GCE

Rewarding Learning

Chief Examiner's Report

Mathematics

Summer Series 2023

Foreword

This booklet outlines the performance of candidates in all aspects of this specification for the Summer 2023 series.

CCEA hopes that the Chief Examiner's and/or Principal Moderator's report(s) will be viewed as a helpful and constructive medium to further support teachers and the learning process.

This booklet forms part of the suite of support materials for the specification. Further materials are available from the specification's microsite on our website at www.ccea.org.uk.

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GCE MATHEMATICS

Chief Examiner's Report

Subject Overview

Candidates are to be commended for their pleasing performance in each of the Pure papers.

However, there appeared to be some challenges in the Applied papers. Each of the Applied units had been optional in 2022 and indeed this was the first year, since the introduction of the Revised Specification, which saw a full cohort of candidates sit the A2 Applied paper.

It would appear that there were gaps in the knowledge of some candidates.

Some common themes which caused concern across all units were poor presentation of graphs and diagrams; inability to produce clearly presented arguments in proof/show that questions; poor use of language in interpretative questions and failure to relate interpretations to the context of the problem.

However, many candidates were able to produce very good responses and achieve well-deserved results.

Assessment Unit AS 1 Pure Mathematics

Unit Overview

This paper allowed candidates of all abilities to demonstrate their strengths in the content studied. The availability of Advanced Information appeared to cement concepts examined in the earlier parts of the paper.

Although candidates' working was generally well structured and accurate, common errors were still evident in basic working such as algebra and trigonometry, and this undermined the ability to generate full solutions.

Q1 This was generally very well answered with most candidates using the most efficient way to solve.

There is still evidence of candidates not checking their answers in a question where this is an option.

Q2 Very well answered given that this is a common type of question.

Issues still exist both in terms of notation and some candidates offering a number of possible options.

Q3 **(a)** This was a straightforward question involving natural logs, but too many candidates did not get any marks due to an inability to get the first line of the solution.

Many candidates gave a decimal solution but were not penalised if the exact solution appeared before that.

(b) This was very well answered with the majority using the Remainder Theorem and solving correctly.

Q4 **(a)** This question needed some thought around how to work out the required area, but most candidates made accurate attempts. Some candidates used work outside the AS specification (area between two curves) alongside those who adopted a geometrical approach.

(b) This was a standard binomial expansion which was generally well answered. Common errors were mostly around the inability to deal with the negative fractional term within the expansion.

Q5 **(a)** This question was structured to enable candidates to focus on separate calculations that linked to a full solution. Most candidates made good attempts.

Part (iv) caused difficulty with the use of bearings.

(b) A standard trigonometric equation with the most common error being the cancelling of $\sin x$ terms and missing out on solutions.

Q6 **(a)** **(i)** The topic of vectors requires little working but errors were evident in the set-up of vector journeys and the linked substitutions.

(ii) Good attempts were made at magnitude, with exact solutions found.

(b) This was a standard nature of roots question with integer solutions onto ranges.

This topic continues to produce improving solutions over recent papers.

There is still an issue with notation, but candidates appear better prepared for this question.

Q7 **(a)** This was a very well answered max/min question in a simple context.

The most common error was not ruling out $x = 6$ when initially found.

The majority ruled it out at the second derivative stage.

(b) Candidates found this challenging and whilst many had a good idea of what they had to show, they lacked the elements of a reasoned argument.

Q8 This question caused difficulty given its unstructured nature. There was some evidence of candidates who produced a well-developed and accurate response but more common was a haphazard approach with work lacking direction and clarity. Algebraic weaknesses undermined correct coordinate geometry approaches.

Q9 **(i)** The majority of candidates attempted to rationalise the denominator and achieved the first 3 marks. The most common error was on the denominator where a missing bracket resulted in a sign error which prevented the algebraic terms from cancelling.

