New Specification

Rewarding Learning

ADVANCED
General Certificate of Education

Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AMT21]

Assessment

MARK SCHEME

(Including Supplementary Mark Scheme to support Teachers)

General Marking Instructions

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters M, W and MW as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

					AVAILABLE MARKS
1	**(i)**	$u = 0\,\text{m s}^{-1}$	$v^2 = u^2 + 2as$	M1	
		$a = 9.8\,\text{m s}^{-2}$	$v^2 = (0)^2 + 2(9.8)(2)$		
		$s = 2\,\text{m}$	$v^2 = 39.2$		
		$v = ?$	$v = 6.26\,\text{m s}^{-1}$	W1	
	(ii)	Conservation of linear momentum:			
		$m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$		M1 M1	
		$8(0) + (42)(\sqrt{39.2}) = 50v$		W1 W1	
		$v = 5.26\,\text{m s}^{-1}$		W1	
	(iii)	$u = 5.259\ldots\,\text{m s}^{-1}$	$v^2 = u^2 + 2as$		
		$a = ?$	$0^2 = (5.259\ldots)^2 + 2(a)(0.05)$		
		$s = 0.05\,\text{m}$	$0.1a = -27.659\ldots$		
		$v = 0$	$a = -277\,\text{m s}^{-2}$ (3 s.f.)	MW1	
		R(↓) apply $F = ma$			
		$mg - R = ma$		M1	
		$50g - R = 50(-276.59\ldots)$		W1	
		$R = 50g + 50(276.59\ldots)$			
		$R = 14319.76\ \text{N}$			
		$R = 14300\,\text{N}$ (3 s.f.)		W1	11
2	**(i)**	**Vertical**			
		$u_y = u \sin 60°$	$s = ut + \frac{1}{2}at^2$	M1	
		$a = -g$	$0 = \dfrac{uT\sqrt{3}}{2} - \dfrac{g}{2}T^2$		
		$s = 0$	$0 = uT\sqrt{3} - gT^2$	W1	
		$t = T$			
			$T = 0$ or $u\sqrt{3} - gT = 0$	MW1	
			$T = \dfrac{u\sqrt{3}}{g}$	W1	
	(ii)	**Horizontal**			
		$d = 12.6\,\text{m}$	$d = u_x \times t$	M1	
		$u_x = u \cos 60°$	$12.6 = \dfrac{u(u\sqrt{3})}{2g}$	W1	
		$t = \dfrac{u\sqrt{3}}{g}$	$u = 11.9$	W1	
			speed $= 11.9\,\text{m s}^{-1}$		
	(iii)	Change the size of the angle that the rocket is launched at to 45°		M1	8

			AVAILABLE MARKS
3 **(i)**	Taking moments about the centre of the see-saw		
	Clockwise Moment = Anti-Clockwise Moment	M1	
	$m_2\,gx = m_1\,g \times 2.5$	M1 W1	
	$x = \dfrac{2.5m_1\,g}{m_2\,g}$		
	$x = \dfrac{5m_1}{2m_2}$	W1	
(ii)	The see-saw is not actually uniform	MW1	
(iii)	Taking moments about centre of the see-saw	M1	
	$m_2\,g(2.1) + 15\,g(y - 2.5) = m_1\,g \times 2.5$ $15y = 2.5\,m_1 + 37.5 - 2.1\,m_2$	MW3	
	$y = \dfrac{2.5m_1 - 2.1m_2 + 37.5}{15}$ metres		
	$y = \dfrac{25m_1 - 21m_2 + 375}{150}$ metres	W1	10

