SHAPE AND SPACE

2

A TEACHING AND LEARNING RESOURCE FOR USING MATHEMATICS

CONTENTS

Introduction

This resource enhances and consolidates pupil understanding of Shape and Space concepts. It uses practical activities and Logo software to promote using mathematical language and concepts related to position, movement and direction.

The suggested activities are designed to help pupils develop Shape and Space skills and enhance those they may have developed by completing the activities in the Part 1 resource .

It is suggested that the teaching methodology and approach remain consistent with the Part 1 resource and that pupils are encouraged to investigate, trial and model their ideas by using their own physical movement (acting as the turtle or 'playing turtle') and a wide range of tangible construction resources, for example plastic building cubes and shape templates.

Logo should not be understood as a drawing program or a drawing environment; rather, it employs the metaphor of drawing a shape to encourage the pupils to become aware of, use and manipulate the properties of the shapes they are drawing.

To achieve the maximum benefit, the methodology should encourage high levels of purposeful collaboration to create an investigative environment, where errors are seen only as a prompt to further investigation and solve problems.

This resource supports the development of the language, perception and understanding of the Shape and Space concepts of Using Mathematics and the Interactive Design feature of Using ICT within the Northern Ireland Curriculum.

The aims of this resource are that pupils will:

- become aware of key Shape and Space concepts;

- develop practical strategies and activities to articulate and explore these concepts;

- be able to use appropriate mathematical language to define and articulate both the concepts and their perceptions of the concepts; and

- appreciate the diversity of these concepts and how important they are to our understanding of our environment and how we interact with it.

This resource also promotes using Logo beyond an assessment instrument and supports Using ICT Interactive Design integration more directly within the curriculum as a mathematical investigation platform.

The target area of learning is Mathematics and Numeracy, and the key Cross-Curricular Skill is Using Mathematics. However, Using ICT and Communication skills are also addressed throughout the activities.

This resource has a number of elements, including:

- practical activities designed to model the skills, thinking and concept development that pupils require to access the Logo environment in a meaningful way;

- modelling activities that introduce specific commands and skills that pupils need;

- Logo activities that integrate specifically with the requirements of the Shape and Space element within Using Mathematics (these activities focus specifically on using and manipulating basic shapes, their properties and the relationships that exist between them); and

- challenges that offer the opportunity for pupils to apply and extend what they have learned.

What is Logo?

Logo is a programming language developed at the Massachusetts Institute of Technology. The word Logo is not an acronym but comes from the Greek word Λόγος (Logos), one of its meanings is 'to reason'.

The principle underlying its use is that it can provide the pupil with a tool to think with, providing an interface between their intention and reality. As the pupil attempts to manipulate the screen turtle, they receive immediate feedback on the consequence of their thinking in the turtle's movement.

An important element in the process of using Logo is that pupils physically take the role of the turtle and 'walk' the path they intend to draw. In this way, they also have the opportunity to internalise the consequences of their actions.

Suggested Teaching Strategies

Practical Activities

A vital element in using and developing Logo within the curriculum is that pupils have sustained opportunities to work away from the computer, with transitional thinking objects that they are familiar with. The primary object is their own bodies as they walk around a floor shape, turning and moving. It is very important that there is sufficient opportunity for them to move and 'act as the turtle', to mimic what they expect the outcomes to be from any command that they give to the screen turtle. Other objects may include plastic building cubes and shape templates.

Modelled Activities

This strategy may include whole class teaching sessions, with the teacher modelling different ideas and discussing the outcomes as a class group.

Paired and Small Group Activities

Discussion is an important element in the process and pupils should have the opportunity to consider and share their ideas and suggestions with each other. You may find **set 2** of our **Thinking Cards** a useful resource here.

Resource Activities

The activities in this resource are presented in a graded progression and make suggestions for supporting pupils' assimilation with practical activities. They are presented in two sections:

1. Using Logo activities; and

2. Extending Logo activities.

Each section has supporting **Activity Cards**, which provide more detailed information for using the activities in the classroom. You may find it useful to give these Activity Cards to the pupils, where appropriate, or use them to structure your lessons.

Key Concepts

- Direction and movement are discrete and independent concepts. Pupils need to understand that it is possible to change position without changing direction and it is also possible to change direction without changing position.

- Left and right are not fixed but dependent on the point of perception for the viewer.

- Complete turns can also be completed with several constituent turns, for example 2 × 180 or 3 × 120.

- The sides and angles are equal in regular shapes.

- A complete revolution requires four right angle turns in the same direction.

- The perimeter of every regular or irregular shape requires a full turn to complete it.

- In every regular polygon, the value of the external angle is always 360° divided by the number of sides.

- As the number of sides in the polygon is increased, the shape will begin to resemble a circle.

Presumed Knowledge

Part 1 and Part 2 have been designed to provide appropriate progression and development of skills. Where possible, pupils should have experienced many of the activities in the Part 1 resource before progressing to Part 2. They also presume an understanding of core Shape and Space concepts detailed in Levels 1 and 2 of the Expansion of the Levels of Progression. In addition to familiarity with the Logo interface, and basic commands and procedures, pupils should have:

- an understanding of the basic properties of squares and rectangles;

- an appreciation that a full rotation measures 360°;

- an understanding that a right angle is 90°;

- an understanding of the syntax used to describe the direction and movement primitives in Logo, such as **forward 100** and/or **right 90**; and

- an understanding of using the **repeat** command to abbreviate procedure lists, for example **repeat 4[forward 100 right 90]** to draw a square.

Class Management

Effective class management will promote small group engagement and prompt discussion, investigation and modelling of ideas and solutions, using a range of resources and strategies, such as:

- playing turtle: physically walking around the perimeter of shapes to assist with directional orientation and to mimic the expected outcomes from any sequence of commands given to the screen turtle or Bee-Bot;

- using Lego, plastic building cubes or any other similar construction materials in addition to a wide selection of shape templates; and

- using a range of presentation materials to sketch what the expected and/or intended outcomes might be, for example grid paper.

Differentiation

It is to be expected that the understanding base within any group of pupils will not be uniform. Appreciation of the concepts within Shape and Space is a process that varies from pupil to pupil. Using differentiation is an important tool to support the pupil in developing their specific understanding and enjoying the process of concept assimilation.

While every effort has been made to make activities as accessible as possible, you should choose activities based on the learning needs of your pupils and the context you are working in. Care should be taken to value every contribution and present any differentiation as integral to the activity and not simply 'easier'. Logo is a learning environment where all contributions are valued.

Strategies

Peer teaching can be an effective strategy and involves pupils who have recently grasped the concept teaching and/or sharing the skill with their peers. Group work is an important strategy (typically small groups of 3–4 pupils), providing an effective model for sharing skills and making goals attainable for everyone.

Resource Overview and Curriculum Links

Key Cross-Curricular Skills	Using Mathematics and Using ICT
Key Stage 2 Processes in Mathematics	**Making and Monitoring Decisions** Pupils should be enabled to: • take increasing responsibility for selecting and using the materials and the mathematics required for their work; • identify and obtain the information required for a task, suggesting appropriate sources to find the information; • plan and organise their work, learning to work systematically; and • develop a range of strategies for problem-solving, looking for ways to overcome difficulties. **Communicating Mathematically** Pupils should be enabled to: • understand mathematical language and use it to discuss their work and explain their thinking; • compare their ideas and methods of working with others; • interpret situations mathematically using appropriate symbols or diagrams; and • present information and results clearly. **Mathematical Reasoning** Pupils should be enabled to: • recognise general patterns and relationships and make predictions about them; • ask and respond to open-ended questions and explain their thinking; • understand and make general statements; and • check results and consider whether they are reasonable.
Shape and Space	**Exploration of Shape** Pupils should be enabled to: • construct a range of regular and irregular 2-D shapes, classify these through examination of angles and sides, recognise line and rotational symmetry, reflect shapes in a line, explore tessellations, name and describe common 2-D shapes, begin to understand congruence in 2-D shapes; and • construct 3-D shapes, investigate the number of faces, edges and vertices on these shapes, name and describe common 3-D shapes, explore the relationship between 2-D and 3-D shapes.

Key Cross-Curricular Skills	Using Mathematics and Using ICT
Shape and Space	**Position, Movement and Direction** Pupils should be enabled to: • understand the notion of angle in the context of turning, recognise right angles, understand clockwise and anticlockwise, know the eight points of the compass, use Logo to understand movement and turning, be introduced to a programming language and use it to create pictures and patterns and to generate shapes; • develop language associated with line and angle, recognise properties of acute, obtuse and reflex angles, investigate angles in triangles and quadrilaterals, measure and draw angles up to 360°; and • use co-ordinates to plot and draw shapes in the first quadrant.
Measures	Pupils should be enabled to: • calculate perimeter and the areas and volumes of simple shapes; • understand and use scale in the context of simple maps and drawings; and • recognise times on the analogue and digital clocks and understand the relationship between the 12 and 24-hour clocks, use timetables.
Handling Data	**Collecting, Representing and Interpreting Data** Pupils should be enabled to: • collect, classify, record and present data drawn from a range of meaningful situations, using graphs, tables, diagrams and ICT software; • explain their work orally and/or through writing and draw conclusions; and • interpret a wide range of tables, lists, graphs and diagrams, create and interpret frequency tables, including those for grouped data.
Using ICT Desirable Features	**Interactive Design** Pupils should be enabled to: • solve problems using a digital device or environment; • investigate and solve problems in a digital environment; • input more complex sequences of commands; • use appropriate ICT tools and features to improve work; • make modifications to improve their work; and • carry out ongoing improvements and evaluate their process and outcome.

Using Logo Activities

In these activities, the progression builds on the accumulated understanding pupils have developed from carrying out activities in the Part 1 resource. The activities can be trialled with both the screen turtle and any programmable robot.

Each activity is presented as a discrete investigation that challenges and extends pupils' knowledge and understanding of specific concepts from the Levels of Progression within the Shape and Space requirements of Using Mathematics from the Northern Ireland Curriculum.

Mathematical puzzles are used to represent the investigations. These are linked to local geographical anomalies and also some others from around the world. This offers the opportunity for extension work in the area of The World Around Us, for example the hexagonal columns of the Giant's Causeway.

You may also wish to adapt these activities for different topics and/or for different programmable devices and/or different PC, tablet or online software applications. Please note that if other software is used, the commands and/or procedures may be different. You may also wish to introduce activities by using an interactive whiteboard to model some aspects of the activities.

Activity 1
Building Shapes

These activities support investigation in constructing regular polygons (shapes with at least three sides). The key mathematical concept is the relationship between the number of sides, the value of the external angle and 360°.

Pupils should be working at: Level 4–5

Requirements for Using Mathematics:

- Describe the properties of regular and irregular 2-D shapes in terms of sides, angles, symmetry and tessellations.

- Know multiplication facts up to 10 × 10 and derive associated division facts.

- Interpret and apply simple rules expressed in words.

- Reflect 2-D shapes in a line.

- Investigate patterns and relationships, using their findings to make predictions.

Reviewing the floor activites from the Part 1 resource, pupils should have the opportunity to revise their investigation of the concept that the sum of the external angles in any polygon is always 360° or a full rotation.

Using a variety of floor size polygon templates and working in pairs or small groups, one pupil should walk around the perimeter of the shape, emphasising the change in direction at each vertex, as a partner mimics the changes while standing in a central position, to illustrate that the journey around the perimeter will execute a complete rotation.

