GCSE

Rewarding Learning

Chief Examiner's Report

Further Mathematics

Summer Series 2019

Foreword

This booklet outlines the performance of candidates in all aspects of this specification for the Summer 2019 series.

CCEA hopes that the Chief Examiner's and/or Principal Moderator's report(s) will be viewed as a helpful and constructive medium to further support teachers and the learning process.

This booklet forms part of the suite of support materials for the specification. Further materials are available from the specification's section on our website at www.ccea.org.uk.

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GCSE FURTHER MATHEMATICS

Chief Examiner's Report

This was the first year that all papers on the new specification were offered.

Unit 1 had a few new topics which were generally answered well. Compared with the corresponding paper on the legacy specification, there were a few more challenging questions this year - familiar topics but tested in a slightly different way, which resulted in good discriminating questions.

The performance of candidates on Units 2 and 3 was very similar to that on Unit 2 of the legacy paper, where the topics of Mechanics and Statistics were combined.

Unit 4 was new to the syllabus. Unfortunately it was not a popular choice as perhaps teachers and candidates preferred to stick to the topics which were on the legacy syllabus. As the number of candidates taking Unit 4 was so small no meaningful report could be written for this paper.

Assessment Unit 1 Pure Mathematics

Candidates generally responded very well on this paper with a mean mark of 61 (out of 100). There were some high scoring papers, though not as many gaining full marks as in previous years. There were also some very low scoring papers.

There were enough difficult parts to test the more able candidates, but the paper also enabled the weaker candidates to display their basic mathematical skills and score marks, particularly in the early part of the paper.

Solutions to questions were set out well, with answers given to an appropriate degree of accuracy.

Unfortunately a lot of basic algebraic errors were evident and this undermined the potential for full solutions in Questions 11 and 14 in particular.

The paper was of the correct length as there was no evidence that candidates were running out of time.

Q1 This question was answered correctly by a majority of candidates. Most errors were simple arithmetic slips, but a number of candidates wrote the terms in $\mathbf{A}^2$ as simply the square of the corresponding terms in **A**.

Q2 Part (i) was a straightforward question and was generally answered well, though more candidates than expected were clearly unfamiliar with the topic of completing the square. If Part (i) was answered correctly then Part (ii) was generally answered correctly as well. However, some candidates were unable to interpret correctly what the minimum value and the corresponding value of x were. Some attempted to find the minimum by differentiating.

Q3 Part (i) was answered well with few candidates having any problems with differentiation. It was encouraging to see the term $\frac{5}{x^4}$ being differentiated correctly by most candidates. At this level candidates should be encouraged to simplify their answers. For example, a large number left the first term as $\frac{6}{8}x$ rather than $\frac{3}{4}x$, though they were not penalised for this. Part (ii) was also answered well. Some candidates had problems getting the correct sign for the second term and some failed to include the constant of integration.

Q4 In Part (a), most candidates drew a correct cosine curve. However, some of the curves tended to be much too straight between the maximum and minimum points. A number of candidates failed to mark an adequate number of points on the axes; in this case -90°, 0°, 90°, 180°, 270° and 360° should be marked on the x-axis and +1 and –1 on the y-axis.

Part (b)(i) was a very familiar question for candidates and it was answered very well. A few candidates used 0.184 instead of –0.184. Part (b)(ii) was answered fairly well, but the most common error was to follow through for both angles from Part (b)(i) and give two answers, not realising that one of the answers was outside the stated range.

Q5 This question was answered very well. Most knew that they needed to find $\mathbf{P}^{-1}$ and to multiply $\mathbf{P}^{-1}\mathbf{Q}$ in the correct order. A few made an arithmetic slip in calculating the determinant.

Q6 Most candidates scored well on this question as they could get most of the marks by simply solving the quadratic equation. Most were able to write down the solution correctly. Not all candidates drew a simple sketch of the quadratic. They should be encouraged to do so to help them identify the correct region for the answer.

Q7 This question was answered well as candidates were familiar with this type of question. Nearly all candidates knew that they needed to eliminate variables and use back substitution. Arithmetic errors were the main cause of incorrect answers, but candidates could still gain most marks for using the correct method.