				AVAILABLE MARKS
4	**(a)**	**(i)** $\mathbf{a} = \dfrac{d\mathbf{v}}{dt}$	M1	
		$\mathbf{a} = 2\mathbf{i} + 4t\mathbf{j}$	W1	
		$t = 4$, $\mathbf{a} = ?$		
		When $t = 4$ s, $\mathbf{a} = (2\mathbf{i} + 16\mathbf{j})$ m s^{-2}	MW1	
		(ii) $\mathbf{s} = \int \mathbf{v}\ dt$	M1	
		$\mathbf{s} = (t^2\mathbf{i} + \frac{2}{3}t^3\ \mathbf{j} + \mathbf{d})$ m	W2	
		$t = 0$, $\mathbf{s} = \mathbf{0} \Rightarrow \mathbf{d} = \mathbf{0}$	MW1	
		$\mathbf{0} = t^2\mathbf{i} + \frac{2}{3}\ t^3\ \mathbf{j}$	M1	
		$t^2 = 0$ or $\mathbf{i} + \frac{2}{3}\ t\mathbf{j} = \mathbf{0}$	M1	
		At start; No value of t for which this can be **0**, therefore never back to O.	MW1	
	(b)	$a = 4t - 11$ $v = \int a\ dt$ $v = 2t^2 - 11t + c$	M1 W1	
		When $t = 5$, $v = 7$ $7 = 2(5)^2 - 11(5) + c$	M1	
		$c = 12$ $v = 2t^2 - 11t + 12$	W1	
		When $v = 0$ $0 = 2t^2 - 11t + 12$	M1	
		$0 = (2t - 3)(t - 4)$ $t = 1\frac{1}{2}$, $t = 4$	W1	
		$s = \int v\ dt$	M1	
		$s = \frac{2}{3}\ t^3 - \frac{11}{2}\ t^2 + 12t + d$ $t = 0$, $s = 0 \Rightarrow d = 0$	W1	
		$t = \frac{3}{2}$ $s = \frac{2}{3}\left(\frac{3}{2}\right)^3 - \frac{11}{2}\left(\frac{3}{2}\right)^2 + 12\left(\frac{3}{2}\right) = 7\frac{7}{8}$ m $t = 4$ $s = \frac{2}{3}(4)^3 - \frac{11}{2}(4)^2 + 12(4) = 2\frac{2}{3}$m $t = 6$ $s = \frac{2}{3}(6)^3 - \frac{11}{2}(6)^2 + 12(6) = 18$m	MW1	
		distance $= 7\frac{7}{8} + \left(7\frac{7}{8} - 2\frac{2}{3}\right) + \left(18 - 2\frac{2}{3}\right)$	M1	
		distance $= 28\frac{5}{12}$m	W1	21

Section B

Statistics

AVAILABLE MARKS

5 **(i)** A change in only one direction is being investigated, so this is a one-tailed test. MW1

(ii) Since Neil's calculated value of r exceeds the critical value for the test, he should reject the null hypothesis and conclude that there is sufficient evidence at the 5% level of significance of a positive correlation M1
between the masses of the animals' brains and their hearts. MW1

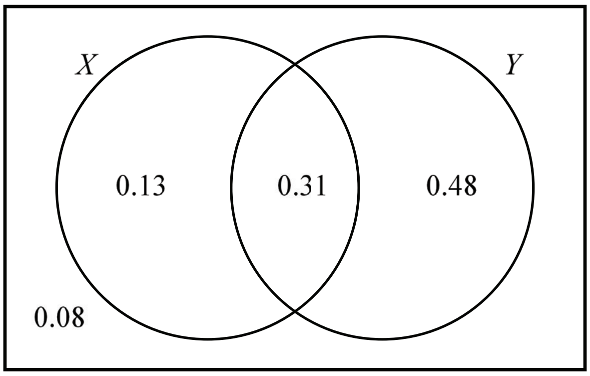
(iii) 0.05 MW1 4

6 **(a)** **(i)** $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$0.92 = 0.44 + 0.79 - P(X \cap Y)$ MW1

$P(X \cap Y) = 0.31$ W1

(ii)