Using the **repeat** command, pupils working in groups are tasked with constructing a series of regular polygons, for example:

repeat 4[forward 100 right 90]

repeat 8[forward 100 right 45]

During modelling, it is important that you draw their attention to the relationship between the **repeat** variable and the size of the turning angle. This, in effect, confirms that they are investigating the factors of 360:
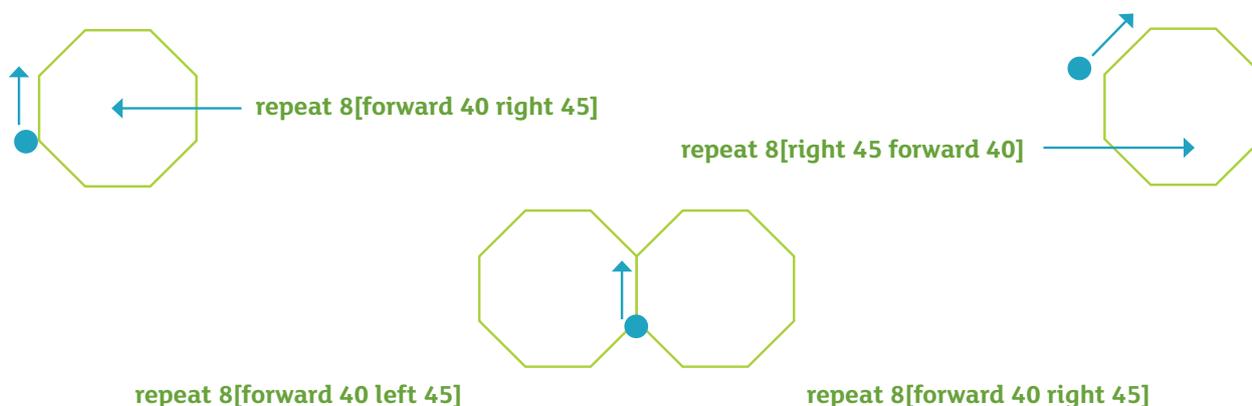
4 × 90 = 360
8 × 45 = 360

(Investigating the factors of 36 can be a useful initial activity: 2 × 18 will correlate either to 2 × 180, 20 × 18 or 18 × 20. This offers the opportunity to explore and develop their knowledge and understanding of the commutative and decimal place value of number.)

Extending the concept, pupils have the opportunity to investigate the visual consequence of increasing the number of sides. It becomes evident as the number of sides is increased, the shape begins to resemble a circle.

repeat 20[forward 20 right 18]

This activity also provides the prompt to investigate the effects of changing the sequence of commands in a procedure and alternating the **left** and **right** commands to create a mirror image of the shape.

repeat 8[forward 40 right 45]

repeat 8[right 45 forward 40]

repeat 8[forward 40 left 45]

repeat 8[forward 40 right 45]

To extend the Key Stage 2 experience, there is an opportunity for pupils to attempt to construct the shapes using graph paper and compasses.

Working in pairs, ask pupils to construct a series of regular polygons using the factors of 360 and then investigate which of them will tessellate.

Pupils also have the opportunity to investigate rotating shapes to complete a pattern and create a design.

repeat 4[forward 50 right 90]

right 45

repeat 4[forward 50 right 90]

right 45

repeat 4[forward 50 right 90]

right 45

repeat 4[forward 50 right 90]

right 45

The solution, as below, is to repeat the sequence, drawing eight shapes in total in a 'fanned' pattern and completing a full rotation (i.e. **right 45 × 8**). This pattern can be used with any of the polygons and the repetition can be varied, as long as the relationship between the repetition, angle of turn and 360° is maintained.
(For example, nine repetitions of 40°, five repetitions of 72° or six repetitions of 60°.)

Activity 2
Remembering Shapes

The key Logo concept is the facility to name and create a procedure to draw various shapes without the requirement to input a list of commands each time you wish to draw the shape during a session. The metaphor is that pupils have the opportunity to 'teach the computer a new word'.

Pupils should be working at: Level 4

Requirements for Using Mathematics:

· Understand and use multiples and factors.

· Understand and use the language of line, angle and location.

At this point, pupils will be familiar with using lists of commands to draw various shapes and then abbreviating these lists using the **repeat** command.

An additional function allows them to create a procedure and then assign a name to it so they can call and use it during their current session without having to retype the commands. The syntax for the procedure is:

To square

>repeat 4[forward 60 right 90]
>end.

You just defined square

Pupils type initially the Logo command **to**, followed by the name they wish to give to their shape.

The lines prompt now changes to a chevron as pupils input the commands, waiting until the **end** command before reactivating the turtle and informing them that the new procedure has been defined.

The question mark prompt now reappears.

Typing in the new command now will prompt the turtle to execute that shape.

Working in pairs, task pupils with constructing a series of procedures to draw several regular polygons and then investigate how they might be tessellated.

Extending this concept gives them the opportunity to investigate a simple iteration *(using a procedure within an existing procedure).*
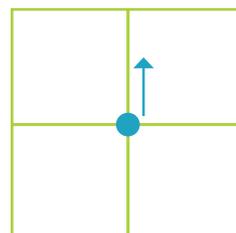
Constructing and executing the procedure **square** as above will produce this shape.

Using the procedure square within another procedure will produce this shape.

**repeat 4[square left 90]**

(Effectively the procedure draws four squares, turning 90° left (anticlockwise) after it draws each one so that the next one is drawn to the left of the previous one.)

This activity requires pupils to appreciate that each vertex has two angles, an external and an internal angle. The exterior angle is the angle that they turned to construct the shape and the internal angle is the angle inside the shape. These angles are supplementary (*their sum is 180°*). Assimilation of this activity will be enabled by significant practical investigation with tabletop shape templates. The solution to this activity involves pupils appreciating that only shapes with an inside angle that is a factor of 360 can be tessellated in this way, i.e. equilateral triangle, square or hexagon only.

Angle A: external

Angle B: internal
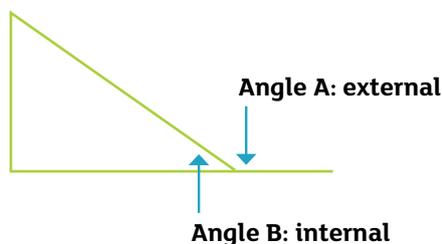
Although not specified in the activity card, you may wish to extend this activity with an investigation to discover the relationship between the external angle and the sum of the other two opposite angles in the triangle. This relationship states that the external angle equals the sum of the opposite two internal angles in every instance. See below for the proof.
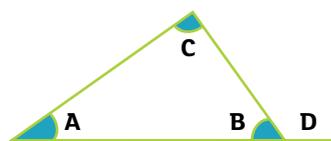
Angle D is the external angle and angles B and D are supplementary.

B + D = 180

Angles A + B + C are the internal angles of a triangle so they add up to 180 also.

A + B + C = 180

Therefore:

A + C = D

Working in pairs, also ask pupils to investigate the range of possible nets for a cube and a tetrahedron. Extension work on this activity offers pupils the opportunity to draw the shapes on card and construct them. (*See below for further details.*)

This activity can also extend into other areas of the curriculum, for example:

Literacy

• Character studies: since the cube has six faces, construct a cube for the character in the novel you are studying and assign to each face an illustration to depict a particular trait.

The World Around Us

• Use the faces to offer an interesting way to present information on a specific region.
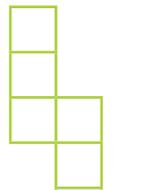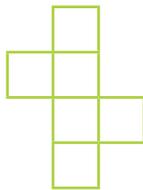
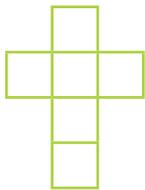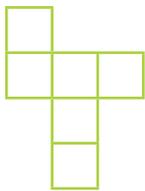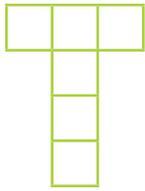**Thinking Skills and Personal Capabilities**

• Use the faces to present advice, preferences and/or comments on self-management or working with others.

Using Mathematics

• Investigate how to calculate visible and hidden faces if the cubes are built into a giant Rubik's Cube.

Solution (nets of cubes)

There are 11 possible nets for a cube, not including reversals.

Please note that pupils may need tabs to stick these together to form cubes.

Activity 3
Enlarging Shapes

In this activity, pupils have the opportunity to investigate and explore the areas and perimeters of squares and rectangles. The key Logo concept is the facility to use a variable that allows pupils to vary the size of the shapes they want to enlarge without having to key in all the commands each time.

Pupils should be working at: Level 4

Requirements for Using Mathematics:

· Estimate area and volume of shapes by counting squares/cubes.

· Understand and use multiples and factors.

· Interpret and apply simple rules expressed in words.

· Work out perimeters of simple shapes.

Pupils should now be familiar with the concept of creating a new command for Logo to execute and this feature will add a flexibility that enables them to explore the concept of area of shapes.

To square :size

>repeat 4[forward :size right 90]
>end.

**You just defined square**

Instead of using a specific value for the **forward** command, pupils can use a variable called **size**.

This allows them to determine the value of the side when they type the command, for example **square 100** will now draw a square with sides of 100.

The syntax of the command requires **:size** preceded by a space.

Typing the command **square 40** will now draw a square with sides of 40, similarly a command of **square 80** will draw a square with sides of 80.

This activity prompts pupils to draw two different sized squares whose dimensions have a numerical relationship. One of the squares appears to be drawn inside the other. The investigation asks them to predict and then calculate how many of the smaller squares are needed to fill the larger space, before prompting them to fill up the larger space with three smaller squares.
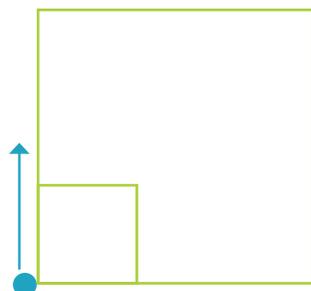
Creating and then executing **square 90** and **square 30** will draw the two squares in the example.

The activity prompts pupils to investigate and calculate how many of the smaller squares can fit into the larger one.

The Number element requires them to understand that if the sides are 90 and 30 respectively then three of the smaller squares can fit along each side giving:

$3 \times 3 = 9$ in total

Using only the **right** or **left 90** command and **forward** or **backward**, pupils are asked to create an efficient procedure for filling the larger space with smaller squares.

To extend the activity, ask pupils to undertake a similar investigation using rectangles and triangles. Side dimensions can be increased while maintaining the ratios to accommodate ability levels.

Consolidating the concept of calculating area, either by counting or by algorithm (*length × breadth*), each of these activities will be enhanced by prompting a practical investigation with tabletop shape templates that pupils can physically manipulate.

A useful activity involves investigating different arrays (*objects arranged in rows and columns*). The activity card will prompt them to explore how many arrays can be created from a fixed value and how the relationships affect the perimeters of the different shapes, for example:

Constructing an 8 × 3 rectangle requires 24 smaller squares to fill the shape. The prompt will be to explore how many other arrays can be constructed from 24 of the small shapes and how the different arrays affect the perimeter while the area remains constant. This provides the opportunity to explore multiplication and division facts, as well as multiples and factors to develop their knowledge and understanding of Number.

Another activity involves presenting pupils with the opportunity to explore how to express relationships in words and symbolic form.

From a blank shape template, ask them to construct a path of smaller squares around the perimeter and consider how many they will need to create the path. The relationship is: the number of tiles needed is always the value of the perimeter **plus** four to accommodate the corners:

The number of tiles is perimeter add 4, or expressed symbolically N = P + 4
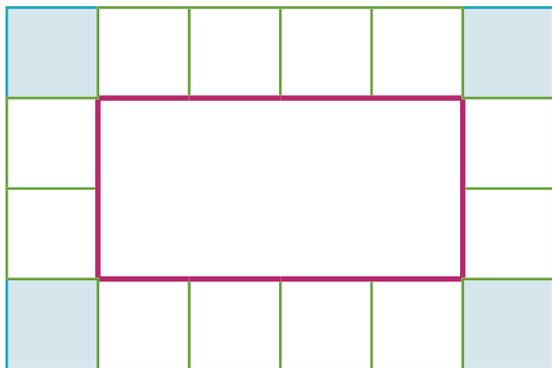
The dimensions of the rectangle are 4 × 2 giving a perimeter of 12.

To create a path around the rectangle, you will need 12 tiles PLUS four to take account of the corner tiles.

An inverse relationship also exists if the aim is to create a path on the inside of the perimeter. In this case, it will be the value of the perimeter LESS four.