(ii) Very few candidates scored full marks in this part as many were working from an incorrect previous part or tried to work from the start. The more able candidates spotted the quicker method for solving and gained full marks with an efficient solution. Other possible approaches were considered and given credit.

Assessment Unit AS 2 Applied Mathematics

Unit Overview

In this unit, candidates were able to demonstrate their understanding of both Mechanics and Statistics. In general, candidates were able to make reasonable attempts for all questions with the Mechanics questions being more confidently answered than the Statistics.

The paper was successful in allowing candidates with different abilities to respond positively; many of the questions could be answered using a number of alternative solutions.

A significant number of candidates found it hard to contextualise their answers and poor use of language was often seen in the theory questions.

The reliance on the calculator was very evident in the Statistics section. Candidates should take care with their use of calculators and ensure that they show adequate method in the full development of their solutions.

Q1 **(i)** Candidates on the whole did very well in this question, although there was an unexpected number of candidates who did not know the relationship between velocity and speed.

(ii) This part was very accessible to most candidates and generally very well attempted. Some candidates tried to mix both scalar and vector quantities within the same equation of motion.

Q2 **(i)** Almost all responses accounted for weight correctly. Labelling both tensions equal was the most prevalent misconception in this question, followed by a number of candidates including a normal reaction R in the opposite direction of weight.

(ii) Candidates who correctly labelled tensions differently, say T_1 and T_2, worked well through this part and generally this question showed good understanding of resolving forces in equilibrium. However, for those that labelled both tensions the same, say T, their subsequent solution was nonsensical and allowed for little follow through marks to be gained here.

A number of candidates resolved along each string due to the strings being perpendicular, which greatly simplified the solution in this case.

(iii) This was well answered. The use of "inextensible" was a common misconception here, but if used alongside the correct answer of "light", this was ignored.

(iv) This part was well answered even if earlier parts were incorrect.

Q3 **(i)** On the whole the shape of the graph was very well drawn. The most common error was with the labelling of time $T + 5$; many candidates incorrectly labelled this as T or $T - 5$.

(ii) The majority of candidates correctly found the maximum velocity of $V = 25$m/s; often this was seen on the graph in part (i) rather than in their solution for (ii).

Follow through marks were awarded from incorrect graphs which allowed for successful differentiation of candidates' abilities.

Q4 **(i)** This question also allowed for good differentiation of candidates' abilities and a spectrum of marks were awarded here. This part was generally approached well, with most candidates resolving the weight correctly and using it in their friction. Most candidates were able to set up $F = ma$ equations for both particles and were awarded follow through marks from these.

A large number of candidates considered the system as a whole and carried out their calculations successfully and efficiently.

Many responses did not make good use of $\sin \alpha = \frac{3}{5}$ as given in the question and opted to work out the size of the angle α. However, rounding errors did not present a problem and full marks were accessible to these candidates.

(ii) This part was more challenging, and a majority of candidates did not attempt to calculate a new acceleration once the string had broken. Many of them simply changed their acceleration in part (i) to negative.

This question proved to be a good discriminator.

Q5 **(i)** Most candidates knew the difference between population and sample, with the majority of candidates being able to access these marks.

Concise language was a real challenge in Parts (i) and (ii). A lot of responses referred to the population as 'people' which was accepted provided entirety was implied.

(ii) Almost all candidates were able to identify that there were distinct strata within the population, but many did not explain the importance of a representative sample. A large number of candidates answered with "more 4-star hotels would be selected" as their reason for why random sampling would *not* be appropriate when indeed this would be a reflective outcome.

(iii) This was answered very well despite many spelling errors being observed.

Q6 **(i)** A surprisingly large number of candidates did not answer this question particularly well despite being a typical question linking means and histograms at both AS Level and GCSE.

Candidates who set up a table to calculate frequencies and list their midpoints went on to score well. As the answer was given in the question, and calculator use is allowed, candidates must "show" how they arrived at 9000/150 rather than simply stating this. A significant number of candidates split the final bar into two in their working which did not affect their answer in Part (i) but was a problem in Part (ii).

