GCE

Rewarding Learning

Chief Examiner's Report

Further Mathematics

Summer Series 2023

Foreword

This booklet outlines the performance of candidates in all aspects of this specification for the Summer 2023 series.

CCEA hopes that the Chief Examiner's and/or Principal Moderator's report(s) will be viewed as a helpful and constructive medium to further support teachers and the learning process.

This booklet forms part of the suite of support materials for the specification. Further materials are available from the specification's microsite on our website at www.ccea.org.uk.

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GCE FURTHER MATHEMATICS

Chief Examiner's Report

Subject Overview

This was the first year in which a full cohort of candidates sat all four papers in this subject. The Applied units were not compulsory in 2022.

Candidates are to be commended for their good performance in each of the units. However, a few key themes were evident in all papers. These are largely issues which relate to examination technique rather than lack of knowledge.

General principles were understood but the presentation of work in many cases was disorganised. This not only caused problems such as transcription errors for the candidates but also difficulty for the examiners in trying to decipher the candidate's work.

Candidates were also prone to lose marks as a result of their lack of attention to details such as the format in which the answer was to be given and also through their lack of rigour in presenting a clear argument in proof questions.

Surprisingly it was often basic knowledge which caused difficulty, such as multiplication of terms involving indices or even knowing the formula for the area of a triangle.

However, many candidates were able to demonstrate their knowledge and achieve very pleasing results.

Assessment Unit AS1 Pure Mathematics

Unit Overview

Although candidates knew the general methods behind each question, their notation was often poor for Further Mathematics students and diagrams were untidy and careless. Many did not look out for the finer details of a question, such as giving an answer as a set of coordinates rather than a vector.

The paper seemed to allow all candidates to demonstrate a good degree of knowledge, the majority scoring highly.

Q1 Almost all students knew how to answer Parts (i) and (ii) with only a few making numerical errors.

Q2 Most candidates were able to find the sum and product of the roots. The majority correctly found the sum and product of the new roots. A few candidates did not finish the question in the appropriate format though. Only a few used the quadratic formula.

Q3 Most candidates made a good attempt at this question through use of inverse matrices, setting up and solving of simultaneous equations, or a combination of both. It was disappointing to see how many were unsure of when to pre and post multiply by the inverses but the fact that they were multiplying by the matrix $2\mathbf{I}$ meant that they could often get the benefit of the doubt and gain full or most marks.

Q4 This was a well answered, but often poorly notated, question. A large proportion of the candidates simply wrote the matrix down without the vertical lines to denote its determinant. However, they knew how to calculate the determinant. A small, but significant number, correctly expanded the determinant but when equating the components, forgot the negative part of the **j** component. There were some careless calculation errors for the simultaneous equations. Otherwise, it was correctly answered.

Q5 **(a)** **(i)** A few students set up the matrix equation in the wrong order. Some set up two separate matrix equations and used simultaneous equations to solve. Most who tried to find the inverse matrix, and use it, got the question correct.

(ii) Generally, candidates knew to find the determinant and use it as the scale factor. The biggest problem they had was actually finding the area of the original parallelogram.

(b) The key starting point of this question requires a clear understanding of the difference between an invariant line and a line of invariant points. If candidates did not set up the correct initial matrix equation, then it generally led to finding two values for their gradient m. If they did not consider $m = 0$ as a solution they lost the last two marks and if they did, they still had to consider that the line $y = 0$, although an invariant line, was not composed of invariant points. Many students scored 3 or 4 out of 5 for this reason.

Q6 **(a)** All candidates knew that the complex conjugate was also a root, and the majority were able to attempt the full solution. However, this question was riddled with careless errors; multiplying out the two factors, poor long division, or, for those who tried to equate coefficients, many did not realise that the original function had a $2z^3$ as its first term and assumed that the third factor was simply $(z + a)$. In their rush, candidates often neglected to actually write down the three factors before showing the solutions for the equation.

(b) **(i)** Almost all knew this locus was a circle, centred at (3, 2), radius 3. However, many of the sketches were small, inaccurate and poorly drawn. Although scale was not essential, some of the x and y intercepts of the circle were blatantly incorrect.