Q8 This question was generally well done as it was a standard curve sketching question which would be familiar to candidates.

Part (i) was answered well with only a few candidates giving x values only instead of coordinates. A number of candidates differentiated and found turning points in this part. Part (ii) was answered well with almost all candidates knowing to differentiate and set the expression equal to zero. Quite a few made an arithmetic slip in calculating the y value corresponding with $x = -\frac{5}{9}$. Part (iii) was well done.

Occasionally, errors in Part (ii) led to the conclusion that there should be either two maximum or two minimum points. This should have led candidates to realise that an error had been made. Almost everyone who answered Parts (i) to (iii) correctly were able to draw a correct sketch. A few lost a mark by not clearly marking the values on the x-axis where the curve crossed the axis and not marking the coordinates of the turning points.

Q9 This was not a standard question and most candidates struggled to complete it successfully.

In Part (a), the most common answer was $\frac{3 \log x}{2 \log}$, which was disappointing to see. Some did get the expression $\log y^2 = \log x^3$ and a number went further to get $y^2 = x^3$, but very few were able to write $y = x^{\frac{3}{2}}$.

In Part (b)(i), the majority of candidates struggled, with hardly any candidates simply using indices. Instead they used logs. Many were able to write $\log 4 + x \log 2 = y \log 2$, but were unable to proceed any further. In Part (b)(ii) not many used the result from the previous part. This part was not answered as well as similar questions in previous years as they found it difficult to deal with the right-hand side of the equation.

Q10 This question was new and it was attempted reasonably well.

In Part (i), almost all candidates knew to take logs, though some did not use values to 3 decimal places as stated in the question. Graphs were generally drawn well, but

the commonest error was to put log r on the y-axis. Candidates should always clearly mark the data points on the graph with a small circle round each point as in a number of cases it was not clear where the points were plotted. Candidates were generally able to find the value of b from the gradient of the graph, but quite a few were unable to find a correctly, often using logs and giving log a as the answer.

In Part (ii), those candidates who had correct values for a and b had no difficulty with this part. However, if errors were made in calculating a and b then the answer in this part was often inconsistent with the given data. The answer should clearly lie between 308.7 and 636.9, but those who found a value outside this range didn't seem to realise that an error must have been made. In Part (iii), incorrect values for a and b again often led to a result inconsistent with the given data. Some candidates gave the radius as the answer instead of the diameter. The final part asked for what assumption was made. A number clearly misunderstood the question and sometimes wrote that they were assuming that the diameter was twice the radius.

Q11 This question was not answered well and proved to be a good discriminator. Most candidates knew that they needed to differentiate, but surprisingly some could not differentiate the term $-3x^{-1}$, even though they had successfully differentiated a similar expression in Question 3. Some wrote the expression as $y = -3x^{-1} + c$ and stated that the gradient was –3, presumably thinking of the expression $y = mx + c$. Even if the gradient of the tangent was incorrect, candidates were given credit if they knew that the gradient of the normal was the negative reciprocal of the gradient of the tangent. It was evident that candidates need more practice in determining the equation of a normal.

Q12 Part (i) was well done, with most candidates able to expand the brackets successfully. In Part (ii), most candidates were able to substitute their answer from Part (i) into the numerator, but then often made errors while gathering together all the terms in the numerator. Those who did get the numerator correct often divided by 3 before factorising, leading to the incorrect answer $\frac{x+2}{x}$. This question showed that candidates generally need more practice with algebraic expressions.

Q13 This proved to be the most difficult question on the whole paper, with very few candidates getting both parts of the question correct.

In Part (i), many candidates knew that integration was required and often knew the correct limits to use. However, while integrating many failed to note that k was a constant and integrated it to get $\frac{k^2}{2}$. Candidates need more practice in questions involving unknown coefficients. In Part (ii), many misinterpreted the question and took the area of B to be $\frac{5}{9}$ times the area of A.