Activity 4
Reflections, Rotations and Spirals

In these activities, pupils have the opportunity to manipulate shapes using only two transformations (reflection and rotation). They also have the opportunity to investigate the construction of spiral shapes. The key Using Mathematics concept is the presentation of shape and how it is altered by either a rotation or a reflection. The key Logo concept encourages investigation into the effect of using direction changes to manipulate shapes.

Pupils should be working at: Level 5

Requirements for Using Mathematics:

- Reflect 2-D shapes in a line.

- Explore ideas, make and test predictions and think creatively.

- Use a range of problem-solving strategies, suggesting and trying out different approaches when difficulties arise.

- Describe the properties of regular and irregular 2-D shapes in terms of sides, angles, symmetry and tessellations.

These activities ask pupils to consider how direction changes can affect the presentation of a shape. Pupils should be encouraged and have the opportunity to trial their solutions using tabletop shape templates before attempting to code them for the Logo turtle.

This activity prompts pupils to consider and replicate the different reflections on both the horizontal and vertical axis for a square.
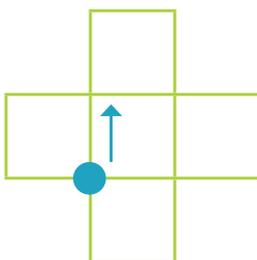
The code for the procedure for the centre square is:

To square

>repeat 4[forward 60 right 90]

>end

Using the procedure **square** the code to draw all the reflections is:

square

left 90

square

left 180

square

forward 60

left 90

square

forward 60

left 90

square

The activity card also asks pupils to consider how two of the reflections can be created at the first corner and then move the turtle to the diagonally opposite corner as the more efficient way to create the reflections.

As an extension, pupils have the opportunity to draw the reflection on the diagonal, adding the reflection line:
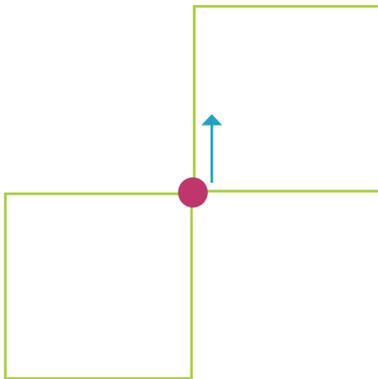
The coding for this activity uses the **square** procedure:

square

right 180

square

Tasked with creating the designs, initially pupils may produce long lists of coding to match each direction and distance discretely. There is advantage in facilitating this and then encouraging economy, asking them to identify patterns and repetitions to produce a more efficient list. This exercise can be completed as a whole class activity.

Investigating other regular shapes, they are tasked with drawing reflections for equilateral triangles. Support is offered as suggestions to alternate left and right commands or reorder the sequence of the commands:
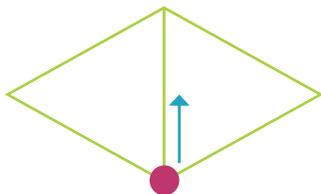
In this instance, it is more convenient to use the Repeat command, avoiding creating separate procedures or excessively using turns.

repeat 3[forward 60 right 120]

repeat 3[forward 60 left 120]
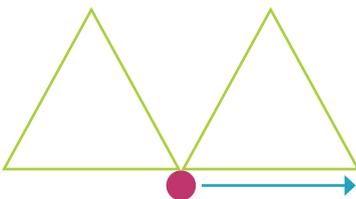
The procedure for this example is:

right 90

repeat 3[forward 60 left 120]

right 180

repeat 3[forward 60 right 120]

This activity is challenging and a partially completed procedure that pupils have to complete will support them. The complete procedure is:

right 90

forward 70

left 120

forward 140

right 120

forward 70

right 120

forward 140

this section will be omitted

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An opportunity to investigate reflecting an irregular shape is offered. The activity card presents the procedure for the shape and asks the pupils to complete two reflections, a horizontal and a vertical one.
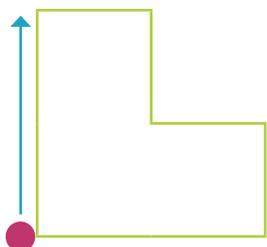
The commands for drawing the shape are:

forward 80

right 90

forward 40

right 90

forward 40

left 90

forward 40

right 90

forward 40

right 90

forward 80

right 90

Pupils are then prompted to investigate constructing a reflection on either the horizontal or vertical axis.

Rotation investigations are also offered. These are presented as guided activities and puzzles.

The guided puzzles revisit the concept from the first activity; the relationship between the number of times the shape is rotated, the size of the rotating angle and 360°.
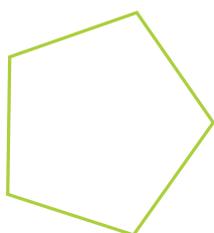
Pupils have the choice of either creating a procedure or using the **repeat** command:

repeat 5[forward 60 right 72]

This creates the initial shape and subsequently creating and using a procedure called **penta** allows them to create a pattern. (They will be prompted to use the **to** command to create the procedure **penta**.)

repeat 20[right 18 penta]

This creates the spiral pattern. The **penta** shape is drawn and then rotated and redrawn 20 times (the 20 and the 18 being factors of 360).
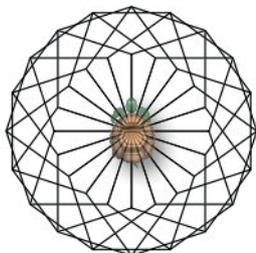
To add an extra dimension to this activity, pupils are offered the opportunity to vary the colour of each shape. The following code allows this.

setpencolour2

repeat 5[forward 60 right 72]

right 18

setpencolour4

repeat 5[forward 60 right 72]

right 18

The **setpencolour** command followed by a number without a space will change the colour of the pen.

Stepping through the procedure, pupils are able to change the colour of the pen after each pentagon has been drawn.

A simple investigation using dominoes is also presented. Ask pupils to choose four dominoes and task them with constructing a square using all four of the dominoes. If a clue or hint is considered appropriate, inform them that their arrangement will have two squares, one around the outer perimeter with a square space in the middle of the shape. To add challenge, the condition will be that the domino dots on each of the sides of their shape must all add to four:

Double two

Double one

Three zero

Zero four
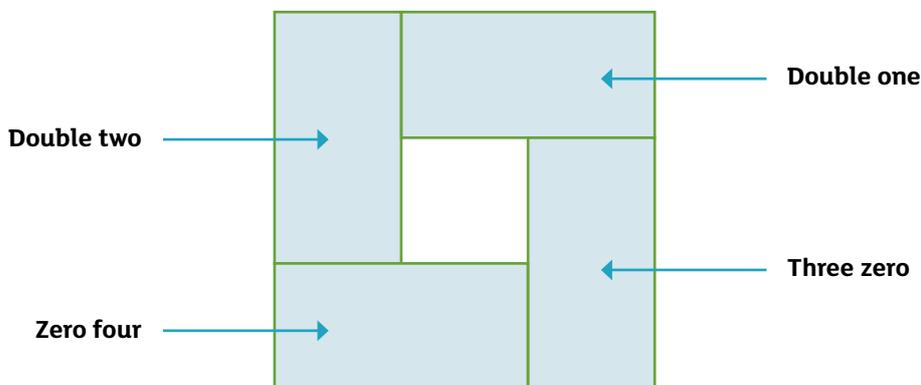
The spiral activity presents an investigation on the effect of changing the length of the forward movement (*either increasing or decreasing it*) while keeping the angle constant:
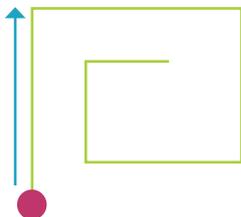
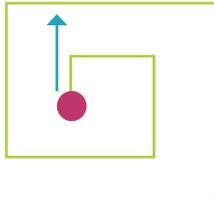A decreasing spiral can be created with the following code:

forward 80

right 90

forward 70

right 90

forward 60

right 90

forward 50... **etc.**

An increasing spiral can be created with the following code:

forward 40

right 90

forward 50

right 90

forward 60

right 90

forward 70

right 90... etc.

Pupils have the opportunity to explore the concept of spirals through considering reflections and constructing a simple game.

Extending Logo Activities

In this section, pupils have the opportunity to investigate and apply the concepts that they have used previously.

The activities are themed around ancient monument sites and extend the development of Logo from the previous section. The class management structure should again encourage pupils to use tangible objects, such as programmable robots and shape templates or tiles, to model the shapes and continue to 'play turtle' by walking around the shapes they are trying to construct.
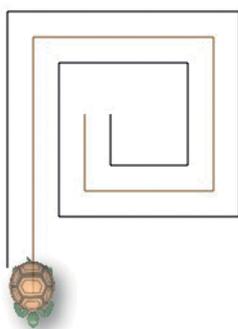
Activity 5
Newgrange

These activities involve the association of the mystery of the strange spiral drawings on the stones at Newgrange and the Winter Solstice day at the site. There are two activities and the context presents pupils with an ancient magical puzzle to solve.

Pupils should be working at: Level 4

Requirements for Using Mathematics:

- Understand and use the language of line, angle and location.

- Investigate patterns and relationships, using their findings to make predictions.

The first activity presents pupils with the code to draw a spiral and the narrative informs them that they have been drawn magically into the centre of the spiral. Their aim is to code an escape. Their path must not cross or touch any of the existing lines, otherwise they will have to stay in the netherworld of ancient Ireland. The solution involves using the **penup** command, then moving the turtle slightly and reversing the commands that they used to reach the centre. All the angles will be right angles.

Complementing the first activity, the idea is presented that as they spiral into the centre, they are really time travelling and can see images from the past, and they are asked to record three events. It is at your discretion to determine the scope of their research. Similarly, as they exit from the spiral it is suggested that they have the opportunity to glimpse three events from their future and are asked to record these.

The second activity draws their attention to the spiral and lozenge shapes carved in the stones and the fact that no information exists on their significance. The activity invites them to use two separate shapes to create a design of their own and leave a short explanation of its meaning. The condition applied is that they must only use vertical, horizontal or perpendicular lines.

When pupils have constructed a successful escape spiral, they can screen dump their image and import it into a document. They can then add text boxes to complete their project, describing the events their imagination has generated.

Activity 6
Finn McCool and the Giant's Causeway

Finn has issued a challenge to Benandonner, the Scottish giant, and together they decide to build a causeway using the hexagonal stones from the coast to allow them to visit each other. However, when they meet they begin to argue immediately. To settle their argument they decide to ask each other a puzzle. The preliminary activity includes work with tabletop templates to manipulate shapes and decide on strategies.

Pupils should be working at: Level 5

Requirements for Using Mathematics:

- Reflect 2-D shapes in a line.
- Explore the properties of common 2-D and 3-D shapes.

The first activity is for two pupils. The aim is to create and connect a causeway using hexagons that run from the diagonally opposite corners of their screen.

Pupils can create a large screen rectangle as a boundary marker. The condition is set that the hexagons have to join with each other on at least one side, not on the vertices. The 'causeway' of hexagons must start at one corner and be laid diagonally towards the other corner:

To position the turtle, the commands are:

penup

backwards 175

left 90

forward 400

A solution for the activity:

repeat 10[forward 50 right 60]

repeat 9[forward 50 left 60]

repeat 9[forward 50 right 60]

(alternating the last two commands to progress the design across the screen)

Complete accuracy is not necessary; a degree of overlap is acceptable.

Benandonner issues the first challenge to Finn. Using only four hexagons, he asks him to arrange them so that 10 of the sides are hidden and 14 are visible. Finn has to draw the solution. Tabletop templates can be used here.
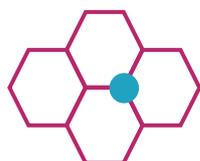
The blue dot represents the most efficient starting point, as the angle of the turtle can be rotated to draw the first three shapes before they need to reposition it.

You may wish to add a 30 degree right turn to offset the shape so that it will draw it with the side as the base.