(ii) Again, candidates generally knew what to draw but were very shoddy in their sketching. The drawing of lines, rather than half lines, was the biggest mistake. Students were also not confident enough just to label the angle as $\frac{\pi}{4}$ and left the $-\frac{\pi}{4}$.

(iii) It was encouraging to see the number of students who correctly found the equations of the circle and line and solved them simultaneously. A small number tried to solve it using their sketches and were penalised.

Q7 **(i)** Again, poor or lazy notation marred this question, although most knew what they wanted to do and correctly answered it. The majority did not put vertical lines around their matrix to denote the determinant. Some did not put a variable matrix after their coefficient matrix for the initial set up. A few put the determinant $\neq 0$ and so scored 4 out of 6.

(ii) Having set $a = -1$ and $a = \frac{1}{2}$ most candidates set up two new sets of equations. However, when eliminating a variable, many did not label or show what they were doing, making it very difficult sometimes to see where they had got new equations from. Some conclusions did not follow on from the previous work. The general solutions were often spread around the working for the last question rather than being collated together using a different variable.

Q8 **(i)** Candidates used a variety of methods to find the line AC. Many tried by simultaneous equations and others by finding the vector equation of the line first. Most made a good start to their solutions but made calculation errors towards the end of the question, either not rearranging correctly for λ or finding an incorrect position vector. Candidates were penalised if their final answer was not written in Cartesian form.

(ii) This was fairly straightforward, based on their previous part, but candidates were penalised if they did not write the answer as coordinates.

(iii) Most candidates knew to use the cosine rule. Again, slack notation was apparent and many correctly got the acute answer of 83.7° without finding the obtuse angle first, or without modulus signs around their initial formula.

(iv) A nice question to finish. The best candidates elegantly found direction vectors and then correctly used the vector cross product to find the area. Many tried to use a formula for volume or made up an incorrect multiple of their cross product as the area. Quite a few tried unsuccessfully to use the dot product. A small number did not show the expansion of their determinants and had obviously used their calculator.

Assessment Unit AS2 Applied Mathematics

Unit Overview

The standard of candidates' responses was pleasingly high this year. The first section was attempted by almost all the candidates. It was excellently answered by the majority, with many candidates scoring very highly. While in the main, candidates were very well prepared, it appeared that some candidates were not adequately prepared in a small number of topics across the specification. As a general rule, candidates fared slightly less well in their second section responses.

There was some evidence to suggest that at the end of their second section a small number of candidates were under some time pressure in their last question or two. That said, nearly every candidate attempted every question in their chosen sections.

Section A Mechanics 1

Q1 The scenario of this question, a body moving up a rough plane, was required to be solved by work and energy considerations, in line with the specification. As such, there were a number of ways to approach a complete solution. Almost no candidates attempted to use forces and accelerations. The main approaches to solve this fell into two categories:

(1) to equate final energy with initial energy plus work done by non-conservative forces, or (2) total work done (by friction and gravity) equals difference in kinetic energies.

Q2 **(a)** Work done as the integral of a variable force was very well carried out, with nearly every candidate obtaining full marks.

(b) **(i)** Work done as the scalar product of total force with displacement was also well understood. Mistakes included not adding both forces and not using the displacement. A small number of candidates did not understand the scalar product at all.

(ii) Work done equals the change in kinetic energy was very well completed.

Q3 The problem involving the fountain in Granny's pond was finished by just under half of the candidates. Most candidates correctly considered the potential energy of the system, but quite a few omitted the kinetic energy component. There were two main approaches to solve this. One approach involved considering the energy of the mass of water moving through the system per second and then the kinetic energy per second plus the potential energy per second being set equal to the power of the pump. The second approach was to calculate the kinetic and potential energies of a particular mass of water and then divide by the time to move this mass to the top. There was clear evidence of candidates who appeared to be completely unfamiliar with this type of problem.