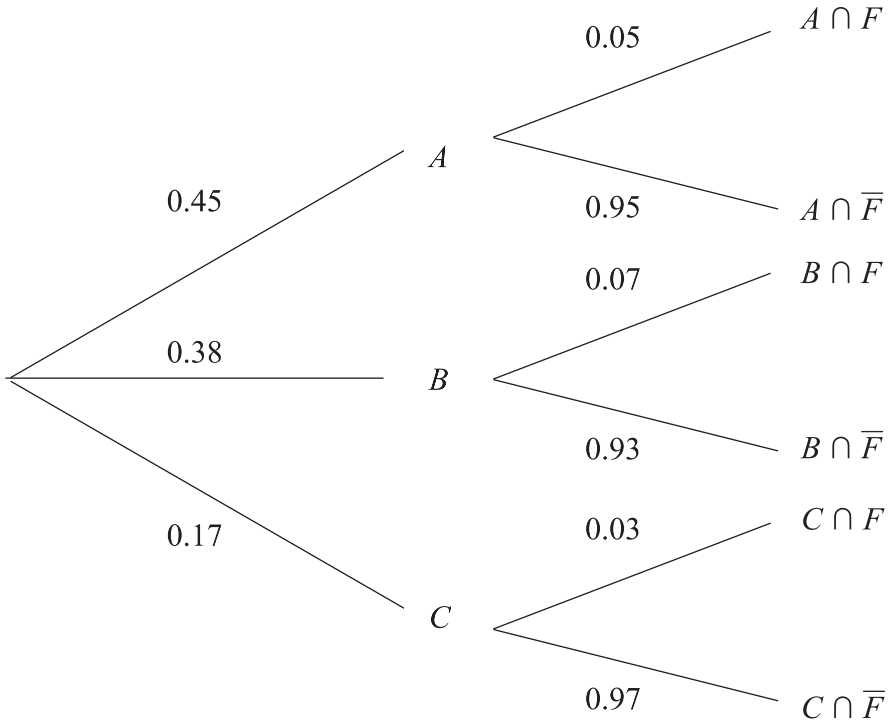
MW1

$$P(Y|\overline{X}) = \frac{P(Y \cap \overline{X})}{P(\overline{X})}$$

$$= \frac{0.48}{0.48 + 0.08}$$ M1

$$= \frac{6}{7}$$ W1

(b)



		AVAILABLE MARKS
(i) $P(F) = P(A \cap F) + P(B \cap F) + P(C \cap F)$		
$= (0.45 \times 0.05) + (0.38 \times 0.07) + (0.17 \times 0.03)$	M2 W2	
$= 0.0542$	W1	
(ii) $P(B \mid F) = \dfrac{P(B \cap F)}{P(F)}$	M1	
$= \dfrac{0.38 \times 0.07}{(0.45 \times 0.05) + (0.38 \times 0.07) + (0.17 \times 0.03)}$	W1	
$= \dfrac{133}{271}$	W1	13

7 **(i)** Let S = standby times, so $S \sim N(180, 8^2)$.

		AVAILABLE MARKS
$P(S > 195) = P\left(Z > \dfrac{195 - 180}{8}\right)$	M1	
$= P(Z > 1.875)$	MW1	
$= 1 - \Phi(1.875)$	M1	
$= 1 - 0.9697$ $= 0.0303$	W1	
(ii) $P(165 < S < 195) = P(-1.875 < Z < 1.875)$	MW1	
$= 1 - 2 \times 0.0303$	M1	
$= 0.9394$	W1	
$= 93.9\%$	MW1	
(iii) $\Phi(-z_T) = 0.975$	MW1	
$-z_T = 1.960$	MW1	
$z_T = -1.960$		
$\dfrac{T - 180}{8} = -1.960$	M1	
$T = 164(.32)$	W1	12
8 $H_0: \mu = 100$	M1	
$H_1: \mu \neq 100$ ('affects' implies non-directional change, so test is two tailed)	MW1	
Let X = intelligence test score, so $\overline{X} \sim N\left(100, \dfrac{15^2}{80}\right)$	MW1	
5% level of significance, two tailed test, so reject H_0 if $\lvert z\rvert > 1.96$	MW2	
$z = \dfrac{104 - 100}{\frac{15}{\sqrt{80}}}$	M1 W1	
$= 2.3851$	W1	
Since $2.3851 > 1.96$ we reject H_0 and conclude that there is sufficient evidence at the 5% level of significance to suggest that drinking coffee before the test affects your score.	M1 MW1	10