Finn replies with his own puzzle and asks Benandonner to create a pattern showing seven hexagons using only six shapes. Again, pupils can use tabletop templates to investigate the shapes needed and their positions. Arranging the hexagons in a circular pattern creates an 'effect' of the seventh as a central empty space bounded by the sides of the other six:
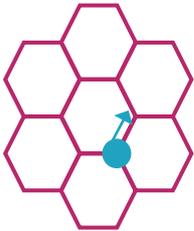
The **repeat** command can be used to draw the individual hexagons:

repeat 6[left 60 forward 80]

One strategy can involve pupils drawing and then repositioning the turtle before drawing the next hexagon. All dimensions are 80 units and angles are either 60° or 120°.

However, when drawing each hexagon with '7' sides, the last side overlaying the first will reposition the turtle to draw the next shape. Start on the blue dot and follow the command sequence below.

right 30

Repeat these two lines six times {
repeat 7[forward 80 right 60]
left 120

Later, after their meal, they are still grumbling at each other and Finn's wife Nuala has had enough. She scolds them and tells them they must work together as a team to solve two puzzles that she will give them. She hopes that the puzzles will keep them too busy to grumble.

The solution to both of Nuala's puzzles requires pupils to understand that the shape can also be turned (rotated). Choice of language is at your discretion.

A separate preparation activity is presented, which can be accessed at Level 4 to familiarise them with the effects of turning the shape.

The activity tasks pupils with finding all the separate ways the hexagon can be presented; either through rotation or reflection (the terms **turn** or **turned over** can be substituted as appropriate). There are *four separate presentations, excluding duplicates*.

She gives them six irregular hexagonal blocks and asks them to arrange two of them to make a rectangle and use the other four to make an identical but bigger hexagonal block:
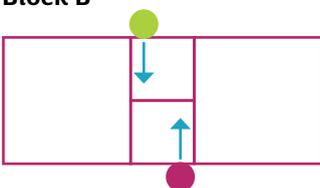
Block A

Block B

Block A represents the block Nuala gave them and Block B represents the first solution. Each of the smaller sides is half the value of the longer sides. The code for the original shape is:

forward 40	**forward 40**
left 90	**left 90**
forward 40	**forward 80**
right 90	**left 90**
forward 40	**forward 80**
left 90	**left 90**

Once pupils have drawn the first hexagon, they need to choose the starting point and rotation to place the turtle on the start of the second hexagon before drawing it. The coloured dots indicate both starting positions. The code to place the turtle to draw the second shape is:
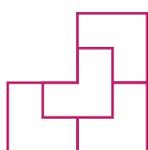
forward 80

left 90

forward 40

left 90

It will be more efficient to use the **to** command to create a procedure to draw this hexagon. This allows pupils to experiment with moving the shape without having to redraw it each time.

Solution

Using the code from the previous section to construct the hexagon, pupils can use trial and improvement strategies to move the turtle to draw the other hexagons.

A lot of time initially should be spent investigating how to assemble the irregular hexagons with tabletop templates that they have access to or have cut out themselves.

**Hexagon
constructed using left turns**

Directed teacher-led investigation can point out the effect of inverting the shapes (turning them over) and how an inverted shape could be constructed by switching each of the turn commands, **left ⟶ right** and vice versa.

Suggesting using and observing pairs of inverted shapes allows them to construct the base of the larger shape.

**Hexagon
constructed using right turns**
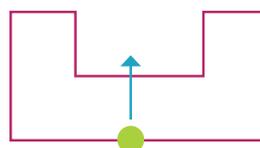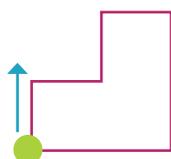
*Using the **to** command to create two separate procedures will avoid a lot of effort trying to position the turtle. You could name the separate procedures as:*

righthexagon

lefthexagon

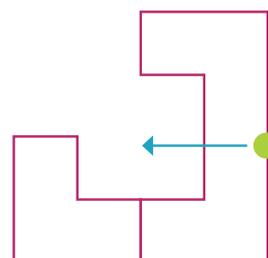To reposition the turtle and draw the third hexagon use this code:

right 90

forward 80

left 90

forward 80

left 90

righthexagon

(The GREEN DOT indicates the position and direction of the turtle after the procedure is completed.)

The following code completes the shape. Pupils have only drawn three hexagons but the positioning has the effect of drawing four.

right 90

forward 80

left 90

forward 80

left 90

forward 80

right 90
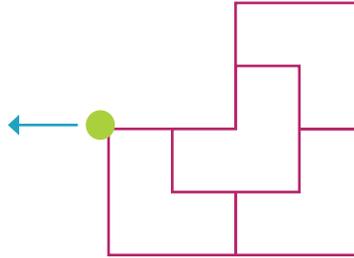
forward 80

TIP: *Remember to encourage pupils to use the 'undo' icon from the top navigation bar as this is useful when they are trialling and investigating different commands and procedures.*

Activity 7
Giza

In these activities, pupils investigate the construction of the Pyramids at Giza. One of the activities is a simple mathematical investigation of triangular numbers and the other is an investigation of drawing a pyramid shape.

Pupils should be working at: Level 5

Requirements for Using Mathematics:

- Draw nets of 3-D shapes.

- Devise and use rules for generating sequences in words and/or symbolic form.

- Present and interpret data using a range of graphs, tables, diagrams, spreadsheets and databases.

- Make and test predictions.

Using identical rectangular tiles, ask pupils to construct a small pyramid. Then introduce them to the concept of triangular numbers and ask them to predict the total number of tiles needed as each successive layer is added. Next, ask them to draw their pyramids, using the **repeat** command to draw a rectangle. You could also ask them to generate and identify triangular numbers under 60.

A simple explantion of the sequence is that the number added to the total increases by 1 with each successive layer added, 1 + 2 + 3 + 4 etc.

Although outside the remit of the Northern Ireland Curriculum at Key Stage 2, you may feel it is appropriate to introduce some of the more able pupils to the formula or method for identifying triangular numbers.

The first step is to rearrange the tiles into an incomplete quadrilateral and then complete the rectangle.
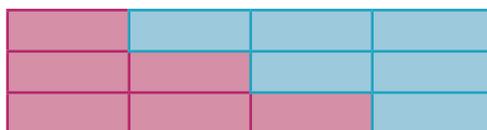
The third triangular number now forms a rectangle that is 3 × (3 + 1) giving 12 tiles in total. To find the number, the total is then halved, 12 ÷ 2 = 6, so 6 is the third triangular number. The formula is n × (n + 1) divided by 2 (when n is the triangular number you are looking for).
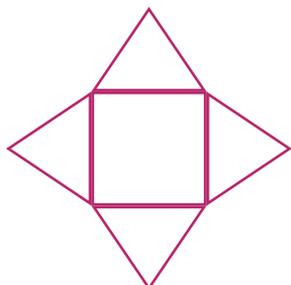
For example, to find the sixth triangular number apply the formula n × (n + 1) ÷ 2

6 × (6 + 1) ÷ 2

6 × 7 = 42 divided by 2 = 21, so 21 is the sixth triangular number.

An additional activity supports them in using Logo to construct a 3-D drawing of a pyramid and construct a net of the pyramid.

The net for the pyramid appears to need a square and four equilateral triangles. These can be coded as below.

To square:

repeat 4[forward 60 right 90]

To equitri:

repeat 3[forward 60 right 120]

The extended code for a pyramid net using the commands to the left is:

square

right 90

equitri

forward 60

left 90

equitri

forward 60

left 90

equitri

forward 60

left 90

equitri

This list can also be abbreviated using repeats:

square

right 90

equitri

repeat 3[forward 60 left 90 equitri]

> **You can remind pupils of and encourage them to investigate the effects of changing the order of the commands in the REPEAT statement.**

Directed observation may also indicate that when drawing the net initially, it is unnecessary for pupils to draw the square as a discrete shape because the base lines of the four triangles will accomplish that for them. This can be demonstrated for them by assembling a net on their own tabletops using four triangular templates only. *Exercise care to avoid the incorrect interpretation that the net for a pyramid requires only triangles.*

The code for this observation can also be either extended or economical.

Extended	Economical
equitri	**repeat 4[equitri forward 60 left 90]**
forward 60	
left 90	
equitri	
forward 60	
left 90	
equitri	
forward 60	
left 90	
equitri	
forward 60	
left 60	

The activity is extended to give pupils the opportunity to construct a shape that gives the appearance of a 3-D drawing.
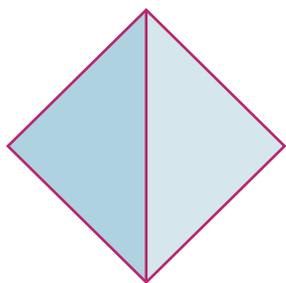
The aim is to construct two triangles, one with left turns and the other with right turns, using the **repeat** command so that the bases of each are adjacent.

Pupils print the triangles and shade each one to create a 3-D effect.
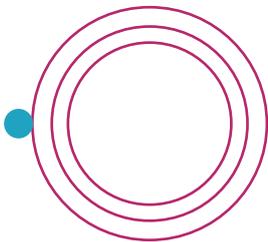
Activity 8
Grianan of Aileach

The ancient Neolithic settlement, Grianan of Aileach, still holds its magic secrets. Dedicated to the sun god, it sits on top of a hill, silent and brooding; the strange concentric circles are all that are left of the former ramparts. This activity prompts an investigation of constructing concentric circles and an investigation of the relationship between the dimensions of the circumference and diameter.

Pupils should be working at: Level 5

Requirements for Using Mathematics:

- Express and use formula in words and/or symbolic form.

- Describe the properties of regular and irregular 2-D shapes in terms of sides, angles, symmetry and tessellations.

Ask pupils to construct a series of concentric circles using the **repeat** command and then investigate the relationship between the circumference and the diameter. The comparison does **not** require them to use the π constant. Instead, direct their investigation so they notice that the diameter is approximately one third of the value of the circumference or, inversely, the value of the circumference is approximately three times the value of the diameter.

The activity prompts pupils to use the **repeat** command and then reposition the turtle using **penup** and **pendown** commands before drawing the smaller circles.

repeat 20[forward 15 right 18] (will draw the circle)

penup
right 90
forward 15
left 90
pendown

This section repositions the turtle to draw each of the circles.

repeat 20[forward 10 right 18] (will draw a smaller circle)
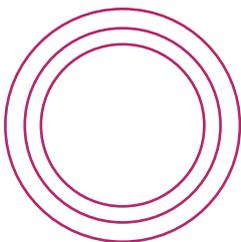
As an alternative, pupils can use the Logo command **circle**. This advice has not been included in the activity card. The list below delivers a similar design leaving the turtle in the middle.

circle 50

circle 60

circle 70

(The number after the command represents the value of the radius.)

This activity relies heavily on using tabletop shape templates, as well as practical work in constructing circles and measuring the circumference and diameter dimensions.

Ask pupils to predict the approximate dimension of the diameter of the largest circle they have drawn.

Using the information from the **repeat** command, you should direct pupils to realise that 20 times a **forward** command of 15 will give the shape a circumference of 300 units. Since the diameter is approximately one third of the circumference, the calculation of 300 divided by 3 will produce an approximate diameter of 100 units.

Activity 9
The Mermaid's Cave at Dunluce

The crumbling walls of the ancient castle at Dunluce cling stubbornly to the cliffs of North Antrim. Below the castle, hidden from view, lies the entrance to the mermaid's cave, a tall rounded opening where the mermaid used to sit and watch the waves break on the shore. Who was she and what was her story? These activities prompt some imaginative writing as well as an illustration. The Logo activity follows on from their investigation of the circles in Activity 8: Grianan of Aileach and asks them to explore the relationship between the dimensions of the circumference and diameter of a hemisphere (half circle).

Pupils should be working at: Level 5

Requirements for Using Mathematics:

- Perform simple calculations using unitary fractions.
- Describe the properties of regular and irregular 2-D shapes in terms of sides, angles, symmetry and tessellations.
- Calculate perimeters of a range of shapes.

Ask pupils to investigate the construction of a hemisphere.