Q4 This question was easily solved by most of the candidates, no doubt due to it being structured into four parts. The last part, which proved to be the only difficulty in the question, depended on the idea that the circular motion described could only happen if the tension in the lower string is greater than zero.

Q5 Both parts of this question proved to be good discriminators.

(i) This part easily succumbs after considering the equilibrium of the $5m$ block and equating friction and the tension in the string. Although the question superficially appears to be of the "bungee jumper" type, use of energy considerations was inadequate to solve this problem. This resulted in a sizeable minority of the candidates being unable to access all of the six marks.

(ii) In the event, this part proved more accessible for a larger number of candidates. Most candidates were able to evaluate all of the component parts of the conservation of mechanical energy equation, and then combine them correctly.

Section B Mechanics 2

Q1 This question on vertical circular motion tested both force and energy approaches.

(i) Most candidates knew how to apply the expression for acceleration in circular motion. A number failed to use both components for the normal reaction at the top of the circle: 2500 N plus 1120g N. Others who tried to use energy only were unable to reach a solution.

(ii) This part easily yielded to an application of the conservation of mechanical energy.

Q2 **(i)** This part was excellently completed by nearly all the candidates. A few, however, failed to use Newton's second law on the earth's surface.

(ii) Most candidates were able to make an attempt at combining Newton's Law of Gravitation with $a = \frac{v^2}{r}$. A significant minority did not use the correct radius: the radius of the earth plus the height of the satellite's orbit.

Q3 **(i)** This part, identifying the dimensions of a quantity by dimensional considerations, was accurately completed by approximately one half of the candidates. There was much inaccurate indices work.

(ii) Candidates fared better with this very well-trodden path, predicting a product of powers using dimensions.

(iii) The final part of this question proved to be discriminating. Many candidates found difficulty, perhaps never having seen such a variation of quantities before.

Q4 **(i)** The first part was trivial.

(ii) Relative velocity continues to prove challenging to many candidates. A good answer starts with a careful velocity diagram illustrating a vector relationship of the form: $v_{AB} = v_A - v_B$. Indeed, there was a close correlation between those candidates who drew accurate velocity diagrams and those who gained most of the marks in the question. Correct attempts at solution fell into two broad categories: taking two dimensional components of the vector quantities and equating; or solving the resulting vector triangle using sine and cosine rules as necessary.

Q5 **(i)** The majority of candidates coped well with solving this right-angle triangle by introducing an unknown.

(ii) This was dispatched by resolving vertically and using Hooke's Law. This was the best completed part in the question. A number did not spot that the tensions on either side of the ring had to be equal and sadly could not proceed.

(iii) This was usually solved correctly by those who obtained full marks in the previous part.

Section C Statistics

Q1 **(a)** Surprisingly, this easy starter, involving a linear relationship between two random variables, proved difficult to a sizeable minority of candidates.

(b) **(i)** A surprising number did not know what a response variable is.

(ii) The majority of the candidates knew that the regression line should not be used to predict the explanatory variable.

(iii) Only a minority spotted that this part addressed the principle that correlation does not necessarily imply causation.

Q2 This simple question testing the geometric distribution was the best answered question in the statistics section. Full marks were awarded to virtually every candidate.

Q3 Some candidates have learned to count, and others not quite so well.

(i) A simple permutation.

Not every candidate spotted that Parts (ii) and (iii) produced fractional answers.

Nevertheless, these parts together with Part (i) were the best completed in the question.

(ii) One arrangement from the number of ways of arranging three horses from six.

(iii) One choice of three horses from the number of ways of choosing three horses from six.

Parts (iv) and (v) use the fact that we must divide the number of arrangements of n items by n as a given horse in a given order could start in any one of n possible equivalent positions.

(v) A number of candidates solved this part exhaustively by simply listing all the 36 possibilities.

Q4 Most candidates made a good attempt at the first two parts of this question.

(i) Correctly solved by the vast majority.

(ii) A disappointingly large number of candidates mistakenly believed that $P(Y > 12) = 1 - P(Y \leq 11)$, obviously being confused by the discrete distribution. This needs to be clearly emphasised to the candidates. Otherwise, the question was well answered.