				AVAILABLE MARKS
9	**(i)**	Each trial has of a pair of mutually exclusive, exhaustive outcomes: 'effective' and 'not effective'.	MW1	
		The probability of effectiveness of the drug is the same between patients.	MW1	
		The outcomes for each patient are independent of each other.	MW1	
	(ii)	$H_0: p = 0.8$	M1	
		$H_1: p < 0.8$ (one directional change only, so the test is one-tailed)	MW1	
		Let X be the number of patients on whom the drug was effective so $X \sim \text{Bin}(20, 0.8)$	MW1	
		5% level of significance, so reject H_0 if $P(X \leqslant 12) < 0.05$	MW1	
		$P(X \leqslant 12) = 1 - P(Y \leqslant 7)$ where $Y \sim \text{Bin}(20, 0.2)$	M1	
		$P(X \leqslant 12) = 0.03214$	MW1	
		Since $0.03214 < 0.05$ we reject H_0 and conclude that there is sufficient evidence at the 5% level of significance to suggest that the effectiveness of the drug has been overstated.	M1 MW1	11
			Total	**100**

Breakdown of Marks

1 **(i)** M1 Attempts a correct equation of motion
W1 Correct answer given

(ii) M1 Uses momentum = mv
M1 Correctly applies conservation of momentum
W1 LHS correct
W1 RHS correct
W1 Correct value for v

Notes
[A] Use of incorrect value from (i) – can award
M1 M1 W1(ft) W1(ft) W0

(iii) MW1 Finds correct value of a
M1 Tries $F = ma$
W1 Correct equation set up
W1 Correct force

Notes
[A] Use of incorrect value from (i) – can award
MW1(ft) M1 W1(ft) W0

2 **(i)** M1 Considers vertical motion using a correct equation of motion
W1 Correct values substituted
MW1 Factorises to get 2 values of T – see [A] below
W1 Selects correct value of T

Notes
[A] Allow MW1 (ft) provided 1st M1 awarded and factors are T and linear function of T i.e. max of 2/4

(ii) M1 Tries $d = s \times t$ for horizontal motion
W1 Correctly substitutes their T from part (i)
W1 Correct speed

(iii) M1 Correctly identifies that modifying the angle will increase the horizontal range of the particle.

3 **(i)** M1 Tries force × distance
M1 Tries conservation of moments
W1 Both sides correct
W1 Rearranges to find x

(ii) MW1 Correctly realises that assumption about uniformity was incorrect

(iii) M1 Takes moments at correct point to eliminate R
MW3 Each correct moment
W1 Simplifies to find y

4 **(a)** **(i)** M1 Attempts to differentiate
W1 Correct differentiation
MW1 Correct value of **a**

(ii) M1 Attempts to integrate
W2 Each correct t term
MW1 Finds $\mathbf{d} = \mathbf{0}$ – see [A] below
M1 Correctly sets $\mathbf{s} = \mathbf{0}$
M1 Correct solution of quadratic in terms of t – see [B] below
MW1 Correct interpretation of both times and statement that the particle does not return to O

Notes
[A] Must see evidence of stating **d** in previous line and an attempt to find it

[B] May see $t^2 = 0$ and $\frac{2}{3}t^3 = 0$ (both must be stated)