If **repeat 20[forward 20 right 18]** constructs a circle, ask pupils how would they edit the procedure to construct a half circle.
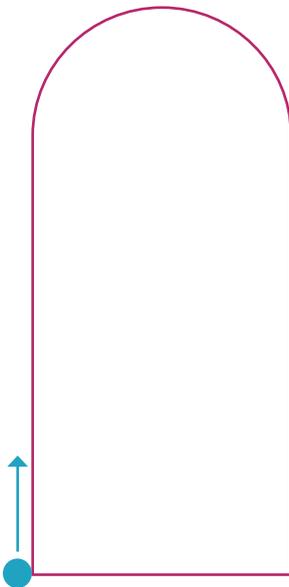
Starting at the blue dot, pupils will draw a line before using the **repeat** command to construct the hemisphere on top of the line and then repeat their forward command to reach the bottom of the arch.

(The dimensions in this activity are approximate and require a trial and improvement technique as the calculation for constructing the hemisphere will not return a completely accurate measurement.)

forward 200

repeat 10 [forward 20 right 18]

forward 200

right 90

forward (here their calculation of the diameter)

Repeating 10 × **forward 20** will move a distance of 200 across the hemisphere, so the circumference of the full circle should be 200 × 2 = 400.

The diameter is approximately one third of the circumference so

400 ÷ 3 = 133 approximately

Using the 'playing turtle' technique, pupils can investigate how far they turn when walking a hemisphere. The concept is that if using the **repeat** command to draw a circle, the 20 repeats of 18° will return a 360° rotation. Therefore, they will need 10 repeats of 18° to return the 180° necessary to draw the hemisphere.

Supplementary activities might involve asking the pupils:

- to imagine the mermaid has a particular memory of the events that took place at the castle, prompting an investigation into the local history of the area;

- to create an image of what she might have seen, prompting them to create an artistic representation of her view; and

- to create a poem of the mermaid's impressions of how she feels, prompting an exercise in imaginative empathy writing.

TIP: *Remember to encourage pupils to use the 'undo' icon from the top navigation bar as this is useful when they are trialling and investigating different commands and procedures.*

Activity 10
The Streets of Derry/Londonderry

By the end of 1689, many of the people who had lived through the seige emigrated from the city to the New World of America. These Ulster immigrants brought many new ideas with them to the settlements and one of these ideas was a revolutionary way to design towns. The city of Philadelphia has a very geometric street design with all the roads crossing at right angles. It is believed that the design originated in the street plan for the Walled City of Derry.

The Logo activity asks pupils to investigate and code a street layout to reflect this geometric design. The practical activity is an investigation on paths through the city, identifying longest, shortest and alternatives, and using the eight points of the compass to direct movement.

Pupils should be working at: Level 4–5

Requirements for Using Mathematics:

- Know the eight points of the compass.

- Understand and use the language of line, angle and location.

- Understand and use scale in the context of simple maps and drawings.

The street design inside the Walled City of Derry is laid out in a grid pattern. This was a new idea in the 1600s when the city was built. After the siege of 1689, many of the citizens left for America and took their ideas with them. One of these ideas can still be seen in the street plan of Philadelphia. Ask pupils to use Logo to show the design of the streets with a central diamond and all the streets leading from it.
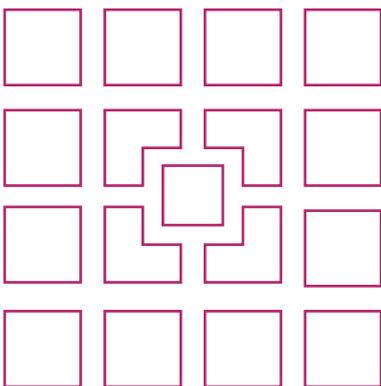
The solution to this activity requires pupils to be able to draw a series of squares laid out in a grid pattern, using the **penup** and **pendown** commands to move the turtle between the squares.

The central diamond requires a smaller square and four irregular hexagons. Pupils can draw these using a single irregular hexagon procedure for convenience, which is then rotated.

(To accommodate skill levels, if appropriate, the activity can be differentiated and the requirement to construct the central square can be omitted. Additional differentiation can also substitute the central hexagonal blocks with squares. The activity card accommodates this choice and facilitates simpler drawing by presenting all the sight lines perpendicular to each other.)

The design brief asks pupils to create a network with five latitude and five longitude avenues or streets. You can suggest the good practice of using as little code as possible and encourage pupils to use a strategy of building a square with five sides. The fifth side is used to position the turtle conveniently for drawing the subsequent square. This suggestion is incorporated in the activity card.

To square

repeat 4[forward 40 right 90] **This will create the initial block for the street.**

forward 40

end

This can be incorporated into a procedure to construct a street level structure:

To street

repeat 5[square penup forward 20 pendown] **This will draw a block of five squares vertically.**

end

The complementary activity asks pupils to use the cardinal points to find position and direct movement within the grid, for example:

- Jack has gone two blocks north from the diamond and three blocks east, can you mark where he is?

- Peter has gone three blocks east and two blocks south, can you give Eva directions to reach him by another route?

- Give Sarah the directions to get from the diamond to the block marked with an X.

Another activity asks pupils to determine the longest or shortest route and quantify alternative routes between positions. The final activity gives pupils the opportunity to compile a simple scale to provide approximate authentic distances.

Activity Cards

(Using Logo) Activity Card 1
Building Shapes (Card 1)

- In this activity, you are asked to investigate and build polygons. (**A polygon is a shape with at least three sides.**) You can use both the Bee-Bot, if you have one, and the Logo screen turtle.

- You will need four different large shape templates or shapes marked out on the floor. One person should walk around the perimeter of each shape, taking care to change direction carefully at each of the corners. Their partner should stand nearby and copy each of the turns they make without moving from their position. What did you notice about the turns in each shape?

- Can you complete this table?

Shape	Name	Number of sides	Number of angles	Size of turn
A				
B				
C				
D				

- Can you now write a sentence to describe how you might calculate the size of the turn you need to draw any polygon if you know how many sides it has?

- Can you decide what angle is needed to complete the **repeat** command to draw each of these shapes?

    repeat 9[forward 40 right ?]

    repeat 10[forward 40 right ?]

    repeat 12forward 40 right ?]

    repeat 18[forward 40 right ?]

    repeat 20[forward 40 right ?]

- As the number of **repeats** increases, what do you notice about the way your shapes are changing?

- Using the **repeat** command to draw a square, you will see that they can tessellate.

    repeat 4[forward 50 right 90]

    right 90

    repeat 4[forward 50 right 90]

    right 90

    repeat 4[forward 50 right 90]

    right 90

    repeat 4[forward 50 right 90]

    right 90
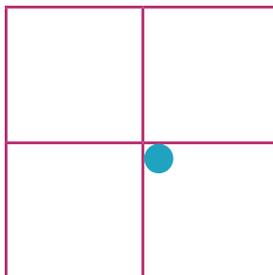
- Can you investigate and find two more shapes that will tessellate and two that will not tessellate? Use the turtle to draw them.

(Using Logo) Activity Card 1
Building Shapes (Card 2)

- How do you think that your shape will change if you reverse the order of the commands?
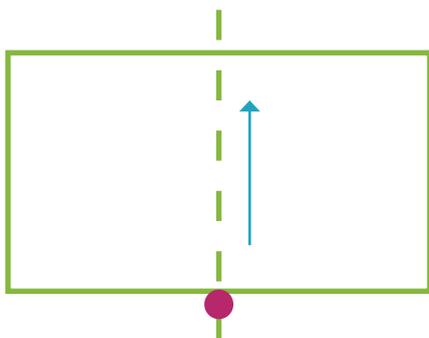 For example:

  repeat 4[forward 50 right 90]

  repeat 4[right 90 forward 50]

- Draw your prediction and then check it by drawing the shape with the turtle.

- When you draw a duplicate shape and change the direction from **right** to **left**, you will see a mirror
  drawing along a vertical reflection line. For example:

  repeat 4[forward 50 right 90]

  repeat 4[forward 50 left 90]

- Can you use the **repeat** command to draw three more polygons and their reflections?

- Could you draw them now with a horizontal reflection line? For example:
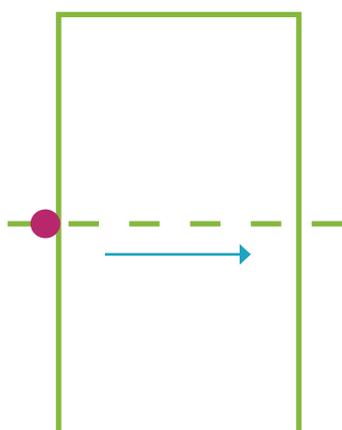
- If you have used the equilateral triangle shape, have you noticed the special shape the mirror has
  drawn? Can you name it?

- Choose your favourite shape and use it to draw an interesting design.

- This procedure is incomplete. It needs another four lines to complete it. Type in the commands below and try to decide what is happening, then add the missing command lines.

  repeat 6[forward 50 right 60]

  right 45

  repeat 6[forward 50 right 60]

  right 45

  repeat 6[forward 50 right 60]

  right 45

  repeat 6[forward 50 right 60]

  right 45

- You can try this pattern with other polygons if you like.

(Using Logo) Activity Card 2
Remembering Shapes (Card 1)

- In this activity, you have the opportunity to investigate some interesting facts about polygons, without the chore of having to type in all the commands every time you want to draw one.

- BlackCat Logo has a really neat command called **to**. This allows you to tell the computer **once** what to do and it will remember the instructions. Try typing the following commands:

To square

>repeat 4[forward 60 right 90]

>end

You just defined square

square

The **to** command tells the computer to wait while you teach it how to do something called **square**.

The computer '**listens**' as you give the **repeat** instruction.

Typing **end** tells the computer you are finished with your instructions.

The computer then tells you that it understands and your new command is '**defined**' or remembered.

Typing **square** will now draw your shape.

- Using the **to** command, can you create four new commands to draw different polygons and arrange them into an interesting design? You can plan your design first using shape template tiles.

- You can also use the new commands you created inside other commands. For example:

repeat 4[square left 90]

- This command will draw four squares, turning the turtle through a right angle to the left before each drawing.
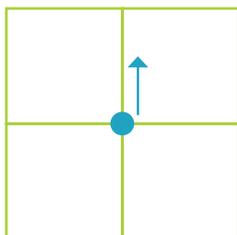
- Using the **repeat** command, can you find two other polygons that can be drawn in the same way, with their corners joining to make a full rotation?

- Can you explain why some shapes will tessellate in this way and some will not? You will need to think about the link between the inside angle and 360°.

Hint Box:

You need to remember that each corner (vertex) has two angles; the angle you turned as you drew it and the angle inside the shape.

- B is the angle inside the shape.

- A is the angle you turned as you drew the shape. (They are NOT always the same.)

- Since a full rotation is 360°, you need to think about the factors of 360.

(Using Logo) Activity Card 2
Remembering Shapes (Card 2)

- Working in pairs or small groups, how many **nets** can you find for a cube? There are 11 in total without counting any reversals.

- Could you use the **repeat** command and the new command **square** that you have taught the computer to create a drawing for as many of them as you can find?

- Choose one of the **nets** and draw it out on a piece of card. You can then cut it out, fold and glue it to make a cube. Before you finally glue it together, you could write an amazing maths fact on each of the faces.
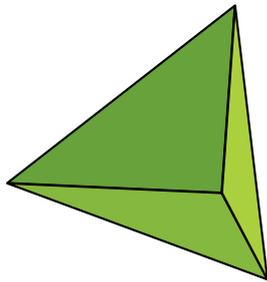

A **tetrahedron** is a solid shape too. It has four faces and each of the faces is an equilateral triangle.

What do you think the **net** of this shape will look like? Could you draw it on squared paper?

It only has two nets.

- Use the **to** command to create a command called **equitriangle**. It will have three equal sides and three equal angles. It is an equilateral triangle.

- The nets below are incomplete. They only have three equilateral triangles. Can you use Logo to copy them and add the missing triangle to complete the **net**?