(iii) Despite the direction to use the Poisson Cumulative Distribution Function table, approximately one half of the candidates were at sea in this part.

Q5 **(i)** Most candidates were able to use the principle that

$E(X) = \int x\mathrm{f}(x)dx$

to find an equation in a and b.

A smaller number knew to also use

$\int \mathrm{f}(x)dx = 1$

to find a second equation in a and b.

Full marks were obtained by just under one half of the candidates.

(ii) Most candidates attempted to use calculus, showing no awareness that the mode fell at one of the two endpoints in this positive x squared curve. This was a discriminating part.

(iii) Most candidates who scored full marks in part (i) were able to write down the equation for the median M,

$\int_{-\infty}^{M} \mathrm{f}(x)dx = \frac{1}{2}$

and solve it to obtain the requested equation.

Section D Discrete and Decision Mathematics

Q1 Candidates displayed a clear understanding of planarity and edge trails and edge circuits of graphs.

Q2 The technique of substituting an indicial term into a recurrence relationship to obtain a closed form solution was not fully practiced by the candidates in the main.

Q3 Proving the equivalence of Binomial expressions was well understood.

Q4 **(a)** Prim's algorithm to find a minimum spanning tree in a weighted graph was well attempted.

(b) Critical path analysis was very well answered and understood.

Q5 The group theory question was again well attempted with solid attempts made in completing a group table in Part (i), finding the identity in Part (ii), listing the periods of elements in Part (iii) and subsets of a graph in Part (iv).

(v) The completion of the group table of a 4-element matrix multiplication group was less well completed.

(vi) This part on the topic of identifying an isomorphism between two of three groups was also less well completed.

Assessment Unit A21 Pure Mathematics

Unit Overview

Generally, candidates knew the methods which they needed to apply in each question. However, their notation was often poor, for example, when using limits. The presentation of their work in a "proof" or "show that" question was often poorly structured and did not always provide convincing evidence for the required proof. In some cases, the handwriting and general presentation of their work was so untidy that it became difficult to decipher the actual methods being employed.

However, the paper allowed all candidates to demonstrate a good degree of knowledge, with many achieving very pleasing results.

Q1 This question was well answered by many of the candidates. However, a significant number proved the formula for u_2 instead of u_1. A few candidates struggled to prove the formula for u_{k+1}. The main problem, however, was the final statement, which was often vague and, in many cases, made no reference to Mathematical Induction.

Q2 Most candidates knew how to approach this question. However, the notation of trying to set up a limit was often poor, or in some cases non-existent. The actual integration of each function was generally not a problem.

Q3 This question was very well answered by most candidates. Almost all got the correct auxiliary equation, complimentary function, and particular integral. A few made minor calculation errors, and some left out the 'y =' at the end, but otherwise the majority of candidates gained full marks.

Q4 The expansion of the brackets was done well, and most candidates were able to separate the terms into correct summations. The most common mistake was the incorrect summations of n^2 and n, which were usually left as n^2 and n. In some other instances, untidy working resulted in candidates losing or incorrectly transcribing algebraic terms. It was evident that some candidates manipulated their incorrect working to produce the answer which was to be shown. However, most candidates achieved at least 7 of the 10 marks.

Q5 Almost all candidates were able to rewrite the fraction using partial fractions. However, candidates then tended to either get the integration fully correct by splitting the second integral into two parts, or completely wrong by trying to integrate using inappropriate methods.

Q6 **(i)** It was surprising, and disappointing, to see so many candidates struggle to set out such a simple proof.

(ii) All candidates were able to make some progress through this question. Those who used a substitution generally got the answer fully correct. Many used the formulae booklet to go directly to an integration giving an answer in terms of $\cosh^{-1} x$ or even straight to logarithmic form. However, a significant number of these candidates omitted the factor of $\frac{1}{3}$. Given that this was a "show that" question, it was clear that a large number of these candidates tried to bluff the final answer simply by writing it as their last line.