(ii) Finding the sum of squares appeared to be the most common error in Part (ii) along with mixing up variance and standard deviation. Commonly carried through was the error of the split bars seen in Part (i).

(iii) A lack of understanding of the meaning of standard deviation/variance was common.

Only the more able candidates made links to spread/variability/consistency, the majority mentioned "longer stays" etc.

Q7 **(i)** This part was answered well by almost all candidates.

(ii) The majority of candidates recognised that it was the Binomial distribution that should be used here and stated at least one condition, perhaps an advantage gained from reading the Advance Information, published prior to the examinations.

Some candidates tried to explain how to apply the Binomial formula and some tried to explain the benefits of using Binomial instead of stating the conditions required and relating these to the context given.

(iii) (iv) Were generally very well attempted.

Any difficulties in Part (iv) arose from a misunderstanding of "less than five".

A heavy reliance on calculator use was evident in these parts. Candidates using calculator functions in these questions should be encouraged to show evidence, the importance of which became apparent in cases where an error was made.

Q8 **(i)** A challenging question which discriminated well. Stronger candidates solved this question using a variety of methods; the best and most efficient solutions came from candidates who used a Venn diagram.

A significant number of candidates left this question blank.

(ii) (iii) Lack of a numerical answer in Part (i) made it difficult to gain marks in Part (ii) and Part (iii).

In Part (iii), many candidates stated $P(X \cap Y) = P(X) \times P(Y)$ with no reference to whether this was the condition for or against independence, or indeed substituting in values to make a conclusion.

A large number of candidates knew that $P(X \cap Y) = 0$ but many stated this in a variety of ways, often equivalently worded such as "no overlap" etc.

Assessment Unit A2 1 Pure Mathematics

Unit Overview

The standard of the question paper was fair leading to a good spread of results. Candidates could access most questions with the most able candidates achieving very high marks. Less able candidates could still gain marks for appropriate methods and within less complex topics. Overall, a reasonable standard of response from candidates.

Q1 Most candidates achieved high marks in this question clearly displaying their understanding of arc length, sector area and triangle trigonometry. A small number of candidates were unable to replicate the correct formulae for arc length and sector area.

Q2 **(a)** This question enabled candidates to display an understanding of the modulus function and how to correctly solve an inequality. Some errors were made in both understanding how the modulus function applies and how to solve inequalities. A good number of candidates achieved full marks here.

(b) **(i)** Most candidates were able to attempt the correct binomial expansion, but a small number of candidates were unable to correctly expand the quadratic term. Most understood the premise of the question, but small numeric errors meant that less accurate candidates did not achieve full marks.

(ii) Candidates needed to display clear understanding of how their solution to Part (b)(i) impacted the range of values for which their expansion was valid. Strong candidates performed well here but others struggled to identify the correct range of x values.

Q3 **(i)** The majority of candidates completed this question well. Some candidates did not have a clear understanding of the connection between the width and the x ordinates.

(ii) This question was well answered by most candidates. A small number of candidates struggled to correctly square the function.

Q4 **(i)** This question was poorly answered by candidates. Few candidates achieved full marks for correctly sketching the required function.

(ii) Good candidates correctly identified the domain and range of the function and recorded this using appropriate notation. This question still poses difficulty to a significant number of candidates.

(iii) Whilst most candidates seemed able to describe the two transformations involved only the best candidates were able to do this whilst identifying the required order. Some candidates were not able to identify the correct direction of the stretch or translation.

Q5 **(i)** This question did not pose difficulty for candidates and was well answered.

(ii) Most candidates were able to correctly identify the connection between Part (i) and this part. Candidates were usually able to perform the correct integration but very few candidates were able to leave a suitably simplified final exact answer. Several candidates did not correctly integrate the $(x + 3)^{-2}$ term.

Q6 **(i)** Most candidates correctly identified the arithmetic series element of this question. The majority were able to correctly answer this part.