- When you have finished, you can print them off and cut them out to check if you were correct.
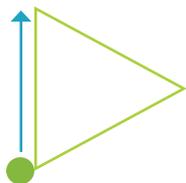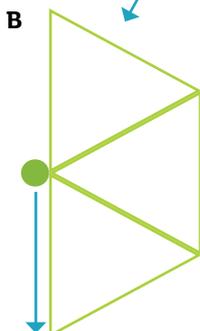

Is this how you have drawn your triangle?
Here is a command to draw it:

repeat 3[forward 50 right 120]

A
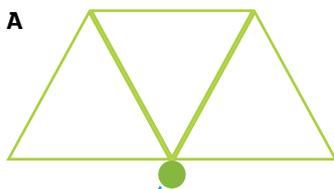
The green dot shows where the turtle is and the arrow shows the direction it is facing when it has finished.

Before you start, check the direction your turtle is facing and whether you used **left** or **right** commands to draw it.

A
right 30
repeat 3[equitriangle left 60]

B
repeat 3[equitriangle right 60]

B

(Using Logo) Activity Card 3
Enlarging Shapes (Card 1)

- In this activity, you will discover a new Logo command and have the opportunity to investigate the areas and perimeters of some regular and irregular polygons.

  (In a regular polygon, all the sides and all the angles are equal.)

- Previously, you discovered how to use the to command **to** create new commands. With this next command, you have the opportunity to change the size of the shape before you draw it.

To square :size

**>repeat 4[forward :size right 90]**

>end

square defined

square 40

Instead of using a number for the **forward** command, we are using **:size**. This is called a **variable** and means we can decide what size the shape will be later.

(The colon : is important)

We can then type in the command **square 40** or **square 60** to create a square with a side of 40 or 60.

- Create and draw two squares using the command style above. The larger square has sides of 90 and the smaller one has sides of 30. When you execute your commands you will see this shape:
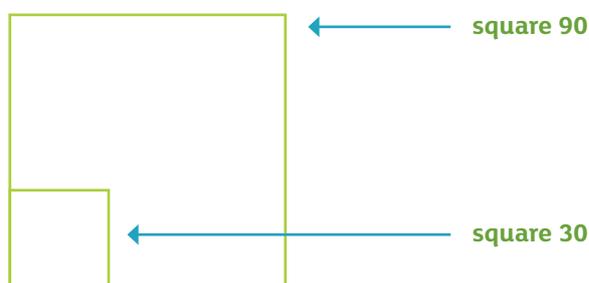
square 90

square 30

- How many of the smaller squares do you predict could fit inside the larger square? How could you calculate the number? Could you create a procedure to fill the larger square with smaller squares?
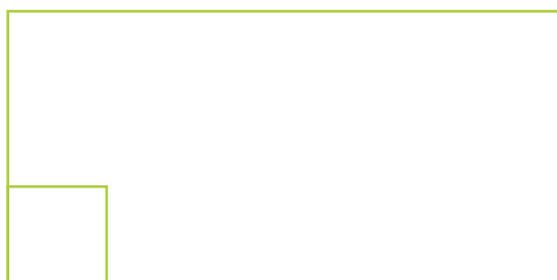
- Now draw a rectangle 240 units long and 90 units wide. How many of the smaller squares do you **predict** could fit inside it? Draw them to check if you were correct.

(Using Logo) Activity Card 3
Enlarging Shapes (Card 2)

- Using all the smaller squares you created for the previous activity, could you arrange them in another array (**arrange them into rows and columns**) to create a different quadrilateral? You can use tabletop shapes to investigate a solution before you draw it using the Logo turtle.

- If you used the turtle to draw the perimeter of a garden that was 120 units long and 60 units wide and you were asked to make a path around the outside of it with smaller squares that were 20 units wide, how many smaller square tiles do you think you might need?

- Could you draw your garden and path with the turtle to check?

Garden

- Could you create some other examples for yourself? Keep the smaller square 20 units and the sides of the rectangle as multiples of 20. Pay careful attention to each of the perimeters and the number of smaller squares that are needed to tile a path on the outside. Did you notice any pattern between them?

- Could you write out the pattern as a rule that might apply to every tiled path? For example, if you knew what the perimeter of the shape was, could you use your rule to calculate how many tiles are needed for the path?

(Using Logo) Activity Card 4
Reflections, Rotations and Spirals
(Card 1)

- In these investigations, you have the opportunity to examine shapes in a different way and also learn a new skill using Logo. You will learn about reflections and rotations and how they can change the way that you see a shape.

- On a sheet of paper, trace out a square shape and then fold your sheet along one of the sides of the square.
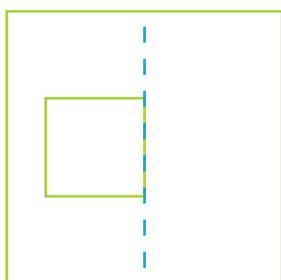
Now try to draw what you think your shape would look like if you flipped it over on that side. This is called a reflection.

How many ways do you think you could flip the shape to get a reflection? Can you draw them?

(Can you find four reflections?)

- Use the **to** command to create a procedure called **square** and draw it. Think carefully about the position and direction of the turtle when it has finished drawing your square. Can you use the turtle to draw the reflection pattern you made on your sheet of paper?

- You can draw two of the reflections without having to change the position of the turtle. You only need to change its direction. Can you draw them?

- Can you draw the other two reflections? You will need to move the turtle. One position allows you to draw both reflections together without having to change direction or move again. Can you discover where that position is and draw all the reflections?
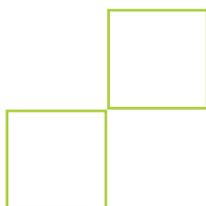
Do you think that these two shapes are reflections?

How could you prove that they are reflections? Where do you think the fold line is? Could you draw it on a sheet of paper? (Hint: It is not a vertical or a horizontal line.)

Using your procedure for **square**, how would you change the direction of the turtle to draw both of them?

- Can you use the **to** command to create two other procedures to draw different polygons and then investigate how many reflections you can discover? Could you draw them? How many can you draw only changing the direction and NOT the position of the turtle? Try to draw the reflections with the turtle changing position as little as possible.

- You might like to experiment by changing the order of the commands in your procedure or changing the directions to draw the shapes to the left or right. Think about the effect these changes might have. You can test draw them to investigate.

(Using Logo) Activity Card 4
Reflections, Rotations and Spirals
(Card 2)

A

B

C
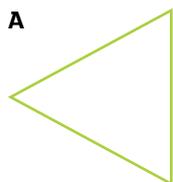
Use the **repeat** command to draw the triangle at A.

Can you draw the shape and its reflection at B? How would you edit the **repeat** command to draw both?

Using the same procedure, can you draw the reflection at C? Think carefully about where you think the starting position and direction of the turtle are.

How are the triangles at A and C similar? Do you think they are drawn in the same way, using the same commands?

Do you think you could draw these reflections?

This list of commands will start the drawing:

right 90

forward 70

left 120

forward 140

right 120

3 × lines missing here...

There are three lines missing at the end of the list. Can you draw the shape and add the missing lines?

The commands for drawing this shape are:

forward 80	**forward 40**
right 90	**right 90**
forward 40	**forward 40**
right 90	**right 90**
forward 40	**forward 80**
left 90	**right 90**
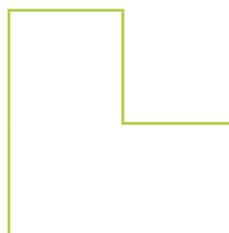
Remember, it may be easier to change direction inside the procedure to get the shape you want. Reverse all the directions to investigate how the shape will change.

Could you use the **to** command to create a procedure and then investigate how many reflections you can discover and draw?

(Using Logo) Activity Card 4
Reflections, Rotations and Spirals
## (Card 3)

- In this activity, you have the opportunity to investigate the effect of rotating shapes and the patterns that can be created.

Can you use the **to** and **repeat** commands to create a procedure to draw a regular pentagon and call it penta?

In this pattern, three pentagons have been drawn and the turtle has been turned 20° to the right after each time one is drawn.

How many times could it be turned 20° to the right before you come back to your start position? Could you try this to see the pattern you have created?
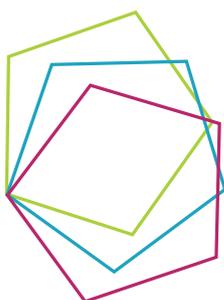
If you turned the turtle 60° to the right or left, how many times could you draw the pentagon before you arrive back at the start?

If you knew how many times you wanted to draw your shape, how could you calculate the size of the turning angle?

You can investigate creating designs using some of the shapes you can now draw.

If you use the command **setpencolour**, you can change the colour of the pen. You must add a number between 0–16 after the command. You can choose from 15 colours.

(See Card 4 for the list of colours.)

- For this activity you will need four dominoes. Take any two of them and try to arrange them so that they make a square. Do you remember how you would recognise a square? How could you prove that your arrangement is a square? Could you write a short sentence explaining your reason why you think it is a square?

- Do you remember the shape that the dominoes are? Could you write a Logo procedure to draw two of the dominoes together, making the square?

- Now try to arrange the four dominoes to make a square and explain why you think your arrangement is square shaped.

- Each of the dominoes has dots on them. Could you investigate the domino dots and choose four dominoes, so that when you have your square assembled all the dots on each side add up to four dots?

- Can you use a Logo procedure to draw what you have assembled?

repeat 2[forward 80 right 90 forward 40 right 90]

- This **repeat** command will draw a rectangle for you. You need to think carefully how to position and turn the turtle.

(Using Logo) Activity Card 4
Reflections, Rotations and Spirals (Card 4)

- In this activity, you have the opportunity to draw and investigate spiral shapes.

- When drawing a spiral, you have to make each side either a little larger or a little smaller than the side before it.

- The procedure below will draw a spiral for you.

forward 100

right 90

forward 80

right 90

forward 60

right 90

forward 40

right 90

forward 20

- Could you try to draw a spiral with the sides increasing each time?

- If you wanted to draw a simple reflection of your spiral, which of your commands do you think you would need to change?

- Could you try to draw a spiral and then add its reflection?

- When you have finished, you could add these lines to your procedure:

penup

home

right 90

forward 10

left 90

pendown

The first list of code will place your turtle beside your starting position. You can then ask your partner to try to follow your path into the centre of the spiral without touching any of your lines.

or

penup

right 90

forward 10

left 90

pendown

The second list of code will position the turtle in the centre of your spiral. You can then ask your partner to try to escape from the spiral without touching any of the lines you have already drawn.

|							
Colour codes for setpencolour							
1	Black	6	Grey	11	Yellow
2	Red	7	Magenta	12	Dark Green
3	Green	8	Orange	13	Light Grey
4	Blue	9	Pink	14	Brown
5	Cyan	10	White	15	Purple

(Extending Logo) Activity Card 5
Newgrange (Card 1)

Newgrange is a settlement site in Ireland where people lived thousands of years ago. There are many strange symbols and shapes carved on the ancient stones. The meanings of these carvings have been lost in the mists of time. Each year at the Winter Solstice, the dawn sun shines right through the ancient doorway of the site, lighting up the interior of the chamber; proof that our ancestors were able to read the star maps in the sky and could predict the time of the shortest day.

- The first activity will spin you into the heart of the mystery at Newgrange.

- This code takes you to the heart of the magical site. Type the code if you dare.

<div>

forward 200   **right 90**

right 90   **forward 100**

forward 180   **right 90**

right 90   **forward 80**

forward 160   **right 90**

right 90   **forward 60**

forward 140   **right 90**

**right 90**   **forward 40**

forward 120

</div>
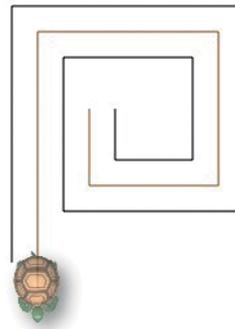
- You need to write code that will take you out of the centre of the spiral safely without touching any of the sides, otherwise you will be trapped forever.

- You could print off the spiral and use a pencil and paper to plan your escape. How many turns do you think you will need?