Q14 In Part (i), most candidates got the area correct. However they should be encouraged to simplify their answers. Many left their answer as $xy + \frac{x}{3} \times \frac{y}{3}$. Part (ii) was answered reasonably well, but again very few simplified their answer. In Part (iii), most candidates knew to set their expression for the perimeter to 96. However, if they hadn't simplified their answer in Part (ii) they often made algebraic slips.

Many candidates did not attempt the final part, perhaps because they didn't realise that they needed to use calculus. Quite a few worked with the area ABCD only and took the area to be xy. This actually led to the correct answer and they were able to get most marks. A few forgot to show that the area was a maximum.

Assessment Unit 2 Mechanics

The overall performance on this unit was very good with a mean mark of 32 (out of 50). A significant number of candidates scored high marks with excellent scripts.

The paper was accessible to candidates at all levels with Questions 3 (i) and (iv) and 5 (iii) being answered well by the majority of candidates. Most candidates seemed to be familiar and confident with the techniques and theory required in this unit and the best candidates were able to achieve high scores. Questions 4 and 5 (iv) were only answered well by the more able candidates.

There was no evidence of candidates running out of time in the paper. The only issue over language used in the paper was the use of the word 'quantity' in Question 1. A large number of candidates used the word quantity in their definitions and hence the markers were instructed to accept the word quantity to mean magnitude or size;

Q1 A surprisingly large number of candidates did not know the definitions of vector or scalar quantities or mixed up the definitions, which was disappointing to see. Most candidates were able to give a correct example of either a vector quantity or a scalar quantity or both which showed some knowledge of the topic. A number of candidates gave a specific definition such as distance or force moved in a specific direction, which was not given credit as a general definition was required.

Q2 This question was generally answered well. In Part (i), a number of candidates used an incorrect equation or used the correct equation incorrectly. It was disappointing to see a lot of basic arithmetic errors in Part (i) – not squaring the value of t in the equation $s = ut + \frac{1}{2}at^2$ or expanding brackets incorrectly. In Part (ii), finding the velocity as a final answer was done by the majority of candidates with only the stronger candidates correctly finding the speed. It was disappointing to see a large number of candidates trying to find the speed using $speed = \frac{dist}{time}$ and attempting to use incorrect magnitudes throughout. Candidates should be made aware that any text in a question given in **bold** means that a hint has been given to take care in the question, read it carefully and answer what is asked. In Part (iii) ,a number of candidates were able to correctly follow through from their calculated velocity in Part (ii) to gain full marks in Part (iii). It was surprising to see candidates finding $\tan^{-1}\left(\frac{x}{y}\right)$ even though the correct formula is given in the formula sheet;

Q3 Part (i) was reasonably well done. Mistakes included adding a frictional force, not labelling the forces or not putting arrows on the forces. In Part (ii), a large number of candidates failed to recognise that there was equilibrium vertically and didn't set up an equation. For those setting up an equation, the equation often only had two terms and seeing $R = 17 \sin 33°$ was very common. Only the stronger candidates scored well here. A number of candidates interchanged sine and cosine and omitted g and used the mass of the block as its weight. Part (iii) was attempted better, with the majority of candidates attempting to use $F = ma$ and gaining at least one mark for having their equation equal to $5.8a$. Unfortunately, some candidates multiplied a by the weight and not the mass. The majority of candidates were able to avail of a straight forward follow through mark in Part (iv).

Q4 This was a very good discriminating question. In Part (i), a large number of candidates were not able to mark the forces correctly. Seeing the tensions labelled as the same letter; not seeing forces being labelled or arrows representing the direction of the force; seeing the 60N, 80N or the 46N forces being omitted; or seeing reactions added were common errors. Also, not putting the weight of the rod in the middle of the rod was common. A large number incorrectly marked the weight of the rod halfway between the strings.

Q5 Part (i) caused some problems in this question. A number of candidates failed to make an equation. Others tried to focus on the forces acting on the car or on the forces acting on the car and trailer as a whole with very little to no success. Those identifying that $F = ma$ needed to be used gained at least one mark for setting their equation equal to 350 × 0.9. Some better attempts at Part (ii) were seen although some poor arithmetic skills were seen; for example, when dealing with the car:

TF – 1020 – 647.5 = 850 × 0.9, leading to TF – 372.5 = 765;
or for the complete system,

TF – 1020 – 332.5 = 1200 × 0.9, leading to TF – 687.5 = 1080

Part (iii) was very accessible for all candidates and the majority of candidates gained full marks.

