Rewarding Learning

ADVANCED

General Certificate of Education

2019

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

AMM41

[AMM41]
FRIDAY 21 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows a framework ABC of light pin-jointed rods suspended by vertical cables at A and B.
AB is horizontal.
A load of mass m kg is suspended from C.

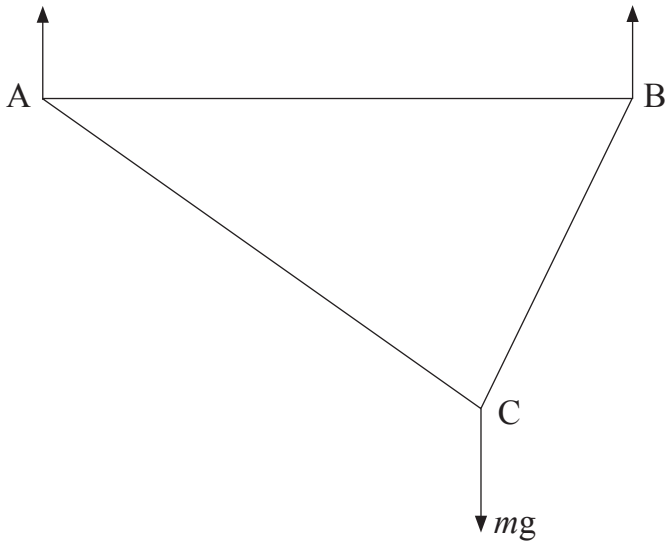

Fig. 1

AB = 5 m, AC = 4 m, BC = 3 m

Find the force in AB, stating whether it is a tension or a thrust. [6]

2 Viscosity η in a liquid is defined as

$$\frac{\text{shear stress}}{\text{velocity gradient}}$$

Shear stress is defined as

$$\frac{\text{applied force}}{\text{area of contact}}$$

Velocity gradient between two layers of liquid is defined as

$$\frac{\text{difference in velocity}}{\text{distance between layers}}$$

Find the dimensions of η. [6]

3 **Fig. 2** below shows a system of forces of magnitudes 3 N, 5 N, Y N and Z N acting on a light triangular lamina ABC.

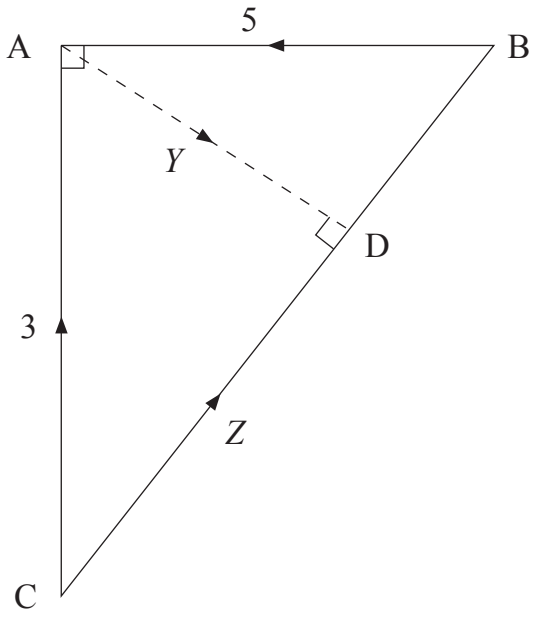

Fig. 2

$B\hat{A}C = 90° = B\hat{D}A$
AC = 4 m, AB = 3 m

The resultant of this system of forces is zero.

(i) Find Y and Z. [6]

(ii) Find the resultant moment of this system of forces. [4]

4 A satellite of mass m kg has a weight of mg N on the surface of the Earth.
Model the Earth as a sphere of radius R metres and mass M kg.

(i) Use the Universal Law of Gravitation to find G, the universal gravitational constant, in terms of M, R and g. [3]

The satellite is placed in a circular orbit at a distance d metres from the centre of the Earth.
The satellite travels at an angular speed of ω rad s^{-1}
The period of rotation of the satellite about the centre of the Earth is T seconds.

(ii) Find T in terms of g, R and d. [6]

(iii) Given that $R = 6.4 \times 10^6$ and $T = 86\,400$, find d. [2]

5 A car of mass m kg is being driven at a speed of v m s^{-1} round a bend of radius 49 m on a level road.
The coefficient of friction between the car's tyres and the road surface is 0.8
The car is about to slip away from the centre of the bend.

(i) Find v. [4]

A bend of the same radius on the same road is banked at 5° to the horizontal.
The coefficient of friction is unchanged.

(ii) Find the maximum speed at which the car can be driven round this bend without slipping. [9]

6 **Fig. 3** below shows a uniform solid hemisphere of radius R metres.
A is the centre of the circular face of the hemisphere.

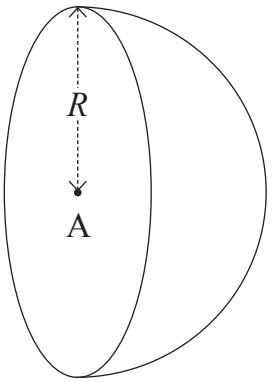

Fig. 3

(i) Show that the centre of mass of the hemisphere is a distance $\frac{3R}{8}$ metres from A along the axis of symmetry. [7]

A paperweight can be modelled by a uniform solid cylinder of radius R metres and length L metres joined to the hemisphere as shown in **Fig. 4** below.

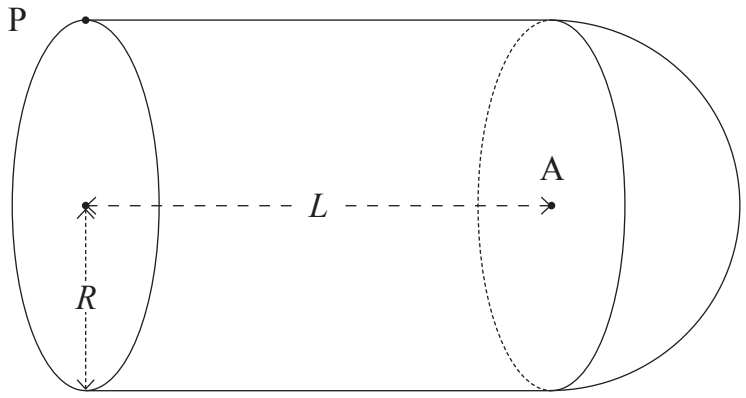

Fig. 4

The hemisphere and cylinder are made from the same material.
The centre of mass of the paperweight is at A.

(ii) Find L in terms of R. [4]

The paperweight is freely suspended from a point P on the circumference of its base.

(iii) Find the angle that the axis of symmetry of the paperweight makes with the vertical. [3]

7 X, Y and Z are three smooth balls of equal radii at rest in a straight smooth horizontal groove, with Y between X and Z.
X, Y and Z have masses m kg, $2m$ kg and $3m$ kg respectively.
The coefficient of restitution between X and Y and between Y and Z is e.

X is propelled towards Y with speed u m s^{-1}
After X and Y collide they have speeds v_1 and v_2 respectively.

(i) Find v_1 and v_2 in terms of u and e. [8]

Y now collides with Z.

(ii) Find the speed of Y after this collision. [3]

Given that $e < 0.5$, X collides with Y again.

(iii) Find the range of possible values of e. [4]

THIS IS THE END OF THE QUESTION PAPER