(ii) A significant number of candidates assumed that the second part of the question was linked to geometric series and so did not gain marks here.

Q7 **(a)** As this question was a proof, candidates had to be careful to clearly display their working and methods.

(b) This question was poorly answered by candidates. Only a very small number of candidates were correctly able to find an exact value for $\tan A$. Most were able to correctly state the double angle formula for $\tan 2A$, but only the most able candidates gained a fully correct solution.

(c) **(i)** This question was well answered by the majority of candidates.

(ii) Candidates correctly used their solution from Part (i) here, but very few correctly identified all solutions to the equation. A number of candidates also struggled to carry the correct level of detail and accuracy throughout the problem giving final solutions that were slightly inaccurate.

Q8 **(a)** This question, based on implicit differentiation, was well answered by most candidates. A number of algebraic errors existed in the application of the negative sign before the quotient rule term and this led to slight errors in the final answer for some candidates.

(b) **(i)** Again this question did not seem to pose difficulty for the majority of candidates. Most correctly identified the use of the product rule and were able to find the equation of the tangent. Some struggled to leave the final equation in the required form whilst others made small arithmetic errors throughout the question.

(ii) Candidates showed a good level of understanding here in relation to the position of a turning point and identification of its nature. Less able candidates struggled to solve the required equations and a small number were unable to correctly find the second derivative.

Q9 **(a)** **(i)** This question was well answered by most candidates with the majority achieving 3 of the 4 marks. Candidates were required to comment on the continuous nature of the function in order to achieve full marks.

(ii) Most candidates were able to correctly apply Newton Raphson formula with the correct functions. A small number of candidates used only one function and so did not gain the majority of the marks available.

(b) Most candidates correctly attempted parametric differentiation. A small number of candidates tried to find a Cartesian form of the equation and differentiate this. Some candidates struggled to display understanding of differentiation of trigonometric terms.

Q10 **(i)** Candidates struggled to correctly interpret this question to form the required differential equation. Only the best candidates did this accurately.

(ii) Candidates displayed understanding of the need to separate variables to solve this differential equation. However, only the best candidates were able to apply all the required methods correctly.

Q11 **(a)** This question was well attempted by most candidates. Some candidates struggled to correctly square the expression in the integrand. Those that did, however, were able to display their understanding of trigonometric identities to arrive at a suitable expression to integrate. Top candidates were able to leave their answer in the desired exact form.

(b) Most candidates achieved some marks in this question. Very few were able to accurately identify the need to use integration by parts, with the subtraction of functions in the correct order. Only the most able candidates were able to achieve full marks here.

Assessment Unit A2 2 Applied Mathematics

Unit Overview

This was the first year that a full cohort of candidates sat this paper. Additionally, many of these candidates did not sit the AS Unit 2 Applied Mathematics last year. Taking this into consideration, there was a good range of outcomes with many candidates achieving pleasing results. It was evident however, that for some candidates there were gaps in their understanding and their ability to apply knowledge in context.

The paper allowed candidates to demonstrate their ability and there was no evidence that timing was an issue for any of the candidates.

Q1 **(i)** Many candidates were able to use conservation of momentum and the correct formula for momentum. However, there were two common errors. One was to give their answer as a velocity instead of a speed. Another was to fail to make it clear that the initial momentum was zero.

(ii) Many candidates were able to use impulse both as change in momentum and the product of force and time. However, only a small number of candidates found the correct magnitude of the force. Some candidates also lost marks for not having the correct direction of their force.

Q2 **(i)** This question was answered reasonably well. As this was a show that question it needed to be clear why $10\mathbf{j}$ had to be added on, and this was not always evident. Another common mistake was writing $23\mathbf{i}$ instead of $23t\mathbf{i}$ in their final answer.

(ii) Surprisingly this question was not answered well. A large number did not consider only the vertical component of the displacement and others that did try to use the vertical component did not differentiate it. Only the most able candidates managed to complete this question correctly.

