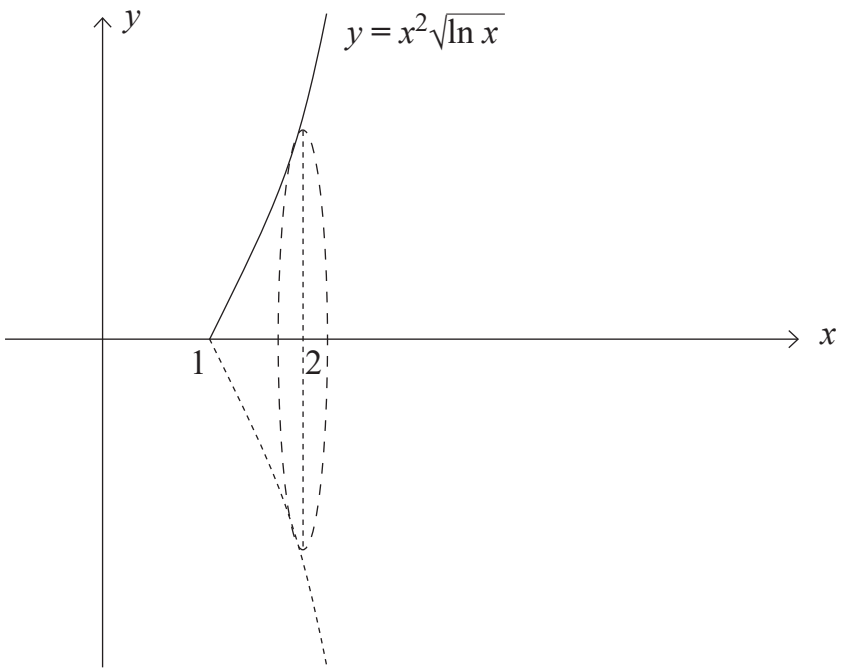

Fig. 1

Find the maximum volume of liquid that the drip tray can hold. [9]

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8 Find the equation of the tangent to the curve with equation

$$y^3 - 3\cos x = 8y\,e^x$$

at the point (0, 3).

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [10]

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For Examiner's use only	
Question Number	**Marks**
1	
2	
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Total Marks	

Examiner Number