Q7 **(a)** **(i)** Many tried to rewrite the equation in terms of x, y, p and q and successfully substituted in the correct values of x and y to obtain q. Many others used the simpler method of setting $r = 3$ and $\theta = \frac{\pi}{3}$. It was disappointing that a small number of candidates had no idea how to approach this question.

(ii) Those who got the first part correct got the second part too, normally using the same methods.

(b) This was generally well done by most candidates. Most were able to replace the double angle; re-write cos θ or sin θ in terms of x, y and r and then r in terms of x and y.

A small number left the answer unsimplified in very inelegant states.

Q8 **(i)** This was intended to be a very simple one-mark answer. Some of the more able students were looking for complicated ways to show the given equation and ended up writing almost a paragraph in their attempt to do so.

(ii) The vast majority of candidates were able to find a correct integrating factor, with just a few missing the negative sign. In other cases, the integrating factor just appeared. Many did not show the stage of multiplying every term by the integrating factor but went straight to the integration. Others omitted multiplying the right-hand side of the equation by the integrating factor. All knew to put in the boundary conditions to try to find c. Nevertheless, the majority of candidates were able to successfully complete the question.

Q9 **(a)** **(i)** This question was reasonably well answered with the majority of students changing to exponential form and successfully rearranging to produce the required result. However, some candidates omitted lines in their calculations which were necessary for a rigorous proof. A few others made algebraic errors, but then proceeded to state the correct final answer despite their incorrect working.

(ii) Most candidates managed to change to the correct cosh format, but many omitted the factor of $\frac{1}{2}$. The subsequent integration was generally correct and almost all candidates obtained at least 3 of the possible 5 marks.

(b) A few candidates rewrote the equation in terms of e but most correctly differentiated the hyperbolic functions using the product rule. They then correctly set their derivative equal to zero, but unfortunately there was a very significant number of careless errors made in the expansion and simplification of the terms. This resulted in many candidates losing the last two marks because, although they had the correct conclusion, it was based on incorrect working.

Q10 **(i)** Some of the differentiation in this question was extremely poor. One of the key problems was that candidates did not simplify at each stage, making the next differentiation a little harder and therefore more prone to error. However, most candidates knew how to structure the question and marks were generally good for this part. Unfortunately, a few did not use differentiation to 'derive' the expansion, but instead simply quoted from the formula booklet and adapted various results. This resulted in a heavy penalty.

(ii) A good proportion realised how to do this part and were able to pick up at least one mark, even if Part (i) was incorrect. Some candidates did not read 'Hence' and tried to redo the whole question using the same methods as Part (i). This resulted in both wasted time and lost marks.

(iii) It was pleasing to see how many candidates actually managed to link Parts (i) and (ii) in this part. However, the structure and flow of their answers were often muddled and not fluent. A few others tried, as in Part (i), to quote results from the formula booklet and piece these together.

Q11 (i) Most candidates correctly separated the integral and substituted $\sec^2 x$. The biggest problem was the subsequent integration of $\sec^2 x \tan^{n-2} x$. The penultimate line of working was not always convincing, and it was unclear whether candidates really did appreciate the significance of n being even. Many students incorrectly split the original function up as $\tan x \tan^{n-1} x$ and could make no further progress. Some were unable to start at all.

(ii) This question should have been straightforward but again, incoherent methods and careless mistakes were rife. Many just tried to find I_6 without initially setting up an integral and a surprising number were unable to find the value of I_0.

Q12 (a) (i) Most candidates made a good attempt to find vertex A. A few set $r = 16\sqrt{2}$ rather than 32. A small number were unable to make any serious attempt at finding the necessary root.

(ii) This was very poorly done. Many candidates did not appear to have any idea how to approach the problem. Others quoted incorrect formulae for the area of a triangle. A few even just tried to find the area of a circle.

