GCE

Rewarding Learning

Chief Examiner's Report

Further Mathematics

Summer Series 2019

Foreword

This booklet outlines the performance of candidates in all aspects of this specification for the Summer 2019 series.

CCEA hopes that the Chief Examiner's and/or Principal Moderator's report(s) will be viewed as a helpful and constructive medium to further support teachers and the learning process.

This booklet forms part of the suite of support materials for the specification. Further materials are available from the specification's section on our website at www.ccea.org.uk.

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GCE FURTHER MATHEMATICS

Chief Examiner's Report

Assessment Unit 1 Pure Mathematics

Unit Overview

Candidates found this paper accessible and were able to successfully complete it within the allotted time. The overall standard of work was very pleasing. Candidates appeared to have a good understanding across all topics although some of the new aspects of complex numbers were less well answered.

Q1 **(a)** **(i) & (ii)** These were very well answered.

(iii) A number of candidates tried an alternative method where they tried to multiply through by the matrix **AB** but in some cases they carried out their algebra incorrectly and were unable to produce the required result.

(b) This was well answered by the majority of candidates. The most common errors were either to map the point (x, mx) onto itself rather than onto another point on the same line or to not realise that the image line also had the equation $y = mx$.

Q2 This question did not pose any difficulty to the vast majority of candidates, although a few tried to use the volume formula for a tetrahedron rather than a parallelepiped.

Q3 **(a)** **(i)** This was well answered, with the only issues being minor calculation errors.

(ii) A significant number of candidates used an alternative solution, whereby they calculated $\mathbf{A}^2$ and then found **AB** which they pre-multiplied by $\mathbf{A}^{-1}$ to produce their answer. This was a perfectly acceptable method although it did lend itself to some calculation errors.

(b) Although this is a new topic on the specification it was extremely well answered.

Q4 **(a)** **(i) & (ii)** were routine examples of complex number arithmetic and were very well answered.

(iii) In this part, however, some candidates did not appear to know how to approach the problem and made little progress. Of those who did try to solve, the most common error was to omit a negative square root and hence lose one of the solutions.

(b) This provided greater challenge to the candidates and a wide variety of methods were seen. In most cases, it was recognised that 1 + i was another root and this allowed candidates to progress with their working. However, simple algebraic errors occasionally led to incorrect solutions.

Q5 **(a)** **(i) & (ii)** Were very well answered.

(iii) & (iv) It became evident in these parts of the question that many candidates were not familiar with the rules for finding the modulus and argument of the product or quotient of two complex numbers. A significant number of candidates found z_2 in a + ib format and then proceeded through lengthy calculations towards the required product and quotient. Since many found it difficult to deal with these numerical values there were a lot of calculation errors and only a few managed to achieve the correct final answers.

(b) The locus problem was well answered. There were only a small number of errors such as an incorrect centre being used or the angle between the half-line and the x-axis not being marked on the diagram.

Q5 **(a)** **(i) & (ii)** were very well answered, with only a few minor calculation errors evident.

(iii) The candidates clearly understood what was required and again the only issues were errors in their algebraic manipulation.

Q7 **(a)** **(i)** The vast majority of candidates achieved full marks, with only a small number forgetting to check that the values of λ and μ satisfied all three equations in x, y and z.

(ii) A number of candidates left their answer as a vector and did not state the coordinates of the point of intersection.

(b) **(i)** This was well answered, although a few candidates did not write their final answer in vector form, as required by the question.

(ii) Most candidates did use the scalar product method shown in the markscheme, but a few used the vector product which was perfectly acceptable. The most common error was to leave the answer as an obtuse angle and not the acute angle as required. A few candidates did try to find the acute angle but incorrectly related their answer to 90° rather than 180°

Assessment Unit 2 Applied Mathematics

Unit Overview

This represented the first Unit 2 (Applied) paper of the new AS Further Mathematics specification and saw the introduction of optional sections. Each section tested a broad range of topics and incorporated a suitable variation in difficulty and style of question, ranging from the application of standard techniques to the more demanding solution of problems set in context, and from highly scaffolded questions to those with very little guidance. While each section included several questions (or part questions) that were accessible to all candidates, there was at least one discriminating question that stretched even the most able candidates. Generally, candidates were well prepared for the paper with methods of solution demonstrating a high standard of development and presentation.

Section A: Mechanics 1

Q1 **(a)** Considering that this question represents a routine application of a standard technique, it proved problematic for a surprising number of candidates. Common mistakes included attempting to calculate the work done by the force, given in vector form, either by taking the vector product of **F** and $\overrightarrow{AB}$ or by finding the product of the moduli of **F** and $\overrightarrow{AB}$.

While candidates fared better in Section (b), it was not without its difficulties.

(b) **(i)** Some candidates did not recognise that finding the work done by a variable force required the use of an integral and incorrectly applied the formula for work done by a constant force.

(ii) The main error was to utilise the kinematics equations for a constant acceleration.