(b) M1 Tries to integrate
W1 Correct integration – to include c
M1 Substitutes $t = 5$, $v = 7$
W1 Finds $c = 12$
M1 Sets $v = 0$
W1 Correct values of t
M1 Tries to integrate v
W1 Correctly integrated
MW1 Finds each of 3 different displacements for $t = \frac{3}{2}, 4, 6$
M1 Tries to combine distances – see [A] below
W1 Correct distance

Notes
[A] Must only use distances for times $t = \frac{3}{2}$, 4, 6 (or their times when $v = 0$) and there must be evidence of consideration of different phases (directions) of motion

5 **(i)** MW1 Correct statement

(ii) M1 Reject H_0
MW1 Correct conclusion in context

(iii) MW1 Correct answer

6 **(a)** **(i)** MW1 Tries to use $P(X \cup Y) = P(X) + P(Y) - P(X \cup Y)$
W1 Correct answer

(ii) MW1 Finds $P(\bar{X} \cap \bar{X}) = 0.08$ (Allow ft of their (i) answer used correctly)

M1 Tries to use $P(Y \mid \bar{X}) = \dfrac{P(Y \cap \bar{X})}{P(\bar{X})}$

W1 Correct answer

(b) **(i)** M1 Knows to find product of $P(F \mid A)$ and $P(A)$ etc
M1 Knows to add each product
W1 Two products set up correctly
W1 Third product set up correctly
W1 Correct answer

(ii) M1 Tries to use $P(B \mid F) = \dfrac{P(B \cap F)}{P(F)}$

W1 Correctly set up (allow ft for using their answer to (i))
W1 Correct answer

7 **(i)** M1 Tries to standardise
MW1 Correct inequality
M1 Correct method for finding area to right of $Z = 1.875$
W1 Correct answer

Notes
[A] Calculator use:
M1 – states $S \sim N(180,64)$
MW1 – correct inequality stated or indicated clearly on sketch graph
M1 – use of 1 – …
W1 – correct answer

(ii) MW1 Values correctly standardised
M1 Correct method used for finding area
W1 Correct area
MW1 Correct percentage

Notes
[A] Calculator use: MW1 and M1 may be awarded if clearly labelled sketch graph shown

(iii) MW1 Identifies correct area
MW1 Correct z or $-z$
M1 Tries to "de-standardise"
W1 Correct answer

Notes
[A] Calculator use: MW1 – identifies correct area on sketch graph
M1 – use of calculator
MW1 W1 – correct answer

8 M1 Correctly stated H_0
MW1 Correctly stated H_1
MW1 Correctly stated distribution
MW1 States 2 − tailed test
MW1 Correct critical value – See [A] below
M1 Tries to use $z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
W1 Correct value of denominator – See [B] below
W1 Correct value of z
M1 Reject H_0
MW1 Correct conclusion in context

Notes
[A] Alternative solution – instead of finding critical values, proceed to find z
Using $z_{test} = 2.3851$
MW1 – for $P(Z > 2.3851) = 0.008537$
MW1 – compare 0.008537 with 0.025
and proceed to make decision and state conclusion

[B] Alternative solution – find $z_{critical} = \pm 1.96$
M1 W1 $\dfrac{x - 100}{\frac{15}{\sqrt{80}}} = \pm 1.96$
W1 $x = 103$, $x = 96.7$
Then compare 104 with these values to make decision and state conclusion

[C] Use of 1-tailed test – can award
M1 MW0 MW1 MW0 MW0 M1 W1 W1 M0 MW0 – max of 5/10

[D] Final 2 marks can be awarded as ft provided 1st 7 marks already awarded

9 **(i)** MW3 One for each correct independent statement

(ii) M1 Correctly stated H_0
MW1 Correctly stated H_1
MW1 Correctly stated distribution
MW1 Correctly stated decision criteria
M1 Identifies Y ~ Bin (20, 0.2) – see [A] below
MW1 Correct value of $P(X \leq 12)$
M1 Reject H_0
MW1 Correct conclusion in context

Notes
[A] Calculator use: may not see this statement. Award M1 MW1 for correct $P(X \leq 12)$