(b) (i) Candidates knew how to go about this question but their methods were often very poorly laid out, with no reference to De Moivre's Theorem or how it linked to their proofs. Many had obviously seen questions like this and had gone into automatic pilot without considering the best way to structure their 'proofs'. A few got the correct expressions for sin 5θ and cos 5θ but then proceeded to try to rewrite them solely in terms of sin θ and cos θ, only to revert back to the original form when they needed to substitute in to find tan 5θ. In spite of their poorly structured approaches, many candidates did muddle through to gain a good proportion of the marks for this question.

(ii) Many candidates left this blank. Others tried to substitute for $\theta = \frac{\pi}{75}$.

Solutions were often numbers substituted into one of the equations given in the question without any thought as to how they could be used to 'prove' what they were trying to do. Of those who did manage to make their way to an answer, many did not finish with an appropriate closing statement.

Assessment Unit A22 Applied Mathematics

Unit Overview

Each section tested a broad range of topics on the specification and incorporated a suitable variation in difficulty and style of question, ranging from the application of standard techniques to more demanding problems set in context, and from scaffolded questions to those with less guidance. While each section included several questions (or part questions) that were accessible to all candidates, there was at least one discriminating question that stretched the more able candidates. Generally, candidates were well prepared for the paper with methods of solution demonstrating a good standard of development.

There was an indication that some candidates dedicated more time to Section A at the expense of their second section, with the responses for the final parts of the latter sometimes appearing to be a little rushed.

Section A Mechanics 1

Q1 A standard question on circular motion involving banked corners, which afforded candidates the opportunity to settle into the examination. A significant proportion gained full marks.

Conceptual errors were rare but included resolving parallel/perpendicular to the banked curve and assuming that friction acted up the slope.

Q2 (i) Considering that this question represents a standard problem involving the seconds pendulum, it was surprising that a significant proportion of the candidates set the period of oscillation to one second throughout. Some candidates incorrectly recalled the relationship between the period and length of a simple pendulum.

For problems of this type, it is important that candidates work to a suitable degree of accuracy in order to avoid rounding errors in their final values.

(ii) While the majority of candidates adopted the expected method of directly comparing the two pendulum lengths, some took an alternative approach of comparing the values of the acceleration due to gravity at the two locations.

Q3 This question was well answered, with a significant proportion of the cohort gaining high marks.

(i) On rare occasions answers given were out by a factor of two i.e., values for the amplitude and period were quoted as 2.5 m and 6.5 hours respectively.

(ii) A common mistake was to set $x = 0.4$ m as opposed to $x = 0.85$ m i.e., candidates failed to recognise that x is the displacement from the centre of oscillation.

Some candidates used the relationship $x = a \sin \omega t$ to find the time taken for the water level to move from the centre of oscillation (at $x = 0$ m) to the specified depth, but then failed to subtract this result from $\frac{T}{4}$ in order to calculate the required time after low tide. It was unfortunate that, where correct working was demonstrated, a small number of candidates lost the final mark by rounding their time down, (instead of up), to the nearest minute.

Q4 **(i)** This was an accessible introduction to the overall question, with most candidates gaining the three marks.

(ii) This part proved to be more challenging for the cohort and there was a notable deterioration in presentation in this question. This resulted in candidates losing marks due to careless algebraic manipulation, inaccurate working or incorrect resolving of forces. The simplest approach was to equate forces at the joints B, C and D, which did not involve a hinge. For candidates who equated forces at joint A, some lost marks by neglecting to include the reaction at the hinge in their calculations.

For these questions, it may be easier if candidates initially assume that the forces in all the rods are of the same type (e.g. all tensions), and then state the type of each force (tension or thrust) based on the sign of the calculated force. Candidates should be advised to draw a clearly labelled force diagram. They should also clearly identify the joint(s) at which they are equating forces, so as to avoid mistakes and ambiguity.

Q5 **(i)** In this part, as with all questions requiring a result to be shown or proven, it was essential that all steps were clearly laid out in order to gain full marks.

(ii) The solution to the 2nd order differential equation should be given using the correct variables from the problem i.e., with x and t as the dependent and independent variables respectively. While many candidates successfully found the general solution of the equation of motion, some lost marks by stopping at this point. Others failed to find the second constant which required differentiation to obtain an expression for the velocity in terms of t.