- Think about how you will need to change the position and direction of the turtle before you begin. You can use **penup** and **pendown** to make sure you leave no trace as you move. Check Activity Card 4: Reflections, Rotations and Spirals to remind yourself of the code needed to reposition the turtle before you attempt to plan your escape.

- As you spiral into the heart of the shape, you are travelling through the past and can see three events that have affected your ancestors. Can you write a brief description?

- When you begin to swirl out, the reverse happens and you see three events that will affect your future. What do you think they will be? Can you write a brief description?

- You can **screen dump** your escape spiral into a Word document and then use **text boxes** and perhaps illustrations to describe the events your imagination has seen.

(Extending Logo) Activity Card 11
Newgrange (Card 2)

The site at Newgrange is very rich with many stones that have strange carvings on them. Our ancestors 5,000 years ago spent a lot of time creating the designs. There are many ideas about the meanings of the designs; however, we are not really sure what they mean. It is reasonable, however, to think that the designs were meant to say something about the world our ancestors lived in.

- In this activity, you are asked to use your Logo skills to create your own design that must last for a thousand years. The design should say something about our world today. You must use only vertical and horizontal lines or lines that are perpendicular to each other (see example below).

- You can use what you have learned about the **to** command to create simple shapes and then use these to create a more intricate pattern.

- To avoid the same misunderstanding at Newgrange, can you write a brief explanation of the meaning behind your design and what it is attempting to say about our world and us today?
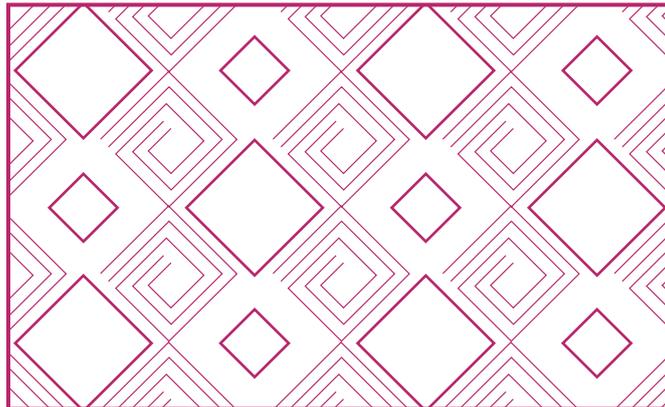
(Extending Logo) Activity Card 6
Finn McCool and the Giant's Causeway (Card 1)

Finn and his wife Nuala have been working in their garden, when they learn that the Scottish giant Benandonner is anxious to visit them. They know that he is very competitive and always tries to do better than Finn at everything. 'Oh dear!' says Finn, 'What a nuisance.'

Nuala advises Finn to go down to the coast and see what Benandonner wants. When Finn reaches the coast and talks to Benandonner they decide to build a causeway. Finn begins on the Antrim side and Benandonner shouts advice to him about where to place each block. Benandonner likes to give advice and Finn hates having anyone tell him what to do. They use regular hexagonal blocks and both are cross and tired when they have finished.

- Use the **to** command to create a procedure to draw a small regular hexagon with each side 60 units long.

- Use these commands to move your turtle to the bottom left of your screen:

  penup

  backward 175

  left 90

  forward 400

  pendown

- Now draw your first hexagon. Look carefully at the position and direction of your turtle when you have finished. Can you draw another hexagon beside it so that the sides and not the vertices touch? You will have to think carefully where to move the turtle to draw your next shape.

- Can you plan and draw a line of hexagons from there to the top right hand corner to make a causeway and allow Finn and Benandonner to meet? It will look like this:
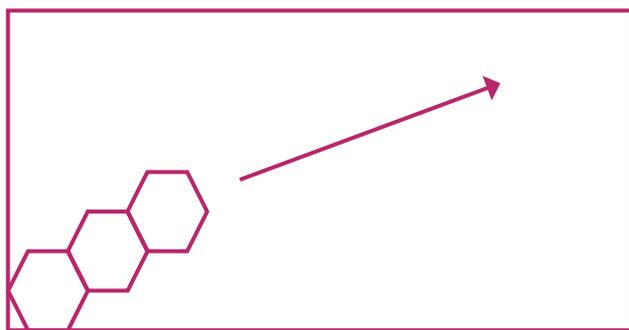

- Look carefully at the commands you used to reposition the turtle. Could you include them in the **repeat** command? Think about how many times you direct the turtle to move along the side of the shape? What effect would building separate hexagons have (using right or left angles)?

(Extending Logo) Activity Card 6
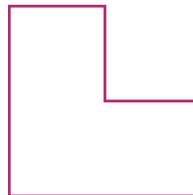# Finn McCool and the Giant's Causeway (Card 2)

They eventually meet face to face when Finn has built the causeway and they are both cross and tired. Benandonner decides to challenge Finn with a Logo puzzle and then Finn replies with his own puzzle. Can you help both of them to solve the puzzles?

- Benandonner says that each stone has six sides; so four stones will have a total of 24 sides. He asks Finn to arrange four blocks in a way that only 14 of the sides are visible. For example, in the pattern below only 10 of the 12 sides are visible. Can you help Finn to create the pattern and then use Logo to draw it?
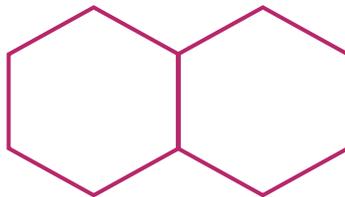
- When Finn has successfully completed the puzzle, he then gives it to Benandonner. He gives him six hexagonal blocks and asks him to arrange them into a pattern that will show **seven** hexagons when you look at it. You will need to investigate different patterns using hexagon tiles to solve this puzzle. Look carefully at your patterns for the '**hidden**' hexagon. Could you use the turtle to draw your design when you have finished?

- Nuala has been trying to cook the dinner and look after the baby and is fed up listening to the two of them squabbling, so she decides to give them a puzzle herself, telling them that if they continue to argue they will not be fed. They will need your help to solve the puzzle.

- She gives them six identical irregular hexagon blocks. She asks them to use two of them to make a rectangle and the other four to make another larger irregular hexagon of the same shape as the blocks she has given them.

- This list of commands will draw the irregular hexagon for you. Use the **to** command to create your irregular hexagonal block.

|	
forward 40	**forward 40**
left 90	**left 90**
forward 40	**forward 80**
right 90	**left 90**
forward 40	**forward 80**
left 90	**left 90**

- You will need to think carefully and decide the direction and position of the turtle after it has drawn each of the shapes before you position it to draw the next shape.

(Extending Logo) Activity Card 6
Finn McCool and the Giant's Causeway (Card 3)

Finn and Benandonner scratch their heads and grumble that Nuala's puzzle is too difficult. They complain that the shapes won't fit together. Nuala overhears them and tells them not to behave like babies and suggests that they should try to turn the shapes in as many ways as possible to see if that will help them.

- Benandonner mishears Nuala and thinks that she said 'Turn the shape *over*', so they look at the shape and turn it over, first to the left and then to the right. Can you draw how the shape looked when it was turned to the right and to the left? Draw three shapes; the original shape in the middle and then what it looked like turned to the left and then turned to the right on either side of the original.

- Could you use Logo to draw the three shapes? How could you change the angles in the shapes to make the shapes easier to draw? Would you begin each drawing from the same position?
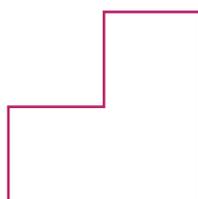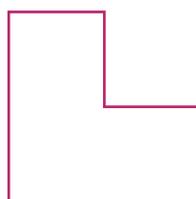

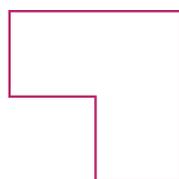**Left turned**　　　　　　**Original**　　　　　　**Right turned**

- Finn understands his wife much better than Benandonner and thinks that she simply said '*turn* the shape', so they try that and look carefully at how the view of the shape changes as they turn it. They discover that if they turn it four times then the shape returns to its original position.

- Can you draw how you think the shape changed? There is one missing from the line below. Use your shape templates to discover what it is and then use Logo to draw all four of them
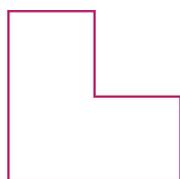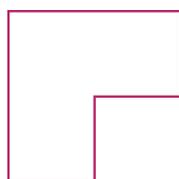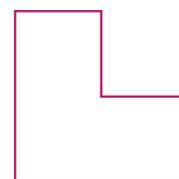

Original　　**After 1 turn**　　**After 2 turns**　　**After 3 turns**　　**Original**

- Finn and Benandonner decide to record all the *different* ways the shape will look if it is **turned** or **turned over**. How many separate ways can you find? Can you draw them and then use Logo to draw them too?

(Extending Logo) Activity Card 6
## Finn McCool and the Giant's Causeway (Card 4)

- Use what you have learned from the previous card to help you to decide if you need to **turn** or **turn over** any of your shapes to solve Nuala's puzzle.

- You will also need to think and look carefully at the shapes and try to decide how your view of the shape might change if you **turn** it or **turn** it **over**. How might the shape look if you changed the right angles to left angles and vice versa?

Can you help Finn and Benandonner to arrange the two hexagons to make a rectangle?

Decide where the turtle is and its directions after it draws the first shape and think carefully where you will place the turtle to draw the second shape.
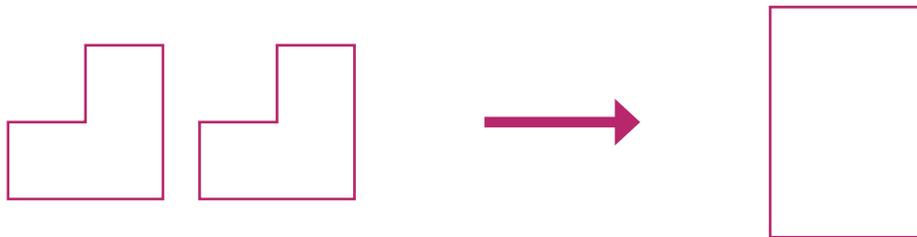
- The second puzzle asks you to use all four remaining hexagons to make a larger hexagon. You will need to plan where each hexagon is placed before you begin to use Logo to draw it.
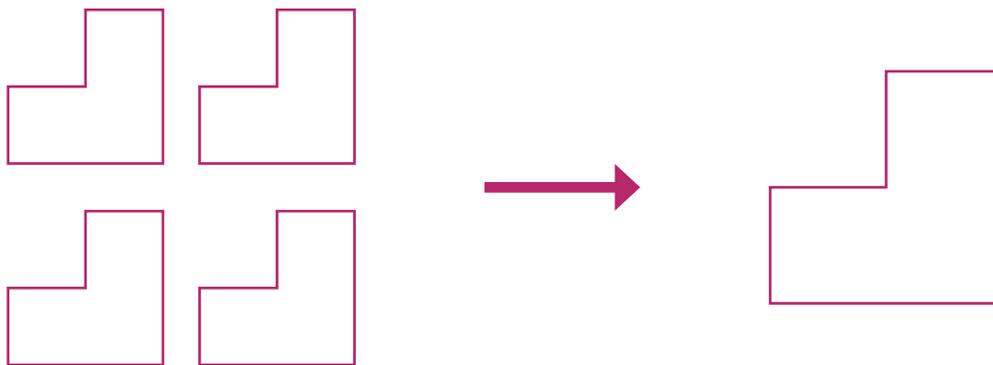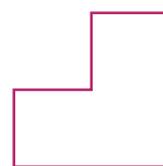

- Remember, some of the shapes might need to be **turned over**. Try building a second shape with all the directions reversed. Now you will have a shape and its reflection. Think about **turning** the shape as well, for example:

| **Shape** | **Shape turned** | **Shape turned over** |

(Extending Logo) Activity Card 7
Giza (Card 1)

The Great Pyramid at Giza is one of the Wonders of the Ancient World. The Egyptians, who built it, were skilled not only with using shapes but also using number. The construction took many years to complete and since each pyramid is built from the bottom up, the builders had to be able to calculate how wide to make the base so that each layer would rise gradually to a point at the top.