(iii) This was very well answered.

Q3 **(i)** Most candidates picked up a minimum of 2 out of 3 marks. The most common error was not knowing the direction of the normal reaction at the sphere.

(ii) The majority of candidates understood how to take moments about a point and knew to use the principle of moments. However, there were some candidates who ignored the stated method and instead started to resolve forces. A further problem existed for those candidates whose diagram in Part (i) was incorrect. However, they were able to be awarded some of the 5 marks.

(iii) The majority of candidates were aware that they should resolve vertically and horizontally in this question and made a reasonably good effort at this which allowed them to gain method marks. Additionally, the correct formula for friction was used in most responses, allowing access to a method mark. However, only the most able candidates had fully correct answers. Again, part of the problem in this question was use of an incorrect diagram from Part (i). This question proved to be a good discriminator.

Q4 **(i) (ii)** These were generally very well answered.

(iii) Most candidates knew to integrate in this question and found the integral of $\sec^2$ to be $\tan$. However, only a few were able to find the correct coefficient of the tangent term. Some omitted the constant of integration and were unable to access the rest of the marks. Only the most able candidates managed to correctly complete this question.

(iv) This was not particularly well answered with many failing to focus on the asymptotic aspect of the displacement and velocity expressions as time tended to 4 seconds. Common incorrect answers stated that the train would have to slow down or change direction or made a reference to the range given in the question.

Q5 **(i)** This was generally very well answered.

(ii) This was extremely poorly answered. Few candidates could provide a clear and coherent definition of a *p*-value.

(iii) A small section of the candidature knew how to deal with this question. While there was some understanding of making a comparison, candidates often struggled with putting it in context.

(iv) This was poorly answered. If the candidate had failed to gain any marks in Part (iii) they generally failed to access any marks in Part (iv). Most candidates who did provide a correct response used half the significance level rather than double the *p*-value in their comparison.

Q6 This was answered well by a significant number of candidates. Typical errors were to use the probabilities of 0.05 and 0.75 as z-scores, using the incorrect area of 0.25, or using sigma squared as the standard deviation in the standardising formula. Most candidates were able to achieve a method mark if simultaneous equations were attempted.

Q7 **(i)** This question was accessible to all and was well attempted. Most calculated the probability with the help of a tree diagram. A small number of candidates lost a mark for giving their answer as a percentage.

(ii) This question proved to be a discriminating part of the paper with only a minority appreciating that the use of conditional probability was required.

(iii) This was reasonably well attempted with most candidates gaining at least one mark, but they often failed to either say the test was not dependable or did not back up their assertion with reference to data to get the full marks.

Q8 **(i)** This was well answered by most candidates. Those candidates who used z-scores were able to achieve at least 3 marks for finding the correct z-scores and those candidates who used calculators proved to be very successful in obtaining full marks.

(ii) This was well attempted, although poor notation meant some candidates lost marks. The most common approach to this question was to use z-scores and many candidates were successful in calculating $z = -2.168$ but some candidates lost marks with their comparison and did not give a conclusion in context.

Q9 **(i)** This was poorly answered. Candidates did not grasp that if Daniel had revised then the probability of getting an answer correct would have exceeded 0.2. The majority of candidates just said they were testing in one direction without any context and answers did not pinpoint the key point and lacked clarity.

(ii) This was not answered well. In some cases, the notation was poor. For many candidates $p < 0.2$ was their incorrect answer for the alternative hypothesis.

(iii) This was poorly answered. Either zero or two marks tended to be awarded in this part. Few candidates could provide a clear and coherent definition of the significance level.

(iv) This part was attempted by most candidates but proved to be a good discriminator. Only the best candidates achieved full marks, as too many had the wrong alternative hypothesis in Part (ii). Final conclusions were not always clear. However, most candidates were able to identify that it was a binomial distribution and gain at least one mark.

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