(iii) To access the final mark in this part, candidates were required to accompany their conclusion with a reason, referencing, for example, the discriminant or nature of the roots of the auxiliary equation.

Q6 This centre of mass question was the key discriminator, with reasonable attempts being made in Part (i), but Parts (ii) and (iii) providing greater challenge.

(i) It is advisable for candidates to clearly indicate the lines relative to which their distances are given. Although not essential, candidates seemed to benefit from including, as part of their method, a table summarising the masses and distances of the components comprising the system. A small proportion of the cohort misinterpreted the problem, incorrectly attempting to include the (light) uniform rods themselves in the calculation of the location of the centre of mass.

(ii) For those candidates who did attempt to equate the moments of the weight of the system and the horizontal force about the point A, some struggled to find the perpendicular distance of the horizontal force from point A.

(iii) The standard approach was to equate the moments of the weight of the original system and the weight of the additional glass bead about the point A. An alternative solution involved using the property that the centre of mass of the new system, (including the additional glass bead), must be along the line AH for the pendant to remain in equilibrium with CD horizontal.

Section B Mechanics 2

Q1 This was a straightforward question that posed no significant problems to the majority of candidates. The most common method of solution involved using separation of variables to solve the first order differential equation in v.

Q2 Part (i) and the first half of part (ii) of this question did not present any major problems, with most of the cohort obtaining full marks. The calculus involved in obtaining expressions for the acceleration and displacement was relatively straightforward.

- **(i)** Candidates should be made aware that, in the context of this problem, acceleration refers to the vector quantity as opposed to its magnitude.
- **(ii)** The second half of this part was poorly answered, with many candidates failing to identify that, in order to find the maximum distance, it was necessary to first of all apply Pythagoras' theorem to the displacement in order to obtain an expression for the distance in terms of time.

Q3 This problem was reasonably well answered by many of the candidates. However, a small number of issues arose, particularly in the latter parts of the question.

- **(ii)** It is worth noting that, as the sum of the moments is a vector quantity, the final result should specify the direction of rotation.
- **(iii)** Several approaches were employed by candidates. The most common was to identify the values of P and Q that would produce a zero resultant force in Part (i), and then to show that these values generated a non-zero value for the sum of the moments in Part (ii). An alternative solution involved setting the sum of the moments in Part (ii) equal to zero, and rearranging to obtain an expression for P in terms of Q. By substituting this into the expression for the resultant force, it could then be shown that it was not possible for a single value of Q to simultaneously produce zero values for both its **i** and **j** components.
- **(iv)** A proportion of candidates struggled to gain full marks in this part. While many picked up some of the marks by setting the single force **F** equal to the resultant force in Part (i) to arrive at expressions for P and Q in terms of α, others failed to appreciate that the moment produced by **F** should be equivalent to the sum of the moments for the system.

Q4 The first two parts of this question required a result to be shown, and most of the cohort coped fairly well with these proofs. When marks were lost, this was usually as a result of carelessness with the directions/signs of the velocities of the spheres. An annotated diagram may be helpful in problems of this type.

- **(iii)** This proved to be a discriminator. Some of the cohort failed to find the final velocities of spheres A and C and so could not progress any further. Others worked through their solutions in terms of e or attempted to find and combine the energy loss in the two separate collisions. While these latter two approaches are not incorrect, they required more complex calculations and, in some cases, marks were lost as a result of errors in the working.

Q5 A good final question, where the latter parts provided challenge.

(i) This was a proof of a standard result and did not present any major problems. It is expected that, for proofs of this type, all components are derived from first principles.

(ii) This proved more challenging. Many candidates failed to appreciate that the height of the large cone could be found using similarity, or equivalent, and therefore made little progress. Other errors included adding the masses of the large and small cones when setting up the moments equation or giving the coordinate of the centre of mass of the small cone relative to its base as opposed to the base of the frustum.

(iii) The crux of this part was identifying that, at the point of toppling, the weight of the frustum would act through the lowest point of contact between its base and the inclined plane.