Q2 **(i)** This was poorly answered since candidates failed to recognise that they needed to refer to the radius-tangent circle theorem.

(ii) It is worth noting that, for marks to be awarded, it is essential that arrows and labels (e.g. W/mg, R) are attached to the force lines on the diagram, indicating the direction and type of the force respectively.

(iii) This did not present any major problems, with the bulk of the cohort obtaining full marks. However, for questions that require a result to be shown or proven, candidates should be reminded of the importance of clearly showing every step of their method.

(iv) The common error was to set the radius of the circular path equal to 40 cm, instead of calculating it by applying Pythagoras' Theorem.

Q3 From the four questions in Section A, this question had the least amount of 'scaffolding' or direction, and as such proved to be a challenge for many candidates. Common misconceptions included multiplying the cross-sectional area by the vertical height, as opposed to the issuing velocity, when calculating the volume flow rate, and mistakenly taking the 0.28 m to be the cross-sectional area itself. However, candidates making these or other mistakes early in the development of their answers were still able to pick up method marks later in the question. Considering that this problem represents a fairly common example of a topic that is explicitly outlined in the specification, it was surprising how much difficulty it caused for some candidates.

Q4 This question discriminated between candidates, with Parts **(i)** and **(ii)** accessible to most candidates and Part **(iii)** stretching the more able.

(i) A number of candidates failed to double the elastic potential energy stored in one of the strings in order to find the total energy of the system.

(ii) For conservation of energy problems, like that found in Part (ii), candidates would probably be more successful if they explicitly stated the start and finish points, as well as the object to which the principle was being applied. Calculating the vertical height increase from point C to the point where the strings go slack proved to be a source of difficulty for some candidates.

(iii) A significant proportion of candidates struggled to gain full marks in this part, with many incorrectly applying the conservation of energy principle to the combined object (pan + ball) from point C to maximum height, hence failing to appreciate that the strings would not remain slack for the entire journey. The two common successful approaches considered the ball only from the point where the strings go slack to maximum height, utilising either the kinematics equations for free motion under gravity or conservation of mechanical energy.

Section B: Mechanics 2

Q1 This was a successful introductory question, which enabled candidates to settle into the section. The problem could have been solved either using a triangle of forces, or by resolving the forces in any two perpendicular directions (vertical/horizontal, parallel/perpendicular to PA, parallel/perpendicular to PB), with the latter proving the most common and successful approach. It was pleasing that very few issues arose when using trigonometry to resolve the forces and when applying Hooke's Law to the elastic string.

Q2 In general for questions involving Dimensions it is important that candidates can confidently derive the dimensions of common quantities such as force, pressure etc. Neither part of the question posed any significant problems for the majority of candidates.

(i) Candidates should work with the correct notation for Dimensions, specifically [M]/[L]/[T] as opposed to kg/m/s. If the answer was left in the latter form, then full marks were not awarded. Worse still, a small number of candidates interchanged between the two forms of notation during their working and confused mass [M] with metres m.

(ii) This represented a routine application of a familiar technique, namely equating indices of common bases. It was encouraging that nearly all candidates realised that the dimensions of pressure difference are the same as that of pressure.

Q3 This question was answered very well, with a significant proportion of the candidates gaining full marks. Lost marks usually stemmed from careless manipulation of the algebra involved. For the small number of candidates who could not make any progress, the most common reason was failure to recognise the need to apply Newton's Second Law to the motion of the planet.

Q4 This question proved very challenging for the cohort, with only the most able scoring full marks.

(i) Many only managed to gain the first three marks for finding the relative velocity. A lot of candidates do not seem to fully understand the concept of applying relative velocity techniques to closest approach problems, as evidenced by their difficulties in constructing a suitable diagram to illustrate the scenario and in the lack of further progress in the question.

There were two possible methods of solution. One involved constructing a displacement diagram and calculating the angle made between the line connecting the initial positions of A and B and the relative velocity vector (either from the relative velocity components directly or from a velocity triangle). The alternative method involved setting the first derivative of the square of the relative displacement (at any time) equal to zero. The latter approach meant that candidates completed most of the work for Part **(ii)** in their working for Part **(i)**.

(ii) It was disappointing that many candidates did not obtain any marks in this part of the question. When calculating the time of closest approach, candidates should be reminded that the distance and relative speed must relate to the same reference frame e.g. the frame of reference where A is stationary. It was also disappointing that, of the few candidates who did manage to get the correct value for t, some lost the final mark by not converting this to an actual time (as stated in the question).

Q5 This question was relatively straightforward and was very well answered by many candidates.

For the few candidates who encountered difficulties the most common mistakes in Parts **(i)** and **(ii)** are stated below:

(i) The principle of conservation of energy was not applied between the correct two points i.e. P and D.

(ii) Newton's Second Law was applied in the incorrect direction, which was surprising given that the relevant direction (OP) should have been obvious from the structure of the given formula for N. As stated previously, where a result is to be shown or proven, it is important that candidates make every step of their method obvious.