- The first activity gives you the opportunity to build your own pyramid. Use the **repeat** command below to draw your rectangle brick shape. Now use the **to** command to create a procedure called **brick**. This will be your building block.

  repeat 2[forward 30 right 90 forward 60 right 90]

- Can you use your **brick** to create this small pyramid? Think carefully about where your turtle is each time it stops and the direction it is facing. What commands will you need to use to change its position before drawing the next **brick**?

- Can you calculate how many **bricks** are needed for your pyramid? If you decided to build a pyramid that had five layers, how many bricks would you need? Could you use Logo to build a pyramid with five layers?

- Could you complete this table for the builders to show how many bricks will be needed in total? You can use Lego or tiles to build them to test your results.

| Number of pyramid layers	Total number of bricks
1	
2	
3	
4	
5	
6	
7	
8	

- Can you see any patterns in the number of total number of **bricks** needed as each layer is added? Use a sentence to explain your pattern.

- John has 46 **bricks** and Aoibh has 54 **bricks**. Does this mean that she can build a higher pyramid than John? Can you explain your answer?
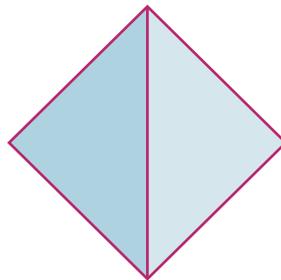
(Extending Logo) Activity Card 7
Giza (Card 2)

The architect who built the Great Pyramid had to design a model first to show the Pharaoh what the structure would look like.

- Could you use your Logo skills to help him to draw the **net** of a pyramid? What shapes do you think you will need and how many of them will be required to complete the **net**?

- You will need to research and investigate the shape of the base and the shapes needed for the faces.

- Sketch your idea on a sheet of paper first and then use the **to** command to create the procedures for the shapes you will need.

- Test draw your shapes and when you are satisfied you can plan how to move the turtle and make any changes to its direction to create the **net**.

- Draw your **net** on a sheet of paper first. How many **different** shapes will you need? **(Choose carefully... you do not need as many as you think.)**

- Below is an incomplete **net** to give you an idea.
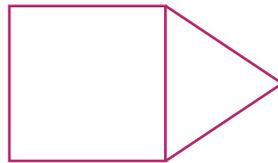
- The architect also wanted to create a 3-D drawing of the shape but he was unsure how to manage it. Fortunately, it is quite simple to achieve this effect using Logo.

- You need to draw two equilateral triangles and join them on a vertical line. Print your design and then shade the sections, making sure that one section is shaded a little lighter than the other.

- Use Logo to draw two triangles like those in the example. Use the **repeat** command and draw a triangle using left turns and then draw the same triangle using right turns before you print them and shade them to show a 3-D effect.

(Extending Logo) Activity Card 8
Grianan of Aileach

The fort at Grianan Aileach was built in the distant past. Dedicated to the sun god, it sits silent on a hill overlooking the River Foyle in Donegal. One of the legends from the site tells of a band of ancient warriors asleep inside the hill. All that remains of the fort today are a circular inner stone wall and the track of some of the earth ramparts that surrounded it.

- In this activity, you have the opportunity to investigate circles, how to construct them and the special relationship between the diameter and the circumference in every circle.

- For the first section of the activity, you will need four or five different sized circle templates, shapes or tiles. You will have to measure their diameters and circumferences. If you measure these to the nearest cm, that will be quite accurate enough.

- Draw a table with two columns. In one column, write the diameter and beside this write the circumference for the same circle. Did you notice any relationship between them?

   (Hint: If you multiply each of the diameters by the same small number, you will get an answer very close to their circumference.)

- Now test your results with some more circles. Can you predict the size of the circumference approximately if you know the diameter?

- Can you use Logo to draw three circles, one inside the other (**concentric**) like the diagram below?
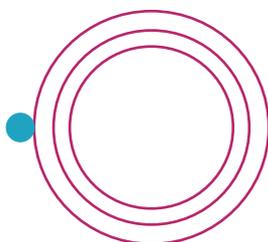
The is the starting position for the turtle and the commands to draw the outer circle are:

repeat 20[forward 15 right 18]

How do you think the commands will change to draw the two smaller circles?

You will need to create another short list of commands to change the position of the turtle to draw each of the circles, otherwise you will leave a track. Remember to use **penup** and **pendown**.

- The stone wall of the fort has a doorway and on each Winter Solstice day the sun shines directly through it and reaches the wall on the other side. Could you use Logo to draw the path of the sun's light?

- Using what you know about diameters and circumferences and the information from the **repeat** command you used to draw the largest circle, could you calculate an approximate distance for the diameter of the circle and then draw it?

(Extending Logo) Activity Card 9
The Mermaid's Cave at Dunluce

The crumbling walls of Dunluce Castle hide a secret. Beneath the cliffs under the castle, there once lived a mermaid. The sea washed her cave as she spun her magic to protect all who lived above her. The entrance to the cave was a mighty curved arch and when she looked through it she was able to see not only the Antrim coast but also every kingdom and time that ever existed.

- Can you use Logo to draw the magic entrance to her cave?

You will need to draw a half circle (hemisphere) for the top of the shape.

Think carefully about the **repeat** command that you use to draw full circle. How would you change it to draw a half circle?

Try out your ideas before you draw the entrance.

It is easier to draw the side first, then the arch and then the other side before drawing the bottom side.

Have you noticed that this bottom side is also the same length as the diameter of the hemisphere? How would you calculate it?

Do you remember from Activity Card 8: Grianan of Aileach that the diameter is always approximately one third of the length of the circumference? (Of course you can also say that the circumference is always three times the value of the diameter.)

So, if instead of drawing the hemisphere (half circle) you had drawn a complete circle, what would its circumference have been?

If you now know the value of the circumference, how could you calculate the approximate length of the diameter?

This will give you the length of the bottom side of the arched entrance.

- When you have drawn the arched entrance, you can print off several copies and stick them side by side on a large sheet of paper to create a panel. In each of the entrances, can you use your imagination to draw what you think the mermaid saw from her time as the guardian of Dunluce?

- Do you think that you could write a description or a poem to describe how she felt at the time of her most exciting view from the arched entrance to her cave?

(Extending Logo) Activity Card 10
The Streets of Derry/Londonderry (Card 1)

The Walled City of Derry in the year 1690 had a very new and interesting street design. All the streets were perpendicular to each other and there was a square or diamond in the middle with the four main streets leading off from this square at right angles. It was very unusual for a city design. After the siege of 1689, many of the people who lived in the city decided to leave and move to the new colony of America. They took many of their ideas with them and one of the ideas was their new idea for designing towns. The city of Philadelphia is one of the American cities whose street design has followed the street design of the Walled City of Derry from 1690.

- Adam Crockett helped to design the street layout in Derry and now has been asked to plan of the streets of Philadelphia. Can you use your Logo skills to help him with drawing the shapes? First, draw your design on a sheet of paper and plan the best starting point and direction for the turtle.
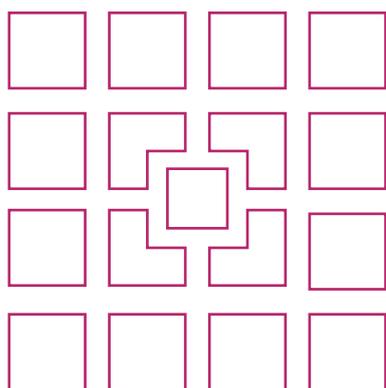
The shapes in this design are familiar to you. However, you will have to think of a way to draw them that will leave spaces between them for the streets. You can use the **penup** and **pendown** commands for this.

Would the middle be a good place to begin?

Is there a **repeat** pattern you could use to create the streets?

Think about how you included an extra side when drawing some of the hexagons previously to position the turtle to draw the next shape.

- In this design, there are three streets running vertically and three running horizontally. However, because there is much more space in Philadelphia, Adam decides to create a plan with five streets running in each direction. Can you help? (*If you think it necessary, then you can omit the centre square and perhaps also change the four central hexagons to squares too.*)

- In the design, there are only two shapes, a square and a hexagon. Do you remember how to use the **to** and the **repeat** commands to draw these?

(Extending Logo) Activity Card 10
The Streets of Derry/Londonderry (Card 2)

- Adam explains that his street plan will allow people to use the **cardinal directions** to give clear instructions. For example, the bakery is two blocks north and one block east of the diamond (*centre*).

- Decide the seven essential buildings and businesses that you think the city should have, then decide which blocks they should occupy. Make sure you distribute them evenly across the city. You can print out your plan and draw them into it by hand. To add character to your shops, you can include a brief description of the owner and an illustration as well. (***You could also take a screen dump of your Logo page and then paste and enlarge it in a Word document for better effect***.)

- Using the **cardinal directions**, can you give the directions to each of the premises starting from the diamond? Can you create what you think a tourist direction pamphlet might have looked like in 1690? Can you also include the directions from one shop to the other to prevent people having to return to the diamond each time?

- Could you create a circuit so that a tourist could visit each business by the shortest route?

- To make your map more useful for a tourist, decide what you think might be the actual dimensions (*length and width*) of the building blocks and then create a suitable scale that they could use to help them calculate the actual distances on your map.
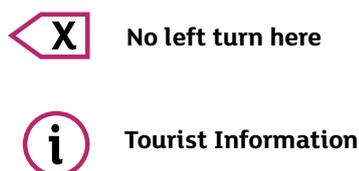
If the length of your block is 40 Logo steps and you think that 4 m might be the actual width of the shop, then you might suggest a scale of:

10 Logo steps represents 1 m or 1 m represents 10 Logo steps.

You could also use a scale line to help them. The dimension of the line would have to be Logo steps and the comment you use would be the actual dimension, for example:

1m

Another interesting feature you might like to add is a key. If you used symbols on your map then the key will help everyone understand what they mean, for example:

X   **No left turn here**

i   **Tourist Information**

Appendix

Appendix
Glossary of Logo Commands

Command	Meaning
forward	This command must be followed by a variable, for example **forward 50** will move the turtle 50 steps forwards in the direction it is facing. The command can be abbreviated to **fd**.
backward	This command must be followed by a variable, for example **backward 50** will move the turtle 50 steps backwards from the direction it is facing. It can be abbreviated to **bk**.
right	This command must be followed by a variable. The variable indicates the size of the turn to the right. The default direction is always facing up the screen, for example **right 90** will turn the turtle 90 degrees in a clockwise direction. The command can be abbreviated to **rt**.
left	This command must be followed by a variable. The variable indicates the size of the turn to the left. The default direction is always facing up the screen, for example **left 90** will turn the turtle 90 degrees in an anticlockwise direction. The command can be abbreviated to **lt**.
cs	This command clears the screen and returns the turtle to the **home** position. The home position is in the centre of the screen with the turtle facing upwards.
home	This command returns the turtle to the **home** position. It will not leave a trail but will leave any existing lines already drawn intact.
penup	This command deactivates the drawing function on the turtle. It allows you to move the turtle without leaving a line drawn to its new position.
pendown	This command reactivates the drawing function, allowing the turtle to resume drawing a trace of where it is going.
to	This command creates a new procedure. The command is followed by a word, which is then recognised by the turtle as a new command. The turtle is deactivated as the commands are entered and reactivated when the instruction **end** is entered and the new command is defined.

Appendix
Glossary of Logo Commands

Command	Syntax
to square :size	This variation of the **to** command allows you to draw squares of different sizes efficiently. The syntax requires using the colon : and a space between the word **square** and the colon. Using the **repeat** command constructs a square template. **repeat 4[forward :size right 90]** This will create a square with a variable side and the following commands can then be used. **square 40** will draw a square of side 40 units. **square 60** will draw a square of side 60 units etc.
setpencolour	This command changes the colour of the pen. It is followed by a number between 0–16 to activate a specific colour.
circle	This command draws a circle and leaves the turtle placed on the centre of the circle. The command is followed by a number that determines the value of the radius. The command **circle 40** will draw a circle with a diameter of 80.

Rewarding Learning