Section C Statistics

Q1 This was a straightforward introductory question, with many candidates gaining full marks.

(i) When marks were lost in this part it was usually in the calculation of the variance, for example, by failing to square the coefficients, or subtracting, instead of adding, the third term.

(ii) When finding the probability from the Normal distribution using a suitable calculator function, candidates should be advised to show some form of method, for example, a sketch of the distribution indicating the required area, or giving the relevant standardised z value (if appropriate).

Q2 This question was answered well by many candidates.

(i) However, this part was not without its issues. A common mistake was to assume that the variance of the sampling distribution of the mean $\overline{X}$ was simply σ^2 as opposed to $\frac{\sigma^2}{n}$. Answers suffered from careless notation, with some candidates failing to properly distinguish between the population distribution X and the sampling distribution $\overline{X}$. When finding the relevant standardised z-value, it is recommended that candidates show some form of method, for example, a sketch of the standard Normal distribution identifying the corresponding region or stating the equivalent equation $\Phi(z) = 0.25$. Care should be taken to ensure that signs are consistent when substituting into the standardisation formula.

(ii) & (iii) These proved to be routine. For the latter nearly all candidates were able to provide the correct justification, (i.e., sufficiently large sample size $n \geq 30$) for adopting the Central Limit Theorem.

Q3 Problems of this type requiring candidates to carry out a statistical hypothesis test are generally well answered, as candidates are able to follow a structured format.

(i) The vast majority of the cohort used a paired-sample test, while a small number carried out a two-sample test. Nearly all candidates correctly identified that the test statistic follows a t-distribution, as opposed to a Normal distribution. As with all hypothesis tests, it is important that all aspects of the test are clearly shown i.e., hypotheses, type of test, degrees of freedom, critical value, etc. For a paired-sample t-test the hypotheses should refer to the mean difference between the populations μ_d, not $\overline{d}$ etc.

Some candidates found $\overline{d}$ and S_d^2 using the statistical functions on their calculator. This is acceptable; however, candidates should be advised to give the d^2 values in the table, quote the summary statistics $\sum d$ and $\sum d^2$ from the calculator, and show the calculation of $\overline{d}$ and S_d^2 from these summary statistics. This enables them to gain some of the method marks in the event that they make a working error. The conclusion should be given in the context of the problem and should refer to the significance level of the test. It should not state that the test proves/disproves the claim, only that there is/is not sufficient evidence.

(ii) & (iii) Candidates continue to find the interpretive aspect of statistics challenging. They should be encouraged to think and discuss these areas in addition to the actual completion of the statistical techniques.

Q4 **(i)** This part caused most of the problems. Common mistakes included assuming that the sampling distribution followed a t-distribution, adopting σ^2 as the variance of $\overline{X}$ and calculating the critical value from $\Phi(z) = 0.99$, rather than $\Phi(z) = 0.995$.

(ii) & (iii) These were well answered.

(iv) The majority of candidates correctly recognised that the sampling distribution $\overline{X}$ followed a Normal distribution with variance $\frac{\sigma^2}{n}$, using the unbiased estimator from Part (iii) as the value for σ.

(v) Any definition of the confidence interval must refer to it enclosing the population mean.

Q5 **(i)** This question proved to be a discriminator, with some candidates failing to appreciate that the mean from the frequency table of observed results could be used as an estimate for the parameter λ of the Poisson distribution. An alternative method, identified by some candidates, for finding the value for λ was to use the table of expected frequencies to set up and solve the equation $e^{-\lambda} \times 52 = 4.3$.

(ii) Candidates appear to be well versed in carrying out the χ^2 test, finding it a reasonably routine task. However, marks were lost by some for failing to combine the $x = 0$ and $x = 1$ classes. It is expected that the hypotheses are sufficiently detailed, including, for example, the parameter of the Poisson distribution. The conclusion should provide similar detail in the context of the problem and also refer to the significance level of the test.

(iii) Very few candidates picked up the final mark which required them to demonstrate an understanding that such a small value for the test statistic should cast doubt on reliability.

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