(iii) Even if candidates failed to show the result in Part **(ii)**, they could still gain full marks in **(iii)**. Almost all candidates recognised that the point where the skier lost contact with the surface corresponded to N = 0.

Section C: Statistics

Q1 This was a standard question on the topic of linear regression that candidates proved to be well versed in completing. The vast majority scored either 10 or 11 from the possible 11 marks.

(i) Although some candidates correctly identified the explanatory and response variables, they omitted a statement outlining their dependence and hence did not gain all marks.

(ii) It was evident that candidates had been made aware of the necessity to include summary values as part of their working in order to gain full marks. Candidates should be reminded to give their final answer in terms of v and t, as opposed to y and x. In a small number of scripts marks were lost as a result of the incorrect input of the raw data into the calculator.

(iii) The only error of note involved candidates inadvertently setting $v = 65$ and solving for t.

(iv) Candidates should be reminded that explanations need to be given in the context of the question and in this case simply stating 'extrapolation' was not sufficient without some reference to $t = 120$ being outside the range of values of time in the table.

Q2 **(i) & (ii)** posed no significant problems to the majority of candidates. However, carelessness with notation when performing an integration still persists. Candidates should also be reminded that it is not sufficient to simply write down the numerical value of a definite integral from their calculator. They must show the actual integration of the integrand.

(iii) This proved problematic for a substantial proportion of the cohort, with some candidates not even making an attempt, and others appearing to try to find the median value. For those who did correctly find the modal value, a few lost the final mark by not using the second derivative to establish the nature of the turning point.

Q3 **(i)** This was an accessible introduction to the overall question, with most candidates gaining both marks.

A significant fraction of the cohort encountered difficulties with Parts **(ii)** and **(iii)**.

(ii) The main mistake in Part **(ii)** was to change the value of λ to 3.5, probably as a result of the phrase 'two successive weeks', instead of keeping λ equal to 1.75 and treating these two weeks as independent events.

(iii) Candidates then made the opposite error of failing to modify the value of λ to account for the two-week period. Candidates should be made aware of the importance of showing all the steps of their working for problems of a similar type. They should not simply write down a numerical value from their calculator i.e. it is expected that the line '$P(X \geq 4) = 1 - P(X \leq 3)$', in addition to the value of $P(X \leq 3)$, would be quoted.

Q4 **(i)** It should have been clear from the context that random stratified would have been the most suitable sampling technique, and indeed this was the most common response. Candidates are expected to include specifics relating to the scenario described as part of any answers given, as opposed to simply making generic statements. For example, a suggestion for the strata types (e.g. gender, age, job role etc.) was required to gain full marks.

(ii) For those who provided an incorrect answer to Part (i), follow-through marks in this part were still available for providing an acceptable advantage and disadvantage. Again, context specific statements are required in such questions e.g. if reference is made to cost or time, it is necessary to explain why the technique in question is relatively costly or time-consuming.

Q5 Since this is a new topic to the current specification, it was pleasing to note the number of candidates who scored highly in this question. The vast majority of the cohort identified that the question involved combinations, with very few scripts making reference to permutations.

(i) The majority of candidates obtained full marks in this this part of the question.

(ii) Quite a few difficulties were evident when an additional constraint was imposed.

(iii) This part was also well answered.

(iv) The majority of candidates obtained full marks in this this part of the question, although a number added the combinations for men and women, rather than multiply them.

(v) As in (ii) the extra constraint posed a challenge for a number of candidates and again some found the sum of the combinations instead of their product.

Section D: Discrete and Decision Mathematics

Q1 **(a)** **(i)** This was a standard definition which was well answered.

(ii) This was generally well answered although there was a common error of omitting the final letter of the cycle.

(b) **(i)** This was also a standard definition which was well answered.

(ii) Although it was evident that candidates knew what they were trying to find, some errors appeared in their final answer.

Q2 **(i)** It would appear that the candidates were unsure of this proof since it was not well answered.

(ii) However, in this part of the question the solutions were extremely well completed.

Q3 This was a routine truth table question which the candidates were very successful in completing.

Q4 **(a)** Candidates knew how to find the required number of edges, with the only error being in counting the edges on the given diagram.

(b) **(i) & (ii)** Candidates clearly understood how to carry out the required procedure. However, an error early in their algorithm could be costly in terms of their marks. If **(i)** was correct, then the answer to **(ii)** was always correct.

Q5 **(i)** If the candidates followed the instructions given in the question, then they were generally able to prove the required result. However, some tried alternative processes and were unable to proceed.

(ii) & (iii) These were routine questions and were very well answered.

(iv) There was a mixed response to this part of the question. It was clear that candidates understood what they were trying to do, but they could not always explain this in a clear and convincing format.

(v) Candidates were able to state a correct subgroup of Q but not all could then find the subgroup of R which was isomorphic to that subgroup of Q.

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