Rewarding Learning

ADVANCED

General Certificate of Education

2019

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

AMF31

[AMF31]

WEDNESDAY 19 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$, where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A crystal of feldspar takes the form of a parallelepiped. Three of its edges are given by:

$$\mathbf{p} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$$

$$\mathbf{q} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$$

$$\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$$

Find its volume. [5]

2 Find a vector equation of the plane which includes the lines

$$\frac{x-5}{4} = \frac{y+2}{1} = \frac{z-3}{-3}$$

and $[\,\mathbf{r} - (\,7\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}\,)] \times (\,2\mathbf{i} - \mathbf{j} + \mathbf{k}\,) = \mathbf{0}$ [6]

3 Consider the function

$$y = \sin^{-1} \frac{x}{\sqrt{1-x}}$$

(i) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - \frac{1}{2}x}{(1-x)\sqrt{1-x-x^2}}$$ [6]

(ii) Hence find the exact equation of the tangent to the curve of the function at the point where $x = -1$ [3]

4 **(i)** Show that

$$\ln(x - \sqrt{x^2 - 1}) \equiv -\ln(x + \sqrt{x^2 - 1}) \qquad x \geqslant 1$$ [3]

(ii) Hence show that if $\cosh y = x$, where $x \geqslant 1$, then

$$y \equiv \pm \ln(x + \sqrt{x^2 - 1})$$ [4]

(iii) Hence solve

$$2\tanh^2 x - 5\operatorname{sech} x + 1 = 0$$

giving the answers in terms of natural logarithms. [5]

5 A computer game designer models a mountain as the shape ABCFED shown in **Fig. 1** below. It comprises planar faces and straight line edges.

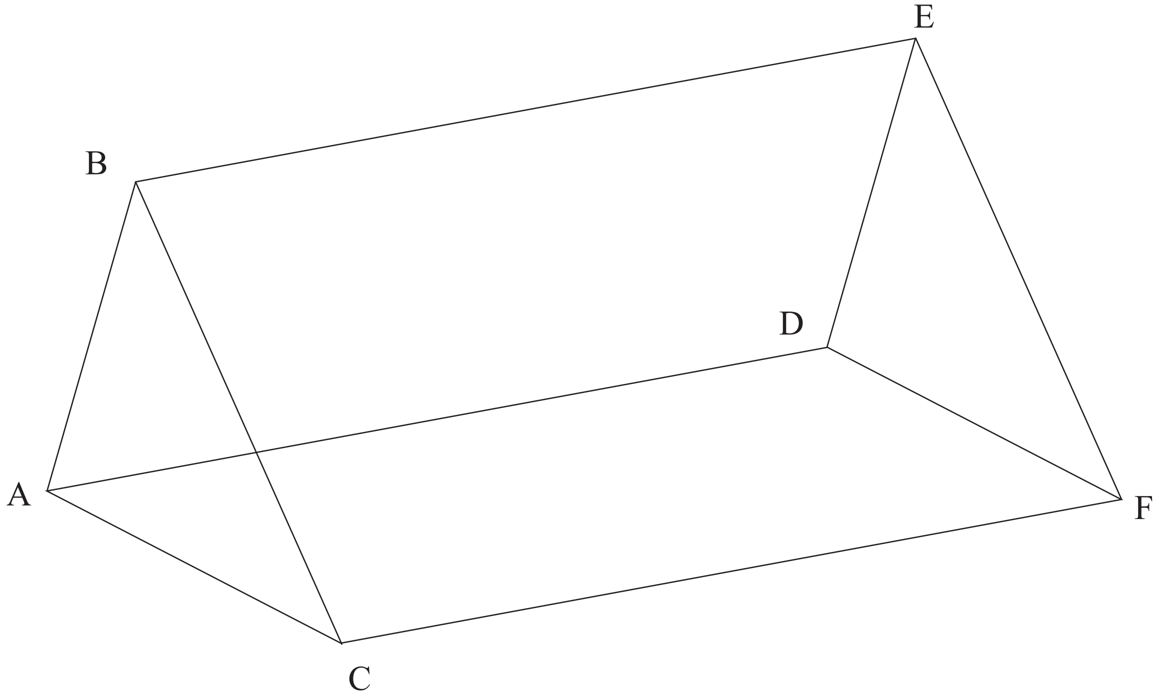

Fig. 1

The equations of the planar faces of two sides of the mountain are:

ABED: $x + y + z = 11$

CBEF: $4x + 2y - 3z = -4$

(i) Show that the ridge BE can be expressed by the vector equation

$$\mathbf{r} = \lambda\begin{pmatrix} a \\ b \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ c \\ d \end{pmatrix}$$

where a, b, c and d are integers to be determined. [6]

(ii) Calculate the angle between the planes ABED and CBEF. [6]

6 **(a)** **(i)** Find $\int \operatorname{sech} u \, du$ [4]

(ii) Find $\int \tanh u \sinh u \, du$ [4]

(b) Using the substitution $p = \sin x$, find

$$\int \cos x \tan^{-1}(\sin x) \, dx$$ [8]

7 **(i)** Differentiate

$$\sqrt{9 - x^2}$$ [1]

Consider the integral

$$I_n = \int \frac{x^n}{\sqrt{9 - x^2}} \, dx \qquad n \geqslant 0$$

(ii) Show that

$$nI_n = -x^{n-1}\sqrt{9 - x^2} + 9(n-1)I_{n-2} \qquad n \geqslant 2$$ [7]

(iii) Hence, find the exact value of

$$\int_0^{1.5} \frac{x^4}{\sqrt{9 - x^2}} \, dx$$ [7]

THIS IS THE END OF THE QUESTION PAPER