Part (iv) was a good discriminator and only the more able candidates gained full marks. A large number of candidates failed to attempt to find the new acceleration of the car and hence gained zero marks for this part. A number of candidates gained full follow through marks from their incorrect answers to previous parts of the question. This question would normally have focused on the trailer coming to rest after the tow-bar breaking and it was pleasing to see a lot of good attempts in calculating the changes to the car.

Part (ii) was only done well by the more able candidates. A large number of candidates were able to gain one mark for correctly giving a moments equation with at least two moments correctly calculated and were able to equate vertically for one mark.

Q6 This question was designed to make candidates make an informed decision on their working and the more able candidates scored very well in it. The solution was split into three parts with three marks for calculating the accelerations for both options and the final mark for a correct conclusion which was only awarded if at least one of the two accelerations of Option A or B were correctly calculated. The majority of candidates were able to get at least one mark for Option A for putting an expression equal to $7a$ and also one mark for Option B was easily accessed for starting with '64.4 –' when finding their resultant force. For Option A, seeing 70 sin 27° = $7a$, or 18.7 = $7a$, each gaining one mark, was commonly seen and for Option B, seeing 64.4 – 18.7 = $7a$, gaining one mark, was often seen. A number of candidates tried to work out accelerations for Option A and B in the same direction and ended up with one positive answer and one negative answer. This was penalised as the question clearly stated the direction of travel of the block for each option and two positive accelerations should have been calculated. Candidates are advised when using $F = ma$ that the resultant force should be calculated in the direction the body is moving leading to an appropriate value of acceleration. Again some candidates resolved incorrectly with sine and cosine being interchanged and others omitted g from the weight of the block. Some poor arithmetic skills were also seen.

Assessment Unit 3 Statistics

Examiners felt that this was a very fair paper, which was successful in permitting candidates to display their knowledge of the specification, while effectively discriminating between candidates of different ability. Candidates generally performed very well on this paper with a mean mark of 36 (out of 50). There were many high scoring papers, but also some low scoring ones.

Questions progressed in difficulty throughout the paper and some topics proved to be good discriminators, namely the evaluation of standard deviation, including pooled samples and conditional probability. The new topics, binomial distribution and normal distribution were generally well attempted.

Candidates need to be reminded that answers should be given to 2 decimal places, unless stated otherwise, and means should not be rounded to the nearest whole number. The level of language used was appropriate and there was no evidence that questions were misinterpreted. Candidates appeared to have sufficient time to attempt all questions.

Q1 Part (i) was an easy opening question to find the mean of a grouped frequency distribution. It was done very well, with 85% gaining full marks, but some marks were lost for minor arithmetical errors. Thankfully there were very few cases of major method errors such as not dividing by 100. In Part (ii), the standard deviation question was generally well answered, with 70% gaining full marks, but it was disappointing to see a considerable number of errors in working out Σfx^2. Some candidates squared the fx column and many arithmetical errors were made.

Q2 Parts (i) and (ii) of this probability question were very well done by the majority of candidates. There was clear evidence of the understanding of a sum of two pair products in Part (i) and triple products in Part (ii). A few candidates multiplied P(BB) and P(GG) in Part (i) and P(BBB) and P(GGG) in Part (ii), instead of adding them. A few rounding errors were made in Part (i) when candidates chose to give the answer as a decimal. Therefore candidates should be encouraged to leave exact probability answers as fractions rather than decimals. Part (iii), which involved conditional probability, caused problems for many candidates, thus proving to be a good discriminator. Less than a third of candidates gained full marks for this part. Many candidates didn't realise that if 3 beads are the same colour then 2 beads have to be the same colour, i.e. they didn't realise that P(3 same and 2 same) = P(3 same).

