Rewarding Learning

ADVANCED

General Certificate of Education

2019

Mathematics

AMM31

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]
THURSDAY 20 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Part of a logo on an advertising hoarding can be modelled by the uniform lamina ABCDEF shown in **Fig. 1** below.

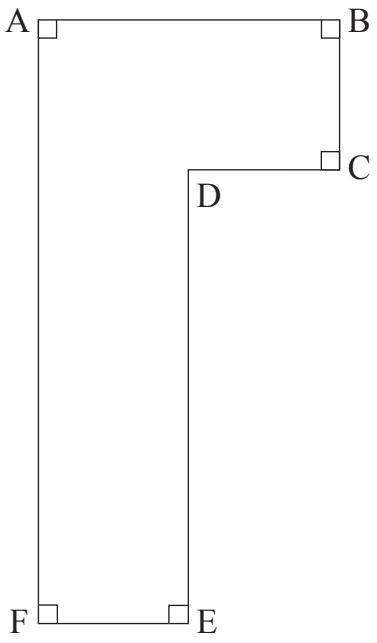

Fig. 1

The total mass of the lamina is 5 kg.
AF = 4 m, AB = 2 m, BC = 1 m, FE = 1 m.

(i) Find the distance of the centre of mass of the lamina from the edges AF and AB. [6]

Particles each of mass 1 kg are fastened at the points A, B, C and D.

(ii) Find the distance of the centre of mass of this new system from the edges AF and AB. [4]

The new system is freely suspended from the point A and hangs in equilibrium.

(iii) Find the angle the edge AF makes with the vertical. [3]

2 A light aircraft flies at 150 km h^{-1} in still air.
The wind is blowing steadily at 30 km h^{-1} from due west.
A is 200 km N30°E from B.

(i) Find the course the pilot of the aircraft should steer to fly directly from A to B. [5]

(ii) Find the time taken for this flight. [3]

(iii) Find the course the pilot should steer to return directly from B to A. [3]

3 A particle P of mass m kg is suspended from a fixed point A by a light elastic string of natural length l metres and modulus of elasticity $2mg$ N.
P hangs in equilibrium a distance d metres vertically below A.

(i) Show that $d = \frac{3l}{2}$ [4]

Take the gravitational potential energy to be zero at the level of the horizontal through A.

(ii) Find the total mechanical energy of the string and the particle, when the particle is hanging in equilibrium. [4]

P is now pulled downwards until it is a distance $\frac{5l}{2}$ metres vertically below A and is released from rest.

(iii) Find the speed of P when it is again a distance d vertically below A. [5]

4 **(a)** A particle P of mass 3 kg moves along a straight horizontal line under the action of a force F newtons;

$$F = \frac{1}{9}(30x - x^2)$$

where x metres is the distance of P from a fixed point on the line.

(i) Find the work done by F as P moves from $x = 9$ to $x = 18$ [4]

When $x = 9$ the speed of P is $5\,\text{m s}^{-1}$

(ii) Find the speed of P when $x = 18$ [3]

(b) A particle Q of mass 0.6 kg moves under the action of forces A newtons and B newtons.

$$A = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} \alpha + 3 \\ \alpha - 1 \\ \alpha + 2 \end{pmatrix}$$

Q moves from the origin O to the point L whose position vector is $\begin{pmatrix} 6 \\ 4 \\ -3 \end{pmatrix}$ metres.

(i) Find, in terms of α, the total work done as Q moves from O to L. [4]

Q is initially at rest at O.
When Q reaches L it is moving at $12\,\text{m s}^{-1}$

(ii) Find α. [3]

5 A particle moves with Simple Harmonic Motion between two fixed points A and B on a straight horizontal line.
A and B are a distance d metres apart.
The particle is instantaneously at rest at A and at B.
The maximum speed of the particle is $10\,\text{m}\,\text{s}^{-1}$ and its maximum acceleration is $25\,\text{m}\,\text{s}^{-2}$

(i) Find d and the periodic time T of the motion. [6]

The particle passes through the point C on the line, where BC = 2 m, while travelling towards A.

(ii) Find the speed and acceleration of the particle at C. [3]

(iii) Find the time taken for the particle to return to C for the first time. [4]

6 **Fig. 2** below shows a particle P of mass m kg at rest on a rough horizontal surface.
P is attached to a fixed point A on the surface by a light elastic string of natural length l metres and modulus of elasticity $3mg$ newtons.
A light inextensible string is fastened to P, passes over a fixed smooth pulley B at the edge of the surface and supports a second particle Q.
Q is of mass m kg and hangs vertically below B.

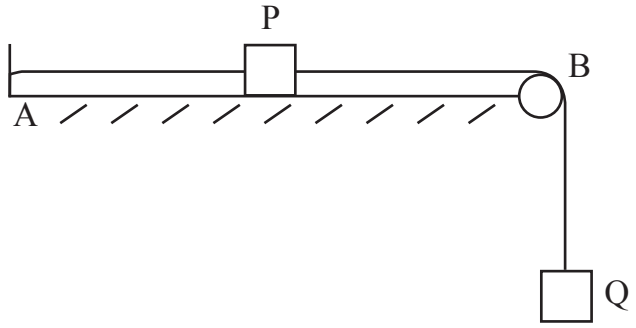

Fig. 2

When the distance AP is d metres, P is about to slide towards B.
When the distance AP is $\frac{3d}{2}$ metres, P is about to slide towards A.

(i) Find μ, the coefficient of friction between P and the surface. [9]

(ii) Find d in terms of l. [2]

THIS IS THE END OF THE QUESTION PAPER