The most common wrong answer was $\left(\frac{1}{5}\times\frac{7}{15}\right)\div\frac{7}{15}=\frac{1}{5}$

Q3 In Part (i), 80% of candidates were successful in finding the mean of all 14 results. However, a significant number incorrectly found the mean of the two boys' results and then found the average of the girls' mean and the boys' mean, giving an answer of 83.75%. In Part (ii), half of the candidates were able to find the standard deviation of all 14 results. Many candidates just gained the first mark for finding Σx^2 for the girls. Then they often just added the two other scores without squaring them or added the square of their sum. Around 30% gained no marks for this part. This question was a good differentiator of ability.

Q4 In Part (a), the expansion, using Pascal's triangle, was done very well by three quarters of the candidates. However, many careless mistakes were made and a few candidates didn't write out Pascal's triangle which cost them a mark. A small number of candidates wrote the expansion to the power of 5 instead of 6. Some candidates left out the plus signs in the expansion and a few candidates put extra plus signs in between the p and q terms.

In Part (b)(i), the majority of candidates were able to find the probability that there were no red pens. One common error was subtracting the correct answer from 1, giving 0.74 and a significant number mixed up p and q, using 0.2 instead of 0.8.

Part (b)(ii) proved to be the most challenging part of this question, with less than half of the candidates getting full marks. This part effectively tested a candidate's ability to select the correct terms from the binomial expansion, while computing the correct expressions for each term. The efficient method of 1 – {P(0) + P(1)} was preferred by the majority, while a number evaluated P(2) + P(3) + P(4) + P(5) + P(6). Many gained the first mark for knowing the method, but then made errors in their calculations. A significant number lost the final mark due to incorrect or early rounding. Some candidates found P(2 red) instead of P(at least 2 red).

Q5 This normal distribution question was very well done by nearly 70% of the candidates. However there was evidence that nearly a fifth of candidates may not have been taught this new topic as it was left blank, or they tried to use the standard deviation formula. A significant number of candidates got the first 2 marks for the correct z score, but then they forgot to subtract the reading from 1. In this question it was obvious that a good diagram of the standard normal curve definitely improved the chances of gaining full marks. A diagram should have the mean and x value below it, as well as the corresponding z values. Only the areas, and consequently the probabilities, should be above the axis. Candidates should be made aware of this use of a good diagram and there is even a hint at the bottom of the standard normal table.

Q6 In Part (i), most candidates gained full marks for the rank orders, but some mistakes were made with the tied ranks. Only a handful of candidates had ranks in reverse order. Spearman's coefficient of rank correlation in Part (ii) was generally well done, but some minor mistakes were made in summing the squares or in substituting into the formula. Some left out the 6 in the formula and some used 9×81 instead of 9×80. In Part (iii), the answer for the coefficient was generally interpreted correctly. Only a small number of candidates lost the mark by omitting the word correlation explicitly and only attempting a description of the relationship in context. In Part (iv), the means were generally correctly found, but unfortunately a significant number lost the mark for incorrect rounding.

In Part (v), lines of best fit were very well drawn by 80% of the candidates, although some candidates didn't draw them accurately through the means. Part (vi) was very well done by nearly half of the candidates. It was disappointing to see many candidates using points not on their line to find the equation of their line. They often used points from the table. Many candidates who used the method of rise/run for finding the gradient didn't remember to make the gradient negative, which illustrates the fact that using the gradient formula is the preferred method to be encouraged. A few candidates just found the gradient and did nothing more. Most candidates used the mean as one of their points to find the gradient and the c value, which is to be encouraged.

Q7 In Part (i), less than a quarter of the candidates gained the full 5 marks for this probability question involving a Venn diagram. Most candidates gained the first 3 marks for placing 17, 12 and 30 correctly on the diagram and for working out that 10 candidates chose just football and hockey. They then gave the answer as 10/120, forgetting to add 25 to 10 before dividing by 120. It was disappointing to see answers greater than 1. A significant number gave the final answer as 10.

In Part (ii), it was a pleasant surprise to see that 60% of candidates gained full marks for this challenging probability question. A small number of those who didn't gain full marks were able to gain the first mark for setting out the correct method. Many candidates got the correct answer with little or no working out, which was not penalised